

Programming Assignment Four – Simultaneous Linear Equations Due Monday, March 17 at 11:59 pm¹

1. Getting started

Click on the link to “Workbook for the fourth programming assignment” on the course home page to download the workbook guassianInputs.xlsm from the course web site. This workbook has the data for a matrix solution problem $\mathbf{Ax} = \mathbf{b}$. The data for the \mathbf{A} matrix and the solution \mathbf{x} were generated randomly. The \mathbf{b} data were then calculated as $\mathbf{b} = \mathbf{Ax}$. The workbook contains not only the \mathbf{A} and \mathbf{b} data usually used for solving the matrix problem, but it also contains the correct solutions, \mathbf{x} . Most of the tasks in this assignment will be based on the data in this workbook. The data are for 100 equations in 100 unknowns. There are 12 sets of \mathbf{b} vectors for which you are asked to find twelve solutions that can be compared to the exact \mathbf{x} solutions. The workbook uses the named ranges A, b, and x, respectively for the matrices \mathbf{A} , \mathbf{b} , and $\mathbf{x}_{\text{exact}}$. (The names b and x refer to all 12 sets of \mathbf{b} and \mathbf{x} vectors; they're are both 100x12 matrices.)

2. Using MATLAB

Copy the commands and answers from the MATLAB command window to a Word file. Use semicolons to suppress all output except for the final results. If you want to see the intermediate results, you can use commands without semicolons, but you should only copy the final results requested below to the Word file.

- Import the workbook data for \mathbf{A} , \mathbf{b} , and \mathbf{x} into MATLAB.² Use following two MATLAB commands to solve for all the \mathbf{x} solutions: (i) use the inv function to compute the inverse and (ii) use the \ operator. Find the RMS error, defined as $\{\sum(x_{\text{correct},k} - x_{\text{MATLAB},k})^2/N\}^{1/2}$, in each of the twelve solutions for both approaches (i and ii). Copy all your commands, but only the twelve RMS error results for each approach to the Word file.
- Determine \mathbf{A}^{-1} and the product of \mathbf{AA}^{-1} . Determine a single RMS error for this product. Note that you can use the MATLAB command eye(N) to construct an NxN unit matrix. Copy all your commands and the final RMS error result into the word file.
- Use the MATLAB command det to determine Det(\mathbf{A}).
- Review MATLAB help, course notes or section 3.6 in the Rao text on condition numbers.
 - Use the cond function of MATLAB to determine the condition number of the \mathbf{A} matrix.
 - Compute your own condition number using the definition of the condition number as $\|\mathbf{A}\| \|\mathbf{A}^{-1}\|$; use each of the four matrix norms available in MATLAB.

¹ Late submissions with a 30% penalty will be accepted up to 11:59 pm Wednesday, March 19. Submissions after this date will be review to help students improve their performance, but will receive a zero grade.

² Different versions of MATLAB on CSUN CECS computers have different import results. On some computers, you will be given the choice of entering one of the named regions into MATLAB. These named regions are copied into a variable called data. Use a MATLAB command like “A = data;” to copy the data read from the worksheet into the named variable you want to use. On other MATLAB installations you will be prompted to enter data from a worksheet which will be stored in the default variable name untitled. Use a command like A = untitled; in this case.

- iii. Define a new matrix which is a submatrix that consists of the first three rows and the first three columns of the **A** matrix. What is the condition number of this submatrix according to the cond function?
- e. Enter the following arrays into MATLAB $A = [1 \ -2 \ 3 \ -5; \ 4 \ 8 \ 6 \ -10; \ -3 \ 2 \ -7 \ 8; \ 6 \ 0 \ 13 \ -18]$ and $b = [-14; \ -2; \ 12; \ -27]$. Determine the number of solutions (one, infinite, or none) for these matrices in the problem $\mathbf{AX} = \mathbf{b}$.
- f. Use the pinv function of MATLAB to obtain a solution to the set of equations from part e, and use the rref function to obtain the reduced row-echelon form for $[\mathbf{A} \ \mathbf{b}]$. Use this reduced row-echelon form to get an algebraic solution for x_1 , x_2 , and x_3 in terms of any arbitrary value for x_4 . Show that the equations that you find here are consistent with the solution you obtained using the pinv function.

3. Using Excel

Use the downloaded Excel workbook for the assignments below. Do all your work in one workbook. Use separate worksheets for the various assignments. (Rename the workbook with your name and the assignment number (e.g. <yourLastName>_4.xlsm.)

- a. Use the data in the workbook and the Excel minverse and mmult functions to find the solutions for \mathbf{x} . For each \mathbf{x} find the root mean square error (RMS) in the each solution defined as $\{[\sum (X_{\text{Correct},i} - X_{\text{Excel},i})^2]/N\}^{1/2}$.
- b. The gaussianInputs.xlsm workbook that you downloaded contains VBA code for an array function that solves $\mathbf{Ax} = \mathbf{b}$ for any number of equations and any number of \mathbf{b} vectors. To use this array function: (1) select the rectangular area for your answers (rows = equations, columns = number of \mathbf{b} vectors), (2) enter the formula =Gaussian(<A array>, <b array>), and (3) press control+shift+enter. This function uses type double and pivoting. In this part of the assignment, you will see the effects of the choice of single or double data types and pivoting or not pivoting on the accuracy of the assignment. Obtain the solutions outlined below.
 - i. Obtain the solutions using the code as written (type double with pivoting). Determine the RMS error in these solutions.
 - ii. Modify the code to obtain solutions with type single instead of type double. Determine the RMS error in these solutions.
 - iii. Modify the code to obtain solutions with type single without pivoting. Determine the RMS error in these solutions.
 - iv. Modify the code to obtain solutions with type double without pivoting. Determine the RMS error in these solutions.

The workbook contains three named ranges: A is the name of the 100x100 matrix **A**; b is the name of the 100x12 matrix, **b**, that contains 12 right-hand-side vectors; x is the name of the 100x12 matrix that contains the 12 exact-solution vectors, **x**. To use the existing code (with type double and pivoting) select the correct number of cells (100 rows by 12 columns) for your solution matrix, **x**, enter the array formula "=Gaussian(A,b)", and press control+shift+enter. Then write Excel cell formulas to determine the RMS error in each of the twelve solutions. This completes task i above.

In doing tasks ii to iv, use a separate worksheet for each set of results and RMS error calculations. In addition, you should place the modified code for each task in a separate module and give the main function, Gaussian, a different name that reflects the task.³ For example, use GaussianSNP for the code that uses the single data type and no pivoting. Note that you have to change several statements in the function where the function name is set to a value (Gaussian = ...) to match the new function name.

³ The functions used in the original Gaussian routine have the heading of Private functions, meaning that they can only be called by other functions in the same module. This means that you do not have to worry about changing the names of all the functions in your new modules, only the name of the function that you are calling from the worksheet.

- c. Copy the original (double with pivoting) code into another module, modify it to compute the determinant of any input matrix, and use this³ code to compute $\text{Det}(\mathbf{A})$. For this part you will replace the back-substitution sub with one which computes the determinant from the matrix in upper triangular form. In order to do this, you have to modify the pivoting code to count the number of times the pivot row is swapped. The determinant is the product of all the diagonal terms in the upper-triangular matrix multiplied by $(-1)^{n\text{Swaps}}$, where $n\text{Swaps}$ is the number of times the pivot row is exchanged with another row.

4. Submission requirements

Submit two files: (1) the Word file with the discussion and the MATLAB commands and output and (2) the Excel workbook with all the results from part 3. (Both files should have the file names <yourLastName>_4 with the appropriate suffix, docx or xlsx.) Your discussion should address the following topics:

- a. The different matrix norms you found in part 2.d.ii. Are any of them the same as the one you found in part 2.d.i? Discuss the difference between the condition number you found for \mathbf{A} in part 2.d.i and the condition number that you found for the 3x3 submatrix of \mathbf{A} in part 2.d.iii. What conclusion can you draw?
- b. The equivalence of the two solutions (using the pinv and rref functions of MATLAB) in part 2.f.
- c. The three values that you found for the determinant using: (1) MATLAB, (2) the Excel function **mdeterm**, and (3) your determinant function from part 3.c.
- d. The differences in RMS error that you find for the six following solutions to $\mathbf{Ax} = \mathbf{b}$: (1) MATLAB, (2) Excel worksheet functions, minverse and mmult, and (3-6) the four VBA codes.