Fourth Quiz, February 26, 2014

1. A matrix, B, is a 3 by 5 matrix. How would you write the last element in the last row of B?

Since **B** has 3 rows and five columns the last element in the last row is b_{35}.

2. For the two matrices below, find the following three components of the product FE: (a) first row first column, (b) last row, first column, and (c) last row, last column

$$E = \begin{bmatrix} 3 & 5 & -6 & 10 \\ 12 & 7 & -9 & 4 \end{bmatrix} \qquad F = \begin{bmatrix} 4 & 7 \\ -2 & 9 \\ 6 & 0 \end{bmatrix}$$

The product matrix will have 3 rows (like **F**) and four columns (like **E**). The last element in the first row of the product, p_{14} = (4) (10) + (7)(4) = 68. The first element in the last row, p_{31} = (6)(3) + (0)(12) = 18; the last element in the last column, p_{34} = (6)(10) + (0)(4) = 60.

3. For the E and F matrices in problem 2, which of the following matrix products are possible? EF, E^TF, F^TE, EF^T E^TF^T, F^TE^T.

For a matrix product to be possible the matrix on the left must have the same number of columns as the number of rows in the matrix on the right. Applying this principle to the proposed matrix products gives the following conclusions:

E has 4 columns and **F** has 3 rows so **EF** is **not** possible.

$\mathbf{E}^T$ has 2 columns and **F** has 3 rows so $\mathbf{E}^T\mathbf{F}$ is **not** possible.

$\mathbf{F}^T$ has 3 columns and **E** has 2 rows so $\mathbf{F}^T\mathbf{E}$ is **not** possible.

E has 4 columns and $\mathbf{F}^T$ has 2 rows so $\mathbf{EF}^T$ is **not** possible.

$\mathbf{E}^T$ has 2 columns and $\mathbf{F}^T$ has 2 rows so $\mathbf{E}^T\mathbf{F}^T$ **is possible**.

$\mathbf{F}^T$ has 3 columns and $\mathbf{E}^T$ has 4 rows so $\mathbf{F}^T\mathbf{E}^T$ is **not** possible.

4. Verify that the equation for A^{-1} shown at the right is correct.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{A}^{-1} = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Do the matrix product ignoring the factor, $1/(a_{11}a_{22} + a_{21}a_{12})$, outside the matrix. The first product, $p_{11} = a_{11}a_{22} + a_{12}(-a_{21})$ which is one when multiplied by the factor. The second product in the first row is $p_{11} = a_{11}(-a_{12}) + a_{12}a_{11} = 0$. The first product in the second row is $p_{21} = a_{21}a_{22} + a_{22}(-a_{21})$ = 0. The final product in the second row is $p_{22} = a_{21}(-a_{12}) + a_{22}(a_{11})$. When this is multiplied by the factor in front of the inverse, it becomes one. So the result of the $\mathbf{AA}^{-1}$ product is a 2x2 unit matrix, confirming the correctness of the expression for the inverse.

5. Apply Gauss elimination to the system of equations shown below. Find a unique solution if it exists; if not determine if there is no solution, or an infinite solution. If there is an infinite solution, find equations for x and y in terms of z.

2x + 3y + z = 13
3x + 7y – 5z = 18
-x – 9y + 19z = –2

Rearrange the equations so that the third equation multiplied by -1 becomes the first equation and it is subtracted from the other equations to eliminate the x coefficient.

x + 9y – 19z = 2
-15y + 39z = 9
-20y + 52z = 12

Subtracting -20/(-15) times the third equation from the second equation gives all zeros for the third equation. Thus we have an infinite solution. For an arbitrary value of z the second equation gives y = (52z-12)/20.

Substituting this into the first equation gives x = 2 + 19z – 9(52z – 12)/20 = (74 – 44z)/10.