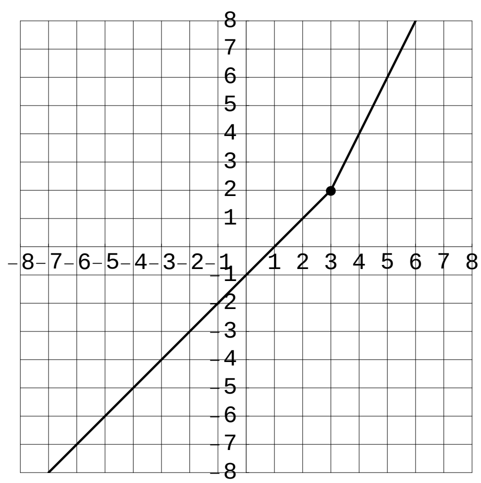
NAME: ______________________________

Math 103L: Continuity (Section 10.2 and 10.4)
These problems are a sample of the kinds of problems that may appear on the final exam. Some answers are included to indicate what is expected. Problems that require a summary statement are marked with Sum . The summary statements should be written in complete sentences and they should include the units of measurement for all quantities mentioned in the summary.

1. Consider the following function.

$$f(x) = \begin{cases} x - 1, & \text{if } x \le 3 \\ 2x - 4, & \text{if } x > 3 \end{cases}$$

(a) Sketch a graph of $y = f(x)$.
Answer:



(b) Where is this function continuous? Explain why using limits.
Answer:
The function is continuous at every value of x. The only possible exception is at $x = 3$ and at that point,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x - 1 = 2,$$

and

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x - 4 = 2.$$

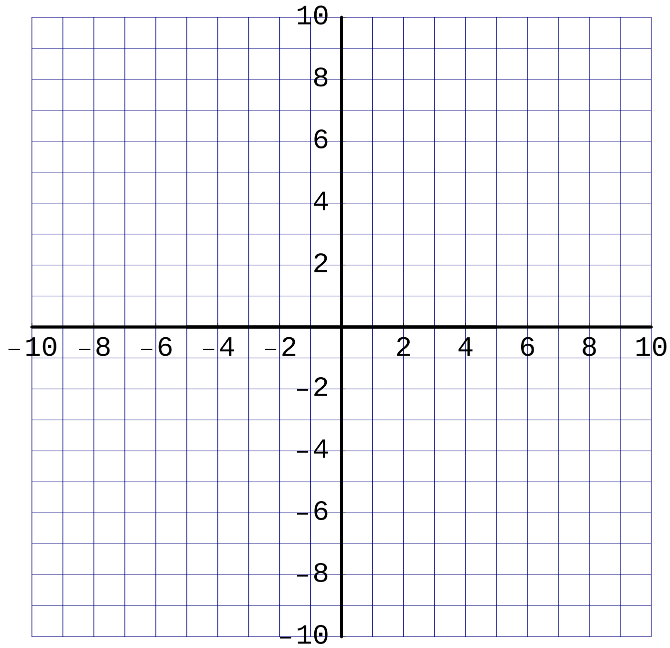
So $\lim_{x \to 3} f(x)$ exists and equals the value of the function $f(3) = 2$.

(c) Where is this function differentiable? (No explanation necessary)
Answer:
The function is differentiable at every value of x except at $x = 3$. The function is not differentiable at $x = 3$.

2. Consider the following function.

$$f(x) = \begin{cases} 2, & \text{if } x < 0 \\ 3x - 2, & \text{if } 0 \le x < 3 \\ x + 4, & \text{if } x \ge 3 \end{cases}$$

(a) On the axes below graph $y = f(x)$.



(b) Using the graph and the function find $\lim_{x\to 0^-} f(x)$.

(c) Using the graph and the function find $\lim_{x\to 3^+} f(x)$.

(d) Where is this function continuous?

(e) Where is this function differentiable?

3. Consider the revenue function $R(x) = 250x - x^2$ for producing x widgets.

   (a) $\boxed{\text{Sum}}$ Find the change in revenue when production changes from $x = 10$ to $x = 20$.

   (b) $\boxed{\text{Sum}}$ Find the average rate of change of revenue for this change in productions levels.

4. The revenue from the sale of x cellphone towers is given by

$$R(x) = 1000x - 10x^2.$$

   The derivative of the revenue function is $R'(x) = 1000 - 20x$.

   (a) $\boxed{\text{Sum}}$ What is the change in revenue if production is changed from $x = 5$ to $x = 6$ cellphone towers?

   (b) What is the (instantaneous) rate of change in revenue at $x = 5$?

5. Use the definition of the derivative to find the derivative of $f(x) = 2x^2 - 3$. Here are some steps.

   (a) Find
$$f(x+h)$$

   (b) Find
$$\frac{f(x+h) - f(x)}{h}$$

   (c) Find
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$