

1a. Finish the work done in class to find a formula for the  $k$ th Fibonacci number,  $F_k$ .

You must show your work.

b. Use the formula in part (a) to find the 25th, 50th and 100th Fibonacci numbers,  $F_{25}$ ,  $F_{50}$ ,  $F_{100}$ .

2. From text Section 8.3: 1, 15, 16.

3a. For the Fibonacci matrix  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ , write down  $A^2$ ,  $A^3$ ,  $A^4$  and then (using the work done in class)  $A^{100}$ .

b. Find  $B^{101}$  if  $B = \begin{bmatrix} 7 & 12 \\ -4 & -7 \end{bmatrix}$ .

4. Suppose Fibonacci had started his sequence with  $F_0 = 1$  and  $F_1 = 3$  and then followed the same rule  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$ . Find a formula for  $F_n$  as we did in class. Show that the ratios  $\frac{F_n}{F_{n-1}}$  still approach the golden ratio. That is, prove  $\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \frac{1+\sqrt{5}}{2}$ .

5. If each number in a sequence is the *average of the previous two numbers*, that is,  $G_n = \frac{1}{2}(G_{n-1} + G_{n-2})$ , set up a matrix  $A$  that shows how to represent this in matrix form. Diagonalize the matrix  $A$  and use  $G_0 = 0$  and  $G_1 = \frac{1}{2}$  to find a formula for  $G_n$  (as we did in class for the Fibonacci sequence). Then compute  $\lim_{n \rightarrow \infty} \frac{G_n}{G_{n-1}}$ .