Appendix I
Computing σ, a large range counting problem

In order to use the bound we obtained for still life measures, we must compute σ. We have the following chart for ranges 1 and 2.

values of σ		
	range	
δ_2	1	2
1	4	4
2	4	8
3	6	8
4	6	12
5	8	12
6	8	13
7	8	15
8	8	16
9	8	17
10	$\times$	18
11	$\times$	20
12	$\vdots$	20
13		20
14		20
15		20
16		21
17		22
18		23
19		24
20		24
21		24
22		24
23		24
24		24
25		24

Values of σ for each δ_2 in ranges 1 and 2.

We have the following chart for ranges $\rho \geq 2$.

range ρ $(\rho \geq 2)$	
δ_2	σ
1	4
2	8
⚠	
$\vdots$	
⚠	
2ρ	$4(\rho+1)$
⚠	
$\vdots$	
⚠	
$4\rho^2-5$	$4\rho(\rho+1)-4$
$\vdots$	$\vdots$
$(4\rho^2+2\rho-5)$	$4\rho(\rho+1)-4$
$(4\rho^2+2\rho-4)$	$4\rho(\rho+1)-3$
$(4\rho^2+2\rho-3)$	$4\rho(\rho+1)-2$
$(4\rho^2+2\rho-2)$	$4\rho(\rho+1)-1$
$(4\rho^2+2\rho-1)$	$4\rho(\rho+1)$
$\vdots$	$\vdots$
$(2\rho+1)^2$	$4\rho(\rho+1)$

Values of σ for each δ_2 in ranges $\rho \geq 2$.

The warning icon is used above to indicate the values of δ_2 for which we have not yet computed σ

Let us explain the first chart listed above, for ranges 1 and 2. By definition, $\sigma = \max\{|y \in (x+\mathcal{N})| : \xi_0(y) = 1, \xi_0(x) = 0\}$. Thus, to compute σ we need to determine the largest number of $1's$ that can coexist in the neighborhood of a dead site. This value depends on δ_2. WLOG, we can consider the neighborhood of the origin, and assume $\xi_0(\text{origin}) = 0$. Let us first consider the range 1 version of this problem.

Range 1

The range one neighborhood, $\mathcal{N}$, along with the number of sites (not including the center) in each site's neighborhood looks like:

3	5	3
5	0	5
3	5	3

If $\delta_2 \geq 5$ then $\sigma = 8$ since no site sees more than 5 other sites (and hence no more than 5 other $1's$). If $\delta_2 = 3$ or 4 then $\sigma = 6$ since in the cases of 7 or 8 live sites, the configurations would look like one of the following (or a rotation of one of these):

0	1	1
1	0	1
1	1	1

1	0	1
1	0	1
1	1	1

1	1	1
1	0	1
1	1	1

In each case, the neighborhood of one of the live sites contains more than 5 live sites. If $\delta_2 = 2$ then $\sigma = 4$ since in the case of 5 live sites no row can contain 3 live sites (otherwise the center site would see 3 live sites), so the remaining choices (possibly rotated) are:

1	1	0
0	0	1
1	0	1

1	1	0
1	0	1
0	0	1

1	1	0
1	0	0
0	1	1

1	0	1
1	0	1
0	1	0

In each case the neighborhood of one of the live sites contains 3 live sites. Similarly, if $\delta_2 = 1$ then $\sigma = 4$ since the maximum configuration in that case is:

1	0	1
0	0	0
1	0	1

(Adding a 1 anywhere will yield a live site with 2 live neighbors.)

Range 2

Let us now find σ for all possible values of δ_2 in range 2.

Assume that the range 2 neighborhood, $\mathcal{N}$, of the origin is filled with $1's$, and the origin is a 0. Let us depict the neighborhood, along with the number of $1's$ in each site's neighborhood:

8	11	14	11	8
11	15	19	15	11
14	19	0	19	14
11	15	19	15	11
8	11	14	11	8

For each $x \in \mathcal{N}$, $x \neq$ origin $8 \leq |(x + \mathcal{N}) \cap \mathcal{N}| \leq 19$. Thus, if $\delta_2 \geq 19$, then $\sigma = 24$. Since the sites nearest the origin see the most other sites, let us assume one of those is a zero. Then all of the others will see fewer than 18 $1's$. Thus, if $\delta_2 = 18$, then $\sigma = 23$. Similarly, if $\delta_2 = 17$, then $\sigma = 22$, and if $\delta_2 = 16$, then $\sigma = 22$. If the four sites nearest the origin are all zeros, then the new count of the non-zero neighbors is:

6	8	11	8	6
8	11	0	11	8
11	0	0	0	11
8	11	0	11	8
6	8	11	8	6

All of the remaining $1's$ see at most 11 $1's$. Thus, if $11 \leq \delta_2 \leq 15$, then $\sigma = 20$. Now suppose one of the sites that sees 11 $1's$ is a zero. Then at least one other site will still see 11 $1's$. However, if the two sites directly to the north and south of the origin are zeros, then each site left will see at most 10 live sites. Thus, if $\delta_2 = 10$, then $\sigma = 18$. If one of the four sites nearest the origin is switched to a zero, then each remaining 1 will see at most 9 $1's$, so if $\delta_2 = 9$, then $\sigma = 17$. At this point, the count is:

4	6	0	6	5
6	0	0	9	7
8	0	0	0	9
6	9	0	9	7
5	7	0	7	5

Switching either the northeast or southeast neighbors of the origin to a zero will have all remaining $1's$ seeing at most 8 $1's$. Thus, if $\delta_2 = 8$, then $\sigma = 16$, and the count will be:

4	5	0	5	4
6	0	0	0	6
8	0	0	0	8
6	8	0	8	6
5	7	0	7	5

Now at least two sites must be switched to zeros before all sites will see at most 7 $1's$. However, once those two sites are turned to $0's$ we can add a site to get the configuration:

4	5	7	5	4
6	0	0	0	6
0	0	0	0	0
5	7	0	7	5
4	6	0	6	4

Thus, if $\delta_2 = 7$, then $\sigma = 15$. Again two sites must be switched to zero before all sites see fewer than 6 $1's$. For $\delta_2 = 6$, $\sigma = 13$ since at least two $1's$ from the above must be turned to zeros and no extra sites may be added. Then we'll have:

3	4	0	4	3
5	0	0	0	4
0	0	0	0	0
5	6	0	0	4
4	5	0	5	3

For $\delta_2 = 5$, $\sigma = 12$ since the above configuration less the site that see 6 other $1's$ contains the maximum number of $1's$. If $\delta_2 = 4$, $\sigma = 12$, using the same configuration. However, for $\delta_2 = 2$ or 3, $\sigma = 8$ since the maximum valid configuration for those is:

2	2	0	0	2
0	0	0	0	2
0	0	0	0	0
2	0	0	0	0
2	0	0	2	2

(Adding a 1 anywhere will give a site that sees at least 4 $1's$.) Finally, if $\delta_2 = 1$ then $\sigma = 4$ since the only valid configuration is that with the four $1's$ in the corners.

In general, if $\rho \geq 2$ we argue as we did above for the higher values of δ_2. That is, the four sites nearest the origin see at most $(4\rho^2 + 2\rho - 1)\ 1's$. Thus, if $\delta_2 \geq (4\rho^2 + 2\rho - 1)$, then $\sigma = 4\rho(\rho + 1)$. If $\delta_2 = (4\rho^2 + 2\rho - 2)$, then $\sigma = 4\rho(\rho + 1) - 1$. If $\delta_2 = (4\rho^2 + 2\rho - 3)$, then $\sigma = 4\rho(\rho + 1) - 2$ and if $\delta_2 = (4\rho^2 + 2\rho - 4)$, then $\sigma = 4\rho(\rho + 1) - 3$. If $4\rho^2 - 5 \leq \delta_2 \leq (4\rho^2 + 2\rho - 5)$, then $\sigma = 4\rho(\rho + 1) - 4$. Similarly, for the lower values of δ_2: if $\delta_2 = 1$, then $\sigma = 4$ and if $\delta_2 = 2$, then $\sigma = 8$.