NAME:_____
CALCULUS II (Math 150B): Final.

General Instructions

1. **You are allowed your cheatsheat (one page, front and back allowed).**
2. **NO Calculators.**
3. **Please show all your work, unless explicitly instructed not to do so.**
4. **You may use the tables in any problem you like on this test.**
5. **Please ask if you are not sure of anything on the exam.**
6. **You have 2 hours.**

Question	Credit	Total Points
1		15
2		10
3		5
4		5
5		5
6		10
7		20
8		5
9		10
10		10
11		5
Total		100

1. Evaluate each of the following integrals.

(a) $\int x^2 e^{x^3}\, dx$

(b) $\int t^{-2} \ln t\, dt$

(c) $\int \sin^6 x \cos^3 x\, dx$

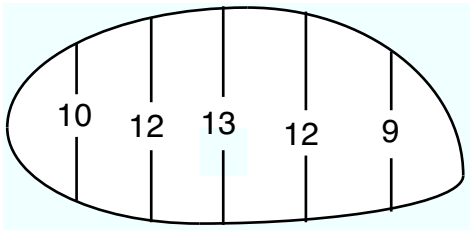
2. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{1}{x(x+2)(x-3)}\,dx$

(b) $\displaystyle\int \frac{1}{(25-16x^2)^{3/2}}\,dx$

3. Show that the improper integral $\int_0^8 \frac{x}{x^2-9}\,dx$ diverges.

4. Use the trapezoidal rule to approximate the area of the pond shown to the right, which has been divided into six lanes of width 5 feet each and lengths as shown.



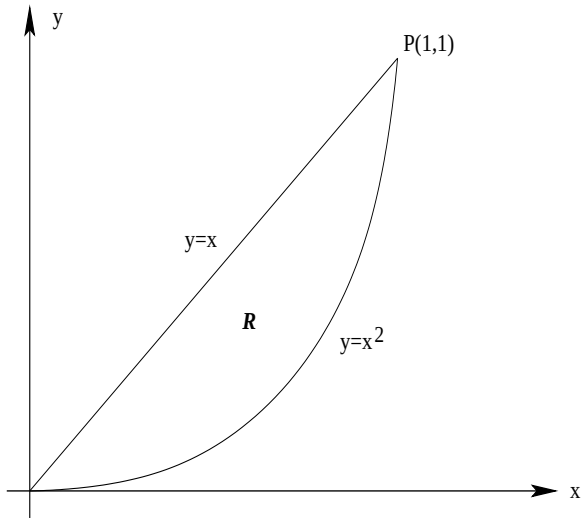
5. For the problem of finding the area of the region $\boldsymbol{R}$, circle <u>all</u> correct answers below:

$\int_0^1 (x + x^2)\, dx$ $\quad$ $\int_0^1 (x - x^2)\, dx$

$\int_0^1 (x^2 - x)\, dx$ $\quad$ $\int_0^1 (y + \sqrt{y})\, dy$

$\int_0^1 (y - \sqrt{y})\, dy$ $\quad$ $\int_0^1 (\sqrt{y} - y)\, dy$

NONE OF ABOVE



6. Let R be the region between the parabolas $y = x^2 - 8x - 4$ and $y = 6 - x^2$. Express as an integral the volume of the solid that results from rotating R about the horizontal line $y = 8$. Do not evaluate the integral.

7. Determine whether the following series converge or diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n^2}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

8. Find the sum for each of the following convergent series.

(a) $5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \cdots$

(b) $1 - \frac{9}{2!} + \frac{81}{4!} - \frac{729}{6!} + - \cdots$

9. Find the convergence set for $\sum_{k=1}^{\infty} \frac{k!(x-1)^k}{(2k)!}$.

10. Sludge fills a conical reservoir of radius 5 meters and hight 3 meters (standing up on its pointy end). The density of the sludge at distance h meters from the top of the rim is $1000e^h$ kilograms per cubic meter. The sludge is being pumped just over the rim of the reservoir until the remaining sludge is 1 meter below the rim. Write an integral describing how much work is done. DO NOT COMPUTE.

11. Below are two approximations to a periodic function $f(t)$ (period 2π).

$$f(t) \approx 2 + .5\cos(t) - .3\sin(2t)$$

$$f(t) \approx 2.5 - .6t - .25t^2$$

(a) Which one is the Taylor series approximation?

(b) If we view $f(t)$ as a signal, the strongest component seems to have period 2π, period π or neither?

(c) Is the function increasing or decreasing at $t = 0$?

(d) Which approximation would you use to find $f(.0001)$?

(e) Which approximation would you use to fine $\int_0^{250} f(t)dt$?