

### Solution to Third Quiz, February 19, 2014

Use the false position method to find a root of the formula  $f(x) = x \ln(x) - \cos(x)$  using initial guesses  $x_0 = 1$  and  $x_1 = 3$ . In the false position method, we maintain two guesses  $x_+$  and  $x_-$  that bracket the root. We find the new guess from the formula below and use  $x_{new}$  to replace  $x_+$  if  $f(x_{new}) > 0$  or use it to replace  $x_-$  if  $f(x_{new}) < 0$ .

$$x_{new} = x_- - f(x_-) \frac{x_+ - x_-}{f(x_+) - f(x_-)}$$

Verify that the initial guesses given bracket the root. Then, get new iterations for the root until the relative difference between two successive calculations of  $x_{new}$  is less than 0.001.

For this problem  $f(x_0) = f(1) = 1 \ln(1) - \cos(1) = -0.5403$  and  $f(x_1) = 3 \ln(3) - \cos(3) = 4.2858$ . Since  $f(1) < 0$ , we define  $x_- = 1$ ; similarly we set  $x_+ = 3$  because  $f(3) > 0$ . Having  $f$  values of different signs shows that the initial guesses bracket the root. The first iteration for the false position method gives

$$x_{new} = x_- - f(x_-) \frac{x_+ - x_-}{f(x_+) - f(x_-)} = 1 - (-0.5403) \frac{3 - 1}{4.2858 - (-0.5403)} = 1.2239$$

We find  $f(1.2239) = -0.9269$  so this value of  $x_{new}$  replaces the value of  $x_-$  and  $x_+$  does not change. The next iteration is solved as follows.

$$x_{new} = x_- - f(x_-) \frac{x_+ - x_-}{f(x_+) - f(x_-)} = 1.2239 - (-0.9269) \frac{3 - 1.2239}{4.2858 - (-0.9269)} = 1.2615$$

At this point the relative error is  $\left| \frac{1.2615 - 1.2239}{1.2615} \right| = 0.0298 > 0.001$ , so iterations continue. Because  $f(1.2615) = -0.01133$ , we replace  $x_-$  with  $x_{new}$  and continue the iterations.

$$x_{new} = x_- - f(x_-) \frac{x_+ - x_-}{f(x_+) - f(x_-)} = 1.2615 - (-0.01133) \frac{3 - 1.2615}{4.2858 - (-0.01133)} = 1.2661$$

At this point the relative error is  $\left| \frac{1.2661 - 1.2615}{1.2661} \right| = 0.00362 > 0.001$ , so iterations continue. Because  $f(1.2661) = -0.01130$ , we replace  $x_-$  with  $x_{new}$  and continue the iterations.

$$x_{new} = x_- - f(x_-) \frac{x_+ - x_-}{f(x_+) - f(x_-)} = 1.2661 - (-0.01130) \frac{3 - 1.2661}{4.2858 - (-0.01130)} = 1.2666$$

At this point the relative error is  $\left| \frac{1.2666 - 1.2661}{1.2666} \right| = 0.000416$ ; because this relative error is less than our desired relative error of 0.001, we have the solution we were looking for:  **$x = 1.2666$** .