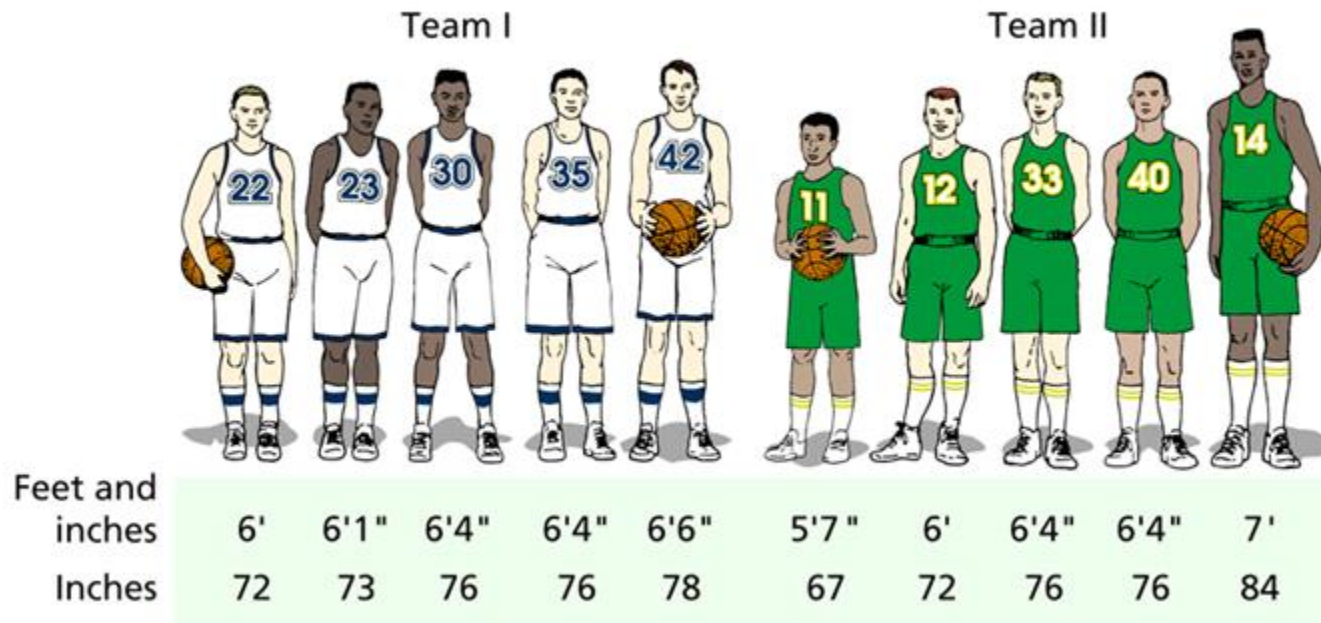# Measures of Variability

Statistics has been called the study of variability.

According to the California Common Core State Standards, what measures of variability must secondary school students learn?

Interquartile range

6th grade, Algebra I, Mathematics I, Statistics and Probability

Mean absolute deviation

6th grade, 7th grade

Standard deviation

Algebra I, Mathematics I, Algebra II, Mathematics III,
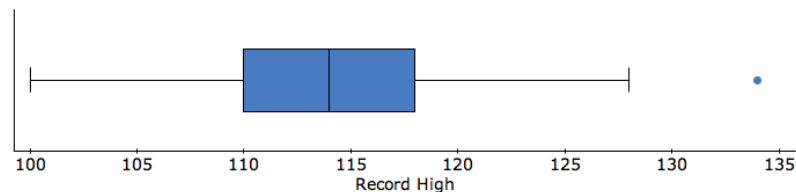Statistics and Probability, AP Statistics

Variance

AP Statistics

Why Not the Range?

1. It's not very descriptive. These distributions have the same range, but very different variability:

   (Some students think the distribution on the left has larger variability. Why?)

2. The range is too sensitive to outliers.

3. No one actually uses the range.

   People give the minimum and maximum, saying,

   The values range from 13 to 23.

   They almost never say,

   The range is 10.

Ann E. Watkins Cal State Northridge September 2014

Interquartile Range

The **interquartile range** (**IQR**) is a measure of the variability of the values in a distribution. It is the difference between the third quartile and the first quartile:
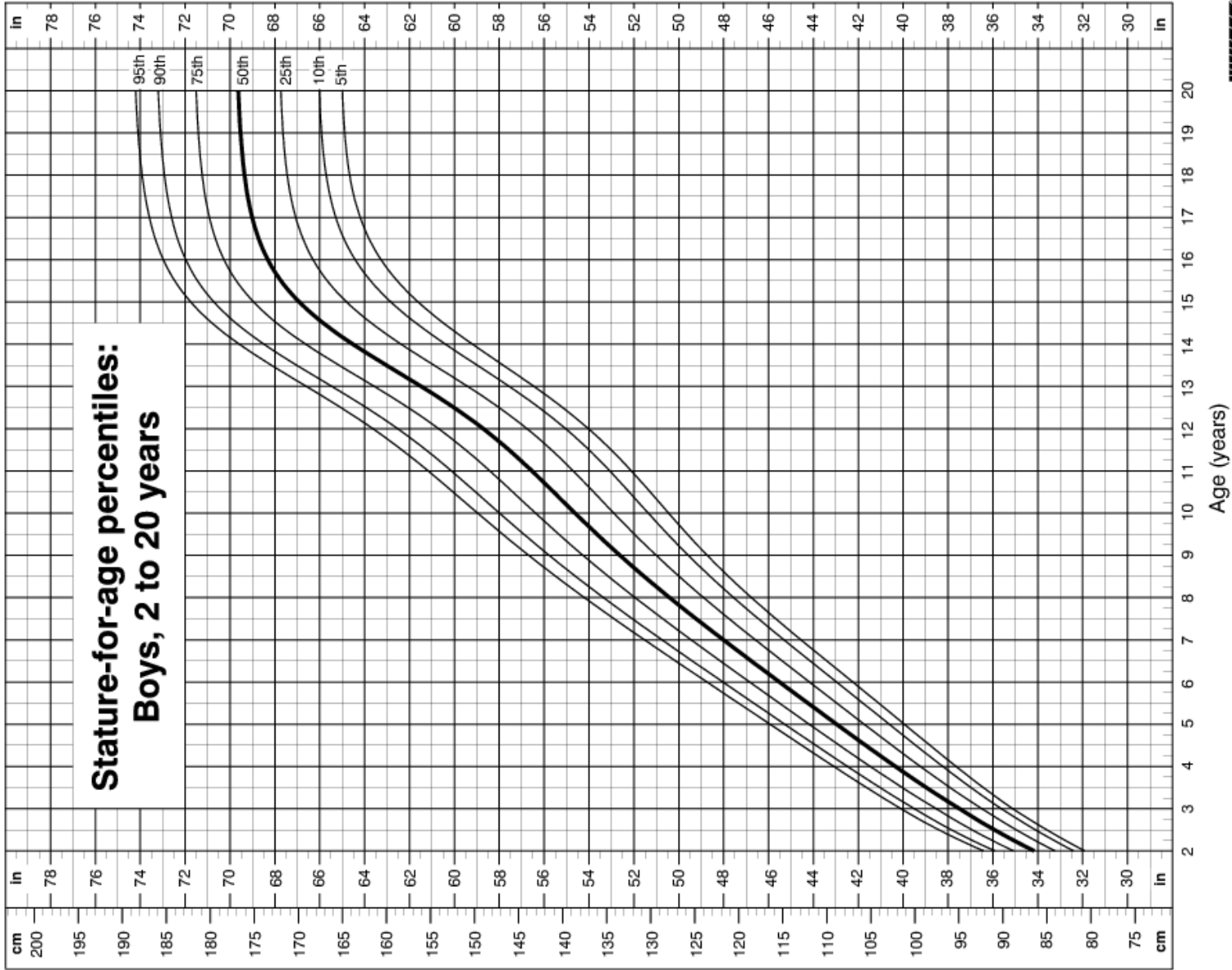
$$IQR = Q_3 - Q_1$$

Example The heights of children the same age are quite variable. CDC growth charts contain an amazing amount of information about that variability.

➢ What is the approximate percentile for a 9-year-old girl who is 49 in tall?

➢ About how tall does a 12-year-old girl have to be so that she is as tall or taller than 75% of the girls her age? How tall does a 12-year-old boy have to be?

➢ How can you tell from the chart when children are growing the fastest?

➢ At what age is the rate of increase in height the greatest for girls? For boys?

➢ Is there more variability in the heights of 2-year-old girls or 14-year-old girls?

➢ Is there more variability in the heights of 20-year-old girls or 20-year-old boys?

➢ Estimate the 25th percentile of height for 4-year-old boys. The 50th percentile. The 75th percentile.

➢ Estimate the *IQR* of heights for 4-year-old boys.

➢ Write a sentence, in context, interpreting the *IQR* for 4-year-old boys.
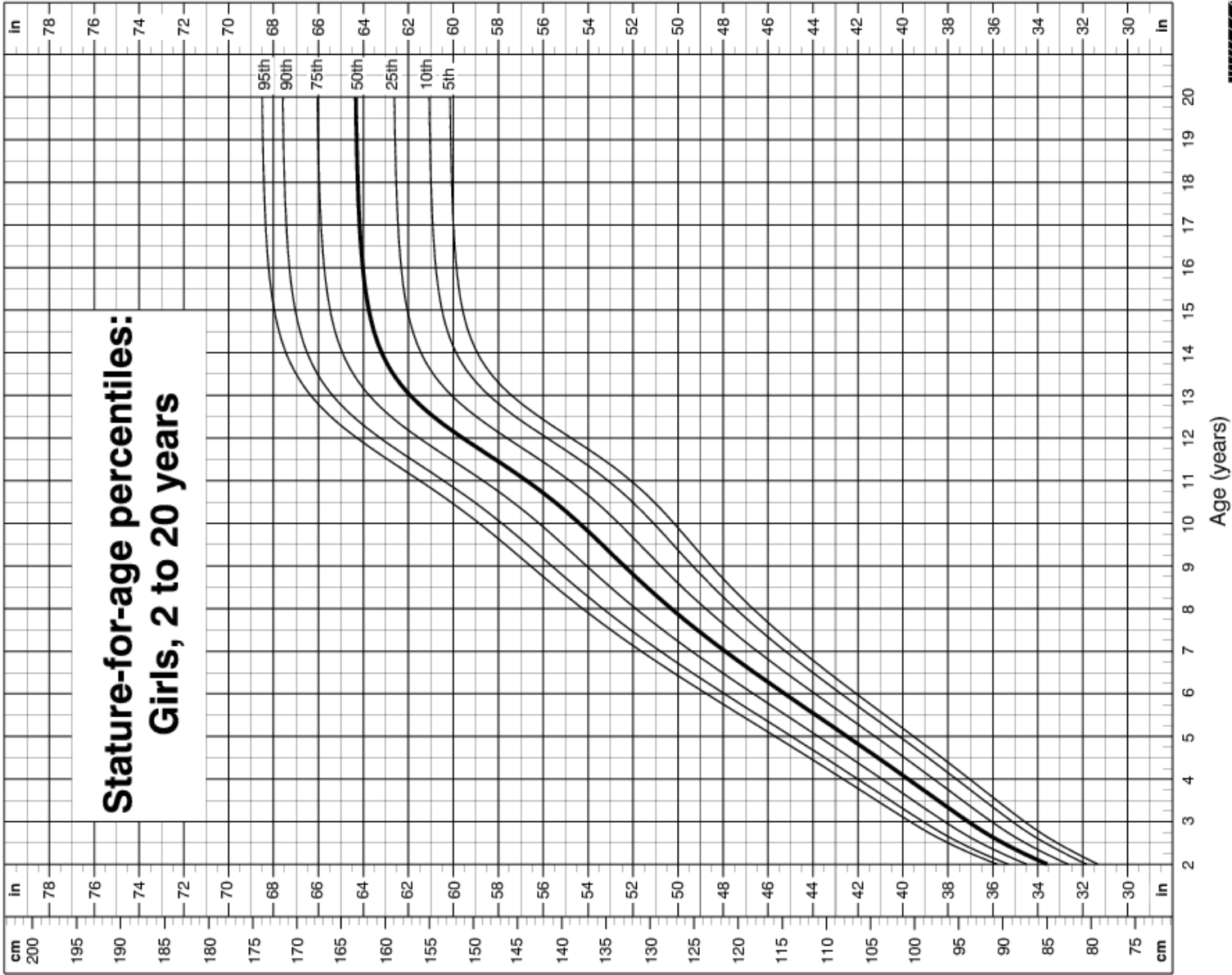
CDC Growth Charts: United States

Stature-for-age percentiles:
Boys, 2 to 20 years

Age (years)

95th
90th
75th
50th
25th
10th
5th

CDC

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CDC Growth Charts: United States

Stature-for-age percentiles: Girls, 2 to 20 years

Age (years)

95th
90th
75th
50th
25th
10th
5th

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The IQR and the Boxplot (Grade 6, Algebra 1, Math I, S&P)

Example Two psychologists randomly divided 139 students into three groups. Group 1 learned the names of the others using the "name game," where the first student states their full name, the second student states their full name and that of the first student and so on. Group 2 learned the names using the name game with the addition that each student stated their favorite activity. In Group 3, each student learned the names by going around and introducing him or herself to each of the other students. One year later the students were sent photos of the others in the group and asked to give the names of as many other students as he or she could remember. The variable recorded for each student was the percentage of names he or she recalled. Here are the results.

➢ Which method would you choose as most effective?

➢ Which group has the most variability in the percentages recalled?

➢ The *IQR* is equal to the length of the _____

Remembering People's Names
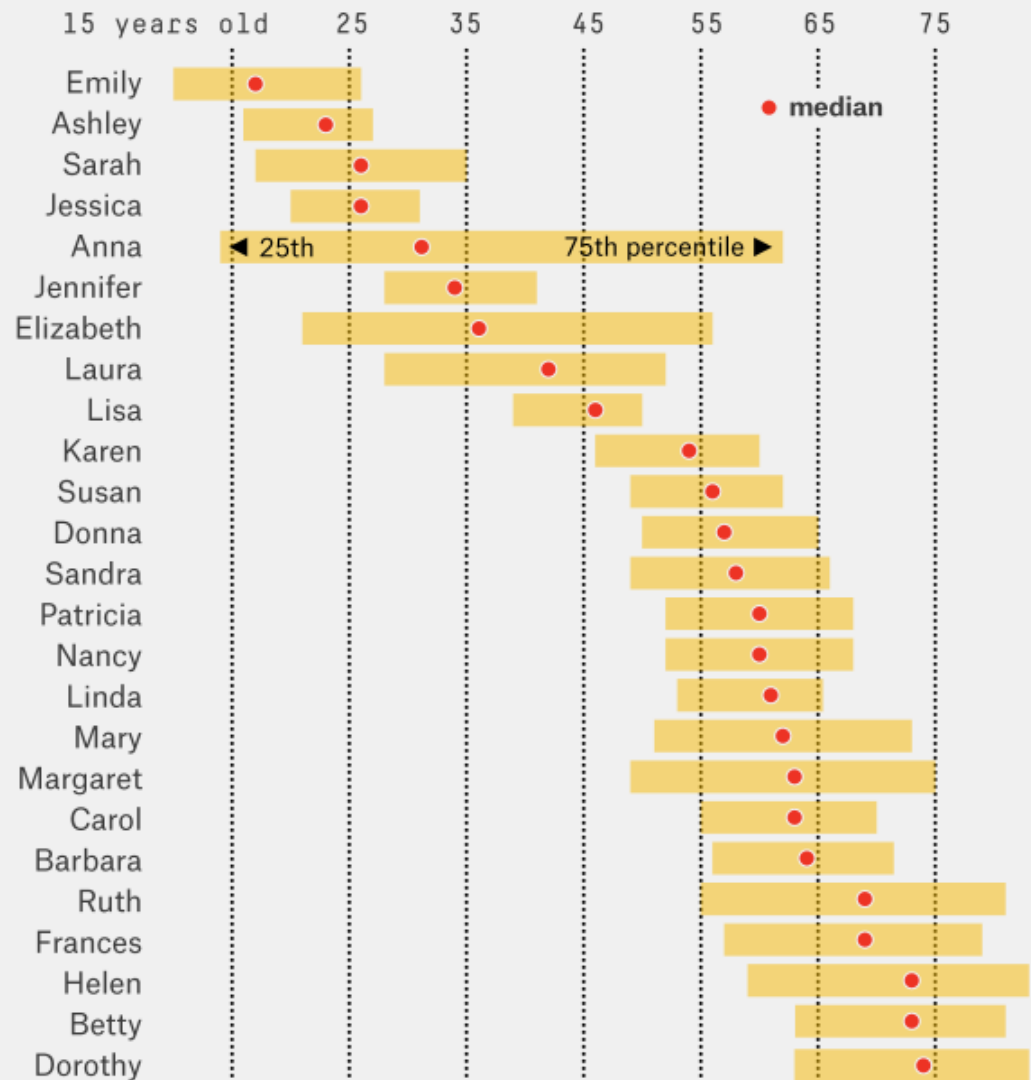
Box Plot

Group

PercentRecalled
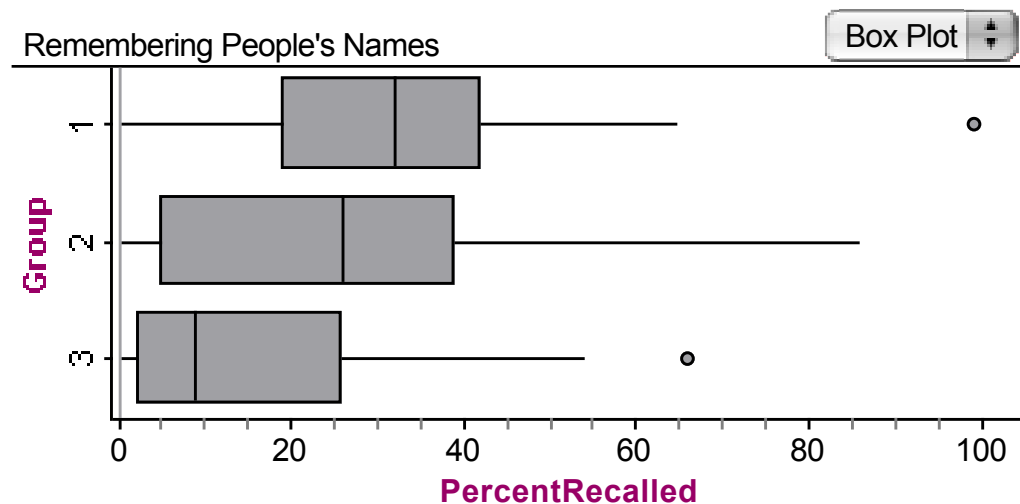
Example Boxplots are especially useful when you have many groups to compare, each with many cases. For example, the graphs of the ages of females and males with various first names on the next two pages contain 25 boxplots (without whiskers), each of which may summarize the ages of millions of people. fivethirtyeight.com/features/how-to-tell-someones-age-when-all-you-know-is-her-name/ using Social Security Administration records

➢ Why do you think the whiskers weren't included?

➢ The *median* for a given name is the age at which half of the females are younger and half are older. Estimate the median age of the Emilys. ___

➢ The endpoints of the boxes are at the 25th and 75th percentiles. The 25th percentile for a given name is the age at which 25% of the females are younger and 75% are older. Estimate the 25th percentile for the Emilys. _____ Estimate the 75th percentile. _____

➢ Use the 75th percentile in a sentence about the Donnas.

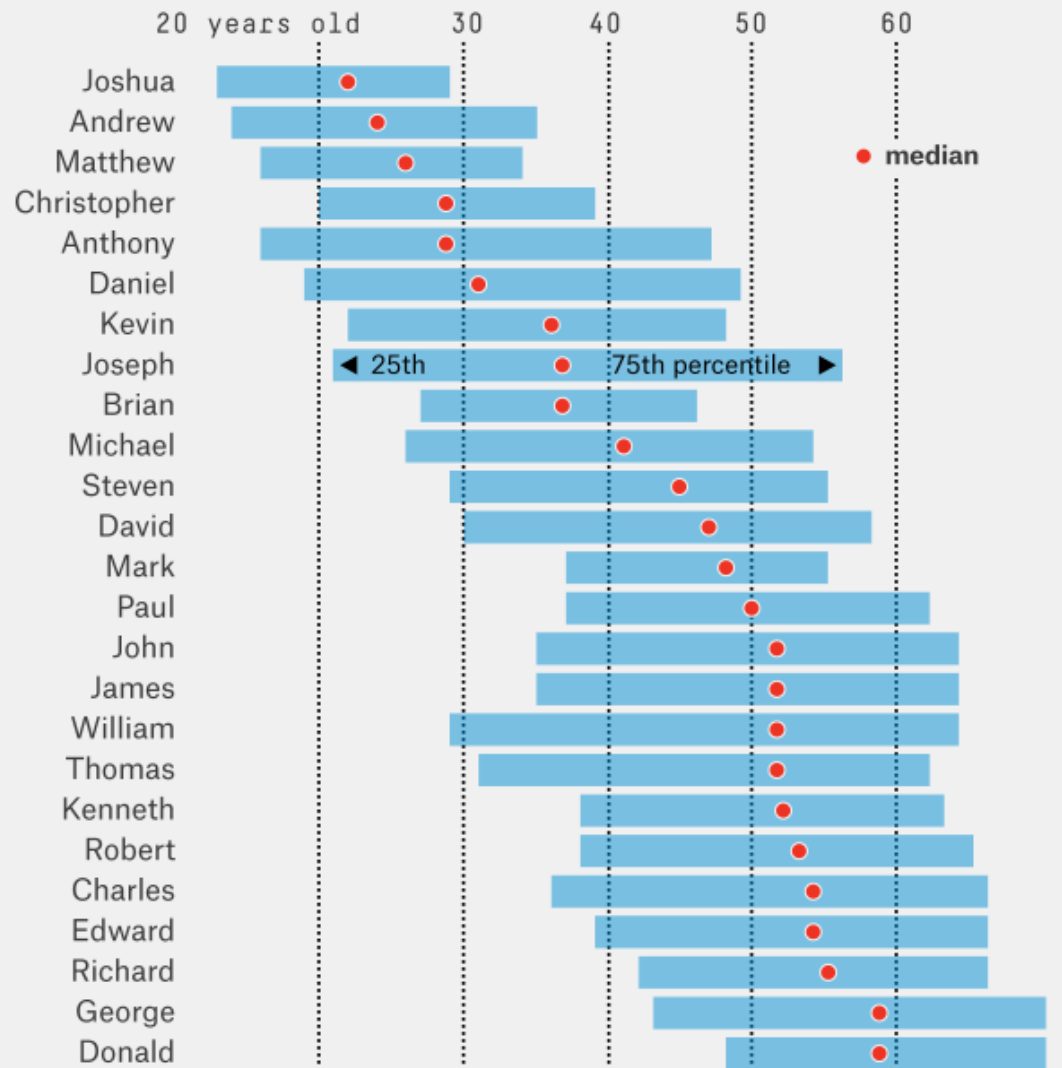➢ For which name is the box the longest? _____ What does this tell you about that name?

Median Ages For Females With the 25 Most Common Names
Among Americans estimated to be alive as of Jan. 1, 2014

Median Ages For Males With the 25 Most Common Names

Among Americans estimated to be alive as of Jan. 1, 2014

SOURCE: SOCIAL SECURITY ADMINISTRATION

Group Activity   The boxplots on the next page show how efficient boxplots can be in displaying a *lot* of information. The researcher who made them explains, "A few years ago I downloaded speed dating data from experiments conducted by Raymond Fisman, et al. (2005), which represents about 8,000 dates by 551 people. On each date, people scored each other on attractiveness, intelligence, ambition, and some other things, along with a *yes* or a *no* to seeing the other person again on a regular date." theoldreader.com/profile/603ece92bbd932a4a9805cdf

For example, the right-most box in the set of boxplots on Attractiveness, represents all of the people who got a median rating of 9 on attractiveness from the other people. The vertical axis gives the proportion of the other people who said "yes" to dating him or her. The outlier is a person who got a median attractiveness rating of 9 from other people, yet only about 0.2, or 20%, said they would like to date him or her.

A. In general, were people who were rated higher on these categories more desirable as a date than those who rated lower?

B. Is there a category that contains an exception to your answer to part A?

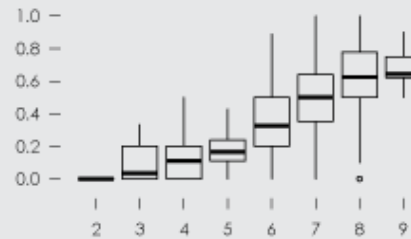**The higher the better**

Generally speaking, people wanted to date others who had higher ratings across all attributes.

Attractiveness

Yes rate from others (1.0 to 0.0)

Median rating from others (2 to 9)
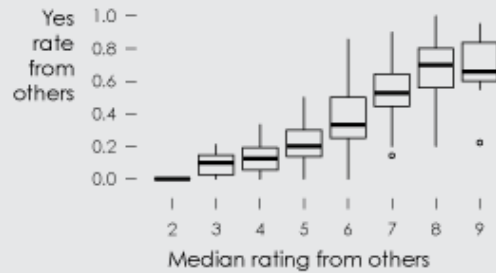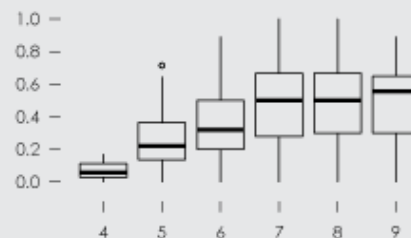
Fun
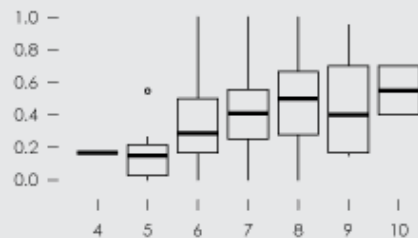
Intelligence

Ambition

Sincerity

C.   What seems to be the worst category or categories in which to have a low rating?

D.   List three things that you find interesting about these data.

STATISTICS IN ACTION 2.2 ▶ Comparing Hand Spans: How Far Are You from the Mean?

There are various ways you can measure the spread of a distribution around its mean. This activity gives you a chance to create a measure of your own.

What You'll Need: a ruler

1. Spread your hand on a ruler and measure your hand span (the distance from the tip of your thumb to the tip of your little finger when you spread your fingers) to the nearest half centimeter.

2. Find the mean hand span for your group.
3. Make a dot plot of the results for your group. Write names or initials above the dots to identify the cases. Mark the mean with a wedge (▲) below the number line.
4. Give two sources of variability in the measurements. That is, give two reasons why all the measurements aren't the same.
5. How far is your hand span from the mean hand span of your group? How far from the mean are the hand spans of the others in your group?
6. Make a plot of differences from the mean. Again label the dots with names or initials. What is the mean of these differences? Tell how to get the second plot from the first without computing any differences.
7. Using the idea of differences from the mean, invent at least two measures that give a "typical" distance from the mean.
8. Compare your measures with those of the other groups in your class. Discuss the advantages and disadvantages of each group's method.

Note: On #6, if a hand span is below the mean, the difference will be negative.

Mean Absolute Deviation (*MAD*)

One way to think about the variability in data is to examine how far the data values are from their mean.

Group Activity   Four Brownie Girl Scout troops each have nine girls. The number of boxes each girl sold on Saturday are given below. (Idea for activity from Jennifer Kaplan and Robert Floden.)

Troop A: 2, 3, 4, 5, 6, 7, 8, 9, 10
Troop B: 2, 2, 2, 6, 6, 8, 9, 9, 10
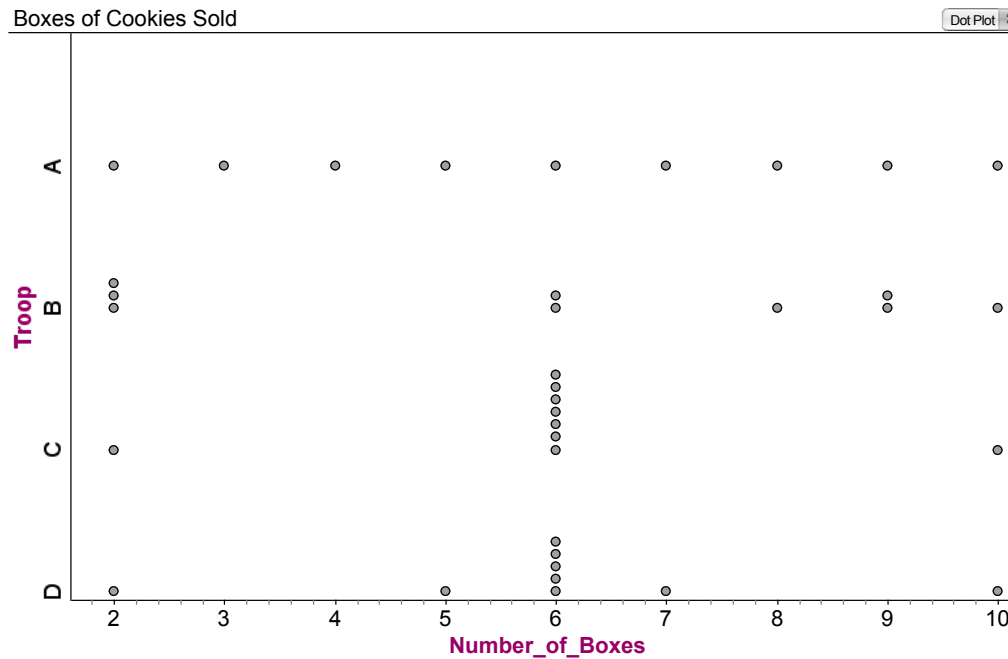Troop C: 2, 6, 6, 6, 6, 6, 6, 6, 10
Troop D: 2, 5, 6, 6, 6, 6, 6, 7, 10

A. Verify that the mean number of boxes sold by the girls in each troop is 6.

B. Which troop is closer to having an equal distribution? Rank the groups from least to most equally distributed. Explain your reasoning with the help of the dot plots and/or with computations.

Boxes of Cookies Sold

Number_of_Boxes

C. Several students used the following method to measure how far each Brownie troop was from an equal distribution. Fill in the table for Troop C.

Troop A

Number of Boxes Sold	Distance from Mean of 6
2	4
3	3
4	2
5	1
6	0
7	1
8	2
9	3
10	4
Total	20

Troop C

Number of Boxes Sold	Distance from Mean of 6

D. Is the distribution of Troop A or Troop C more variable according to the method devised by these students?

E. The students' method works out well because the troops are all the same size. Now suppose that Troop E has 40 girls, 20 of whom sold 5 boxes and 20 of whom sold 7 boxes. What is the mean for Troop E?

F. Using the students' method, what is the sum of the distances from the mean for Troop E?

G. Does it seem sensible to say that the sales from Troop E are more variable than from Troop A?

To account for different group sizes, the sum of the distances from the mean is divided by the group size, resulting in the mean absolute deviation.

Mean Absolute Deviation

The mean absolute deviation (*MAD*) is a measure of the variability of *n* values from their mean:

$$MAD = \frac{\sum |x - \bar{x}|}{n}$$

$x - \bar{x}$ is called a *deviation from the mean.*

➢ Compute and then compare the *MAD* for Troop A and Troop E.

Examine the formula for the *mean absolute deviation.*

➢ Define a deviation from the mean using words only.

➢ Why are the absolute value signs needed? Using Troop A as an example, what would happen if $\sum(x - \bar{x})$ were computed rather than $\sum|x - \bar{x}|$?

➢ What is $\sum(x - \bar{x})$ always equal to?

The Standard Deviation

The standard deviation is a third measure of variability. Like the *MAD*, the standard deviation is based on the deviations from the mean, but instead of taking the absolute value of each deviation, each deviation is squared.

Standard deviation? Is not that an oxymoron?

Standard Deviation

The standard deviation (*SD* or *s*) is another measure of the variability of *n* values from their mean

$$SD = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Example Complete the table to compute the standard deviation of the number of boxes sold for Troop A.

Number of Boxes Sold, x	Deviation from the Mean, $x - \bar{x}$	Squared Deviation from the Mean, $(x - \bar{x})^2$
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total	0 boxes	_____ squared boxes

$$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} =$$

Why Divide By $n - 1$ in the Formula for the Sample Standard Deviation?

Suppose you have a population that consists only of the three numbers 2, 4, and 6. You take a sample of size 2, replacing the first number before you select the second.

a. Finish the list of all 9 possible samples you could get in the first column of the chart below. (Order matters so 2, 4 is different from 4, 2.) Then complete each column.

Sample	Mean	Variance, dividing by $n = 2$	Variance, dividing by $n - 1 = 1$
2, 2	2	0	0
2, 4	3	1	2
Average			

b. Compute the average of each column.

c. Compute the variance of the population {2, 4, 6} (dividing by n) and compare to your answers in part (e).

d. On the average, do you get a better estimate of the variance of the population if you divide by n or by $n - 1$? Explain.

e. If you divide by n, rather than $n - 1$, does the variance tend to be too big or too small on average?

Comparing the Measures of Variability

Students must learn three ways to measure the variability of a distribution: the *IQR*, the *MAD*, and the standard deviation. Often, but not always, when comparing distributions each of these measures will give the same answer as to which has the most variability and which has the least.

- Generally, the *IQR* is the most useful when the distribution is strongly skewed or has outliers.

- The standard deviation is the most commonly reported measure of variability. It has important mathematical properties and is most useful with symmetric distributions.

- The *MAD* is used mainly to introduce the idea of deviation from the mean to middle school students as a prelude to the standard deviation.

➢ What can you conclude if the *MAD* of a set of data is 0?

   What can you conclude if the *SD* is 0?

   What can you conclude if the *IQR* is 0?

Sample Homework

1. Refer to the growth charts.

    a. What is the approximate percentile for an 8-year-old girl who is 50 inches tall?

    b. About how tall does a 10-year-old girl have to be so that she is as tall or taller than 75% of the girls her age? How tall does a 10-year-old boy have to be?

    c. What is the 25th percentile of height for 6-year-old boys? The 50th percentile? The 75th percentile?

    d. Compute and interpret the *IQR* of heights for 6-year-old boys.

    e. Is there more variability in the heights of 6-year-old boys or 12-year-old boys? How can you tell from the charts?

    f. Is there more variability in the heights of 5-year-old girls or 5-year-old boys? How can you tell from the charts?

2. Refer to the cookie sales for the four Brownie troops.

   a. Compute the mean absolute deviation and the standard deviation for the four troops (some were done in class). Round to the nearest tenth and record your results in the table below.

Troop	MAD	SD
A		
B		
C		
D		

   b. Rank the four troops according to their variability from the mean from smallest to largest. Does it matter whether you use the *MAD* or the *SD* to do the ranking?

3. You may have learned about the **range** as a measure of variability:

$$\textbf{range} = maximum - minimum$$

a. What is the range of the number of boxes sold by the Brownies in Troop A? Troop B? Troop C? Troop D?

b. Why are the *MAD* and the *SD* better measures of variability in a distribution than is the range?

c. Is the range resistant to outliers? _____ Is the interquartile range resistant to outliers? _____

d. Does the distribution on the left or the right below have the larger variability? _____
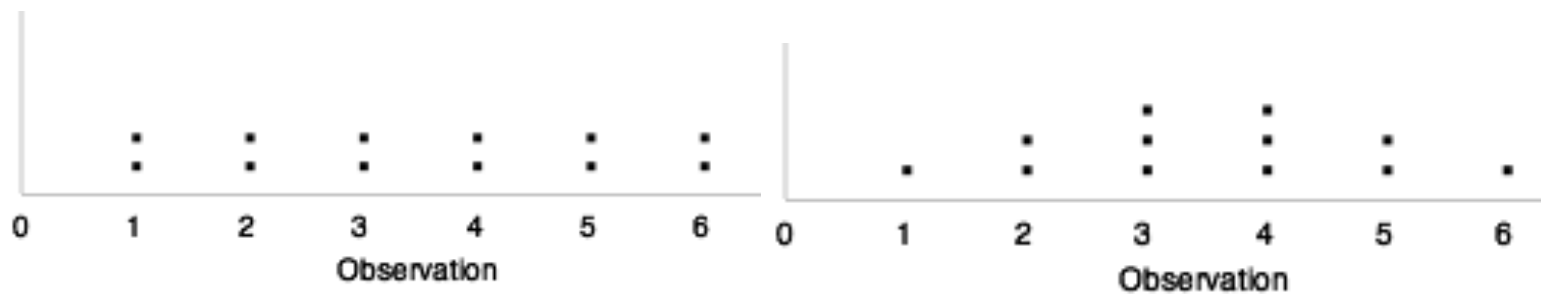
e. Many middle school students pick the wrong distribution in part c. Can you explain what they are thinking?

4. You may have used the distance formula to compute the distance, d, between two points in the plane, (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

a. Use the distance formula to compute the distance between the points (4, 4) and (6, 2).

b. Compute the standard deviation of the values 6 and 2.

c. Compare the processes you used in parts a and b. What's the same? What's different?

5. Does the statement below describe the mean absolute deviation or the standard deviation? _____

   **It is the sum of the distances from the mean.**

6. The dotplots below give the pesticide concentrations at three depths in a river. Answer the questions without computing.
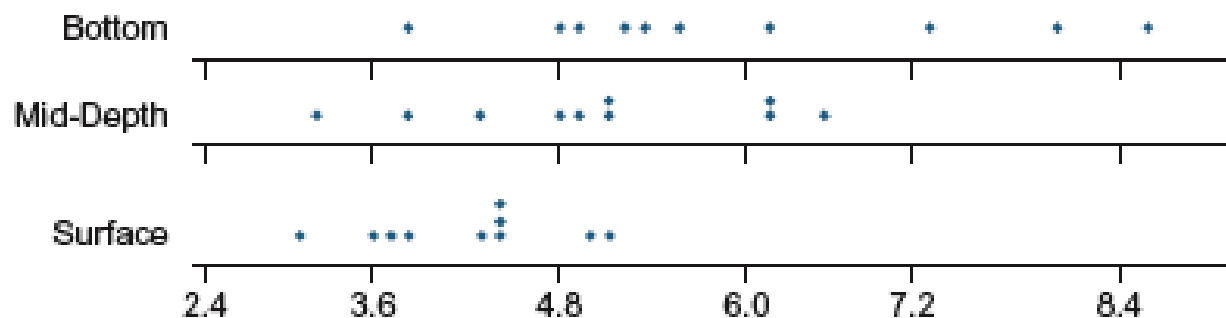
a. Which depth has the largest mean? _____ The largest standard deviation? _____

b. Which has the smallest mean? _____ The smallest standard deviation? _____

7. Match each of the following histograms of test scores in Classes I, II, and III to the best description of class performance.

n = 27 **Test Scores for Class I**

n = 27 **Test Scores for Class II**

n = 27 **Test Scores for Class III**
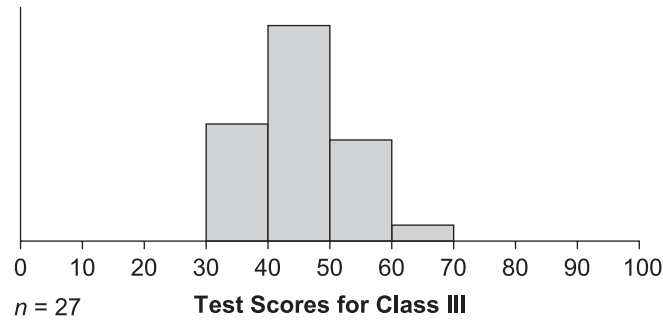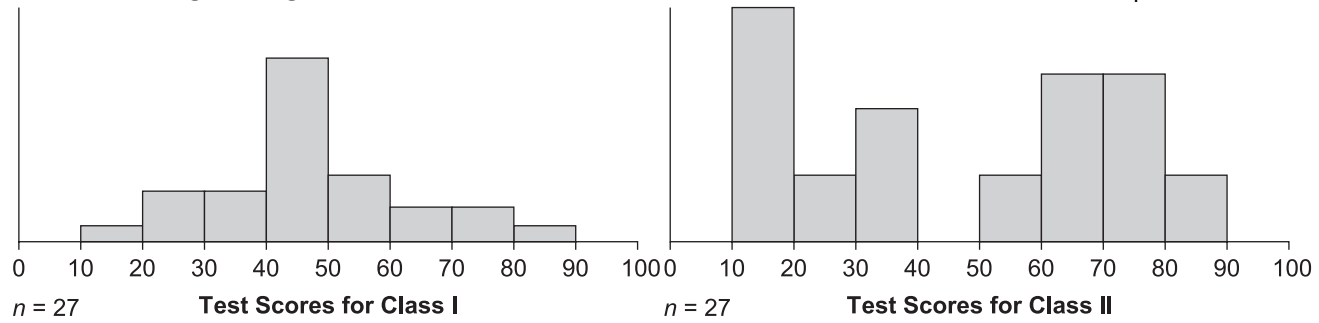
a. The mean of the test scores is 46, and the standard deviation is 26. _____

b. The mean of the test scores is 46, and the standard deviation is 8. _____

c. The mean of the test scores is 46, and the standard deviation is 16. _____

8. No computing should be necessary to answer these questions.

   a. The mean of each of these sets of values is 20, and the range is 40.
      Which has the largest standard deviation? _____ Which has the smallest? _____

      I.   **0 10 20 30 40**

      II.   **0 0 20 40 40**

      III. **0 19 20 21 40**

   b. Two of these sets of values have a standard deviation of about 5. Which two? _____

      I.   **5 5 5 5 5 5**

      II.   **10 10 10 20 20 20**

      III. **6 8 10 12 14 16 18 20 22**
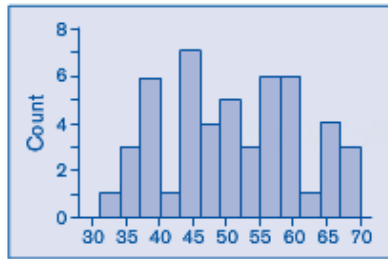
      IV.   **5 10 15 20 25 30 35 40 45**

   c. The standard deviation of the first set of values listed below is about 32. What is the standard deviation of the second set? Explain your answer.
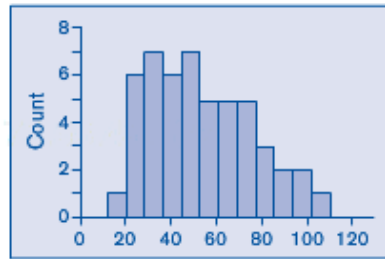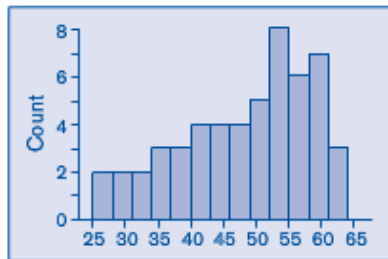
      16 23 34 56 78 92 93

      20 27 38 60 82 96 97

9. Write the letter of the histogram next to the appropriate set of summary statistics. From *Activity-Based Statistics*, Scheaffer et al.
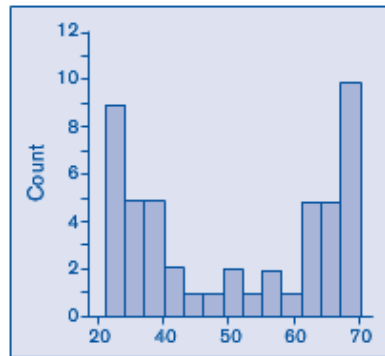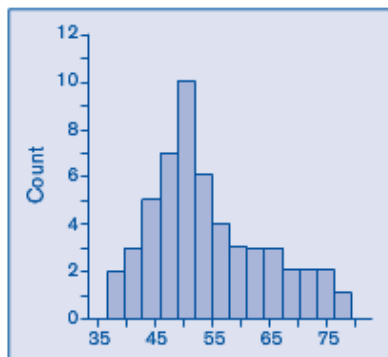

Histogram a

Histogram b

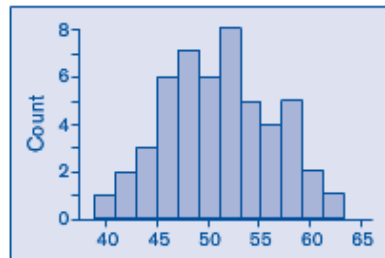| Variable	Mean	Median	Standard Deviation
1	60	50	10
2	50	50	15
3	53	50	10
4	53	50	20
5	47	50	10
6	50	50	5

Histogram c

Histogram d

Histogram e

Histogram f

Search the site...

"Science is the best way to satisfy your own curiosity for the governmental account."
-Soviet physicist Lev Artsimovich

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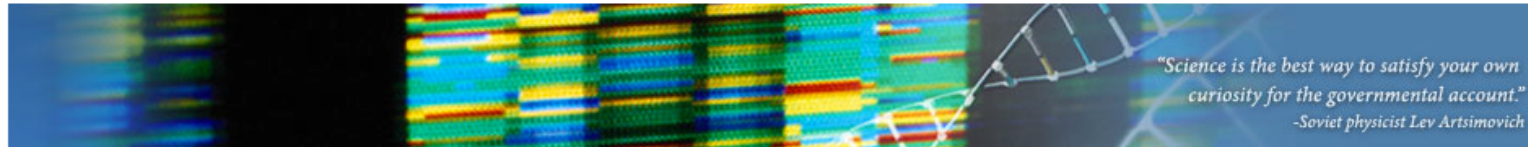
Education > K-12 Education

K-12
Workshops for Teachers
Online Resources for Teachers
Publications in Statistics Education
Student Competitions
Useful Web Sites for Teachers
Census at School
ASA-NCTM Joint Statement on Preparing Pre-K-12 Teachers of Statistics

Undergraduate
Graduate
Continuing Education
Caucus of Academic Representatives
Educational Ambassador
Education Home

Useful Web Sites for Teachers

With so many web sites available, it is hard for teachers of statistics to select those that are most useful. The following is a compilation of information by members of the ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability. You may wish to visit these sites and see which are appropriate for your teaching level.

General Sites

Statistics Education Web (STEW) is a free, online bank of peer-reviewed lesson plans for K-12 teachers of mathematics and science. Lesson plans showcase the use of statistical methods and ideas in science and mathematics based on the framework and levels in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework (www.amstat.org/education/gaise) and Common Core State Standards.

Census at School - U.S. is a free international classroom project that engages students in grades 4 to 12 in statistical problemsolving using their own real data. Students complete an online survey, analyze their class census results, and compare their class with random samples of students in the United States and abroad.

K-12 Statistics Education Webinars The American Statistical Association offers free webinars on K-12 statistics education topics. This series was developed as part of the follow-up activities for the Meeting Within a Meeting (MWM) Statistics Workshop for Math and Science Teachers (www.amstat.org/education/mwm).

Careers in Statistics

Visit the ASA's **Career Center** for information about career opportunities in statistics.

http://www.amstat.org/education/usefulsitesforteachers.cfm