

Practice Math 103 Spring 2007: Midterm III.  
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1. Find the derivatives of the following functions and simplify.

(a)  $q(x) = 4x^5 + 3x^2$

(b)  $n(x) = \frac{4x+3}{(7x-1)^2}$

(c)  $m(x) = \left(\frac{4x+3}{(7x-1)^2}\right)^{1/3}$

2. Use the definition of the derivative (that thing with the limit) to find the derivative of  $f(x) = 2x^2 - 3$ . Here are some steps.

- (a) Find

$$f(x+h)$$

- (b) Find

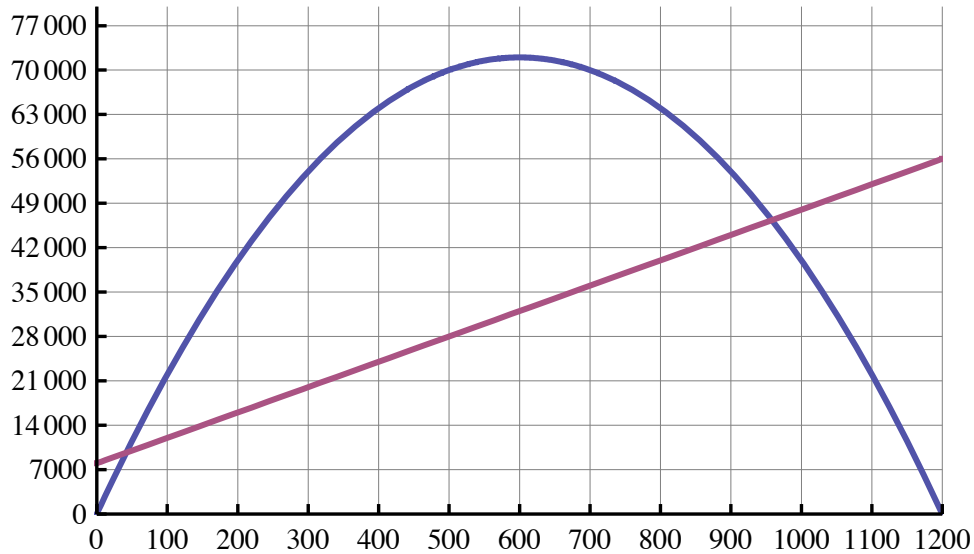
$$\frac{f(x+h) - f(x)}{h}$$

- (c) Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

3. Let  $f(x) = 2x^2 - 3x + 4$ . Find and graph the equation of the line tangent to the graph of  $y = f(x)$  at the point  $(0, f(0))$ .
4. Let  $f(x) = x^3 - 27x$ . If they exist, find the absolute max and min of  $f(x)$  on the intervals below. Give explanations and if an absolute max or min does not exist, then say why.
- (a)  $(-\infty, \infty)$
- (b)  $[0, 10]$

5. The graphs of the revenue and cost functions for the production and sale of  $x$  units are shown below. The cost function is the straight line and the revenue function is the curve.



- Use the graph to estimate the production level  $x$  that maximizes the profit.
  - Mark the points  $(x, C(x))$  and  $(x, R(x))$  on the graphs of the cost and revenue functions corresponding to the value of  $x$  that maximizes profit.
  - What is the maximum profit?
  - If the fixed costs increase by ten dollars, should the production level be raised, lowered, or remain the same to maximize profit? Explain in terms of the graph.
6. A company manufactures and sells  $x$  wigits per week. The weekly price demand and cost functions are:

$$p(x) = 20 - x$$

$$C(x) = 25 + 5x.$$

- Find the marginal revenue function.
- Use marginal revenue to estimate the revenue earned by producing the 6th wigit.
- Find the maximum revenue. Continuing with a company manufactures and sells  $x$  wigits per week. The weekly price demand and cost functions are:

$$p(x) = 20 - x$$

$$C(x) = 25 + 5x.$$

- Draw some axes and graph the following:
  - Draw in the axes, label them, and mark a scale;
  - Label  $y = R(x)$ , its  $x$ -intercepts, its maximum point with coordinates;
  - Label  $y = C(x)$ , its  $y$ -intercept and one other point on  $y = C(x)$ .
  - Shade the areas which correspond to the company making a profit make a bold line to represent the maximum profit graphically.

- (e) Find the production *level* that will realize that maximum profit and the *price* the company should charge for each wig.
- (f) Explain why profit is maximized when marginal revenue equals marginal cost.
- (g) If marginal revenue is at \$2.50 per unit produced, explain using math and words why and how production should be changed.
- (h) If the fixed costs to produce wigs increases from 25 to 26, does the output level which maximizes profit increase? decrease? or remain the same? Why? (Note: You do not have to compute here.)
- (i) If the variable costs decrease to \$0 per item, where will profits be maximized?

7. The demand equation  $p$  is given by

$$x + p = 4800.$$

- (a) Write demand as a function of price.
- (b) Find the elasticity of demand at a price of \$800?
- (c) If the price increases 10% from a price of \$800, what is the approximate (percentage) change in demand? State whether demand will increase or decrease.