Larger than Life
it's so nonlinear

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Larger than Life (LtL) is a four-parameter family of two-state cellular automaton (CA) rules that generalize John Conway's celebrated Game of Life as follows:

At each time t each site $x \in \mathbb{Z}^d$ is either *live* or *dead*. The neighborhood of the origin is $\mathcal{N}$, a finite subset of the *d-dimensional* integer lattice $\mathbb{Z}^d$; the translate $x + \mathcal{N}$ is the neighborhood of x. If a dead site sees between β_1 and β_2 live sites in its neighborhood at time t, it becomes live at time $t+1$; otherwise it remains dead. If a live site sees between δ_1 and δ_2 live sites (including itself) in its neighborhood at time t, it remains live at time $t+1$; otherwise it becomes dead.

In most of this dissertation two-dimensional LtL rules with range ρ *box neighborhoods* are considered. The rules are defined on $\mathbb{Z}^2$ with $\mathcal{N} = \{y \in \mathbb{Z}^2 : ||y||_\infty \leq \rho\}$ $(\rho \in \mathbb{N})$. In this framework, Life is a range-one LtL rule with parameters

$$(\beta_1, \beta_2, \delta_1, \delta_2) = (3, 3, 3, 4).$$

Two approaches are taken — one from the perspective of probability theory and particle systems. Interest is in the ergodic behavior of the infinite system started from a random initial configuration. Such limiting behaviors as almost sure convergence to all dead or all live sites are proved. If ξ_t denotes the state of the system at time t started from a random initial configuration, then ξ_t can be thought of as a Markov process since the sites update independently from all preceding times except the current one. The Markov process is degenerate since the transitions are deterministic. Nevertheless, it has a compact state space, so there exists a measure, μ that is invariant under the rule. Upper bounds, sometimes sharp, are obtained for such measures.

The other approach is to think of the time evolutions of finite initial states as iterated maps. Various finite configurations such as *replicators* and large-range analogues to Life's *gliders* are presented. Replicators are configurations of live sites that make copies of themselves so that their space-time diagrams are isomorphic to Pascal's Triangle Mod 2 in an

appropriate dimension. Replicator theory is developed for *d-dimensional* CA rules and applications to the infinite systems are also described.