

Preface

This dissertation studies the rich dynamics and complex self-organization exhibited by *Larger than Life*, a family of cellular automata. Larger than Life is a generalization of John Horton Conway's celebrated *Game of Life* (which we often refer to as Life). Life is the most famous example of a *cellular automaton* (CA). Its fame comes from the fact that its *update rule*, which is very simple, generates extremely complicated dynamics. Life's simplicity enables ease of simulation: "movies" of Life are popular screen savers for personal computers, and its complexity continues to generate challenging problems. Conway's original challenge — to find a finite Life configuration that generates an infinite population — was introduced in Martin Gardner's *Mathematical Games* column in 1970. Since then, "Life Enthusiasts" have been seeking, and finding, new digital creatures admitted by Life. For example, last January one of the enthusiasts found the *snail*, which is the slowest moving known Life configuration.

Not only are CAs fun games, they are becoming a popular tool for modeling problems in fields such as chemistry, mathematical biology, and population ecology. As one example, they are being used to study the extent to which spatial variation in ecological systems influences the ability of a species to survive. The Larger than Life family of CA rules contains some nonlinear population dynamics which are prototypes for such spatial models.

What *is* a CA? Begin with a *lattice* of sites (this can be finite or infinite). Let each site be in one of a finite number of *states*. This is the *initial configuration* of the system. Define a *neighborhood* that consists of a finite number of other sites. At each *time step*, each site checks the states of its neighbors. It then makes a computation, which is based on the status of its neighbors, to decide what state it will be in next time. All of the lattice sites do the same thing, and after making their computations, they *update* (meaning change states or not) simultaneously. A CA rule is said to be *local* because each site in the system is only affected by the sites in its finite neighborhood. It is *homogeneous*, or *translation invariant*, because the neighborhood for each site in the system is defined in the same way, and each site updates according to the same rule. It is *deterministic* because its rule is so. It is *discrete*, in time, because the sites update at discrete times. It is discrete, in space, because its universe is a lattice. It is *dynamic* because the sites update at each time step and it is *parallel* because the sites update synchronously.

Let us introduce the most famous example of a CA by defining Life. The universe is an infinite two-dimensional grid, or \mathbb{Z}^2 . There are two states, 0 and 1, which we think of as *dead* and *live*, respectively. The neighborhood of a site consists of the eight sites surrounding

it. Each time step a dead site will become live if there are exactly three live sites in its neighborhood. A site that is live will stay live if two or three of its neighbors are also live. At time zero the player gets to decide which sites are live and which are dead. After that, the rules of the game take over: every time step all of the sites update according to the deterministic rule we have just described.

For Life, as is the case with most CAs, the initial state has an enormous impact on the resulting dynamics. If the player chooses a *random* initial state, complex dynamics emerge. For example, there are *gliders* that walk across the infinite grid. If not stopped by some other Life pattern, the gliders will walk on forever. These *digital creatures* were the inspiration for Larger than Life. The original question, asked by David Griffeath, was: Will digital creatures, analogous to Life's gliders, emerge from random initial states if we use larger neighborhoods and rescale Life's update rule appropriately? Not only was the answer affirmative, but the Larger than Life (LtL) family of rules introduced a rich collection of two-dimensional CA dynamics. Being *nonlinear*, the rules are fascinating to study. However, penetrating their emergent dynamics is an extremely challenging endeavor.

The most popular CAs to study are those which have some "nice" property, making them amenable to fairly standard mathematical analysis. For example, in the case of two-state CAs, such as those in the LtL family, there are rules which are *symmetric* in the states. That is, the conditions for changing from each state to the other are the same. Another fairly understandable class of two state rules are those which are *monotone*. This means that if A and B are sets of live sites and $A \subset B$ then after one time step, the set of live sites generated from having B as an initial state will contain the set of live sites generated if A is the initial state. Another simplifying feature is to work with small neighbor sets such as the *Von Neumann neighborhood*; in that case, each site in the grid sees its neighbors to the north, south, east, and west.

However, most LtL rules have no such regularity properties. As a consequence, the mathematics behind many of the most intriguing LtL rules is rather impenetrable. Therefore it is exceedingly difficult to obtain rigorous results for them. The question thus arises: Should we study only the restricted class of rules that have sufficient regularity, so that rigorous results are within reach? If we were to take this stance then for starters we would miss out on Life itself. New technology such as the *Cellular Automaton Machine* (CAM), as well as ever faster clock speeds on personal computers, enables us to make ever greater empirical progress in understanding how such systems behave. At present, however, mathematically rigorous methods are not growing as quickly as the technological advances. As mathematicians, we are conflicted — do we stop seeking nonlinearity and rules that attempt to model real-life

problems, for the sake of rigor, or do we seek complexity and consequently work harder to attain rigor? In this dissertation, we attempt the latter, but remain conflicted.