When I first started teaching middle school students about circles and the number \( \pi \), my task as I understood it was straightforward: Teach them that \( \pi = 3.14 \) or \( 22/7 \), and teach them the formulas for finding the area and circumference of a circle. I poured in the information, but a lot of it spilled out and was lost. Here are two examples of my "wisdom": as it came out at the other end of the spout: (1) "\( \pi \approx 3.14 \), since nobody likes fractions anyway" and (2) "There are two formulas that you have to use: \( \pi r^2 \) and \( \pi d \). Sometimes you use one formula and sometimes the other. If you are lucky, you will guess right."

I found this understanding unsatisfactory, so my first recourse was to try to drill the knowledge into my students harder. The result was frustration for them and me. They had already had lots of experience in memorizing, and the more nonsensical the information was, the more difficulty the students had getting it right. Because nothing was more nonsensical to them than circles and \( \pi \), we were getting nowhere.

I knew that the solution to this problem was not to teach students to memorize better. I needed to give them tools for understanding the concepts instead. At about that time, an article called "The Mountains of Pi," by Richard Preston (1992), appeared in The New Yorker. The article described the state of research at that time into the number \( \pi \), and it started me on a program of reading about mathematics and its history that has not stopped yet. Although I have no intention of emulating the Chudnovsky brothers in New York City, who have devoted their lives to chasing the next million digits in the decimal representation of \( \pi \), I was fascinated by their accomplishments. I also discovered that \( \pi \) turns up in many situations that have nothing to do with circles, a fact that was not mentioned in any of my undergraduate mathematics courses, and that people have been aware of \( \pi \) and its relation to circles for several millennia. My most interesting discovery was that my students found the underpinnings of mathematics fascinating, too. Once they understood where the concepts came from, learning the essential ideas was easier. I have since been told that more than one class was "awesome!"
Activities for Learning about $\pi$

OVER THE YEARS, I HAVE INVOLVED MY STUDENTS in the number $\pi$ in a variety of activities. The first involves a carpenter's 25-foot steel tape measure and a classroom set of 60-inch tape measures. We make a tour of the school, measuring circles as we go. Having discovered that the circumference and diameter are the only two parts of the circle that we need to measure, we have devised a chart for the class scribe to fill in as we go. On our return to the classroom, we measure a variety of circles that I have accumulated there, including oatmeal, salt, and baking powder packages; some borrowed pots from the kitchen; and so on. With the two left-hand columns on our charts for circumference and diameter completed, everyone sets to work with calculators to fill in the right two columns; the ratio of circumference to diameter as a fraction for each circle and the decimal equivalent of the fraction. Usually a student comments that he or she keeps getting almost the same number for every circle. Someone else agrees, observing that the number always seems to be a little more than 3. When everyone concurs, I give that number a name: $\pi$.

Then I ask my students to consider the "ball" that they carry on top of their shoulders, that is, their heads. People buy hats in different sizes. If students go home and ask their parents or grandparents what hat size they wear, they will say something like 6 3/4, 7, or 7 1/2. Where do those numbers come from? We then use a tape measure to record sample circumferences of heads in inches. When we divide those numbers by $\pi$, we get such numbers as 6 3/4 or 7. Even milliners use the number $\pi$!

The History of $\pi$

HOW LONG HAVE WE KNOWN ABOUT $\pi$? THAT question takes us back in history to find that the ancient Babylonians and Egyptians and many other early civilizations had a number that they used for $\pi$. They recognized that the ratio of the circumference and diameter of a circle was constant and that it allowed them to figure out the circumference of a circle if they knew its diameter. My classes follow the development of $\pi$, from the ancient Babylonians, who used 3 1/8, or 3.125, in about 2000 B.C.; to the early Greeks, who used $\sqrt{10}$, or about 3.162; to Archimedes (287-212 B.C.), who found that $\pi$ was between 3 10/71 and 3 1/7.

Then I show students a selection from 1 Kings 7:23 in the Old Testament: "And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and its height was five cubits. And a line of thirty cubits did compass it round about." The calculation in the quotation makes $\pi$ about 3. The Old Testament measurements are not as accurate as ours, but people at the time did not have steel measuring tapes! In fact, when 1 Kings was written, sometime before 500 B.C., a more precise value for $\pi$ was generally known. Nehemiah was asked about this discrepancy in about 150 A.D., and he said that it was the result of the difference between the inside measure and the outside of the vessel, since clay is thick.

In China in about 500 A.D., $\pi$ was known to be approximately 3.1415929, and in Persia in 1424, it was known correctly to sixteen decimal places. The Dutchman Ludolph van Ceulen found $\pi$ correctly to twenty decimal places in 1596; for that discovery, the number $\pi$ is often called the Ludolphian constant in Europe. Since then, mathematicians have found millions and billions more digits of the decimal expansion of $\pi$, more than anyone could possibly find useful.

Student Interest in $\pi$

WHEN WE TALK ABOUT THIS TOPIC, USUALLY AT least one student wants to memorize some of the digits of $\pi$. Almost always, another student volunteers information about someone who has memorized $\pi$ out to 100 or 500 or 1000 digits to the right of the decimal point. My students are impressed by such a feat, and some of them start memorizing right then, although I am quick to point out that such an exercise serves no useful purpose. Every calculator has a value of $\pi$ that is good enough for any computation that they will ever do. I mention,
Because π is a constant, it is used to check computer chips

The greater the number of rectangles, the less area is wasted inside the circle and the closer the enclosed area comes to the actual area of the circle.

Fig. 1 Archimedes' method of exhaustion

However, that a couple of years ago, I found a Web site that had mnemonic devices for remembering π. It includes the classic "See, I have a number,..." By simply counting the letters in each word of this phrase and using the comma as the decimal point, students can remember 3.1416, which is π rounded to the thousandths place.

One question that my students ask about π is "How do they find those digits?" After all, we tried pretty hard to measure precisely, but the best we could do was to find that π is probably between 3.1 and 3.2. Archimedes did much better than that, and he did not have a steel measuring tape. Their question leads, first, to an explanation of Archimedes' method, which leads to the limit as one of the foundations of calculus: the exhaustion of the circle. My students try to figure the area of a circle by getting closer to the area using rectangles that become progressively narrower (see fig. 1) or by trying to catch the area between two polygons (see fig. 2). By the sixteenth and seventeenth centuries, people were rearranging the sectors of two circles to form a rectangle that approximated twice the area of a circle (see fig. 3). Convincing students that precision in measurement has limits is not difficult. We could not use either of these methods to get a truly precise value for π.

The answer to the students' question is that the number π also turns up in number theory—in ways that have nothing at all to do with circles. In the seventeenth and eighteenth centuries, mathematicians began experimenting seriously with infinite series.

The following formulas for the infinite series of Wallis, Leibniz, and Euler are intelligible to middle school students:

\[
\frac{\pi}{2} = \frac{2}{1} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdot \ldots \\
(\text{John Wallis in 1655})
\]

\[
\pi = 4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots \right) \\
(\text{Leibniz in 1673})
\]

\[
\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \ldots \\
(\text{Euler in 1736})
\]

I let my students play with these formulas, using calculators long enough to discover that the formulas do lead to π. I also mention that one of these formulas for π is used to check the functioning of computer chips, because π truly is a constant. In 1997, a pentium chip was recalled because it failed to find π. One class several years ago programmed my classroom Apple II computer to follow one of these formulas for a weekend. When we returned on Monday, we found that π is just a little more than 3.14.

Eventually, in every class, someone asks why we call this number π. In my introduction, I have already told them that π is the letter P in the Greek alphabet. The obvious question is why P? Students also wonder whether the ancient Greeks were the first to use that symbol for 3.14158978. . . . The answer to the first question is purely conjecture: Peri in Greek means around, and π was first derived from the distance around a circle. However, the Greeks did not use the symbol π for the number. William Jones, an Englishman, was probably the first to use it in 1675. The symbol gradually became accepted only a little more than 300 years ago.

"Proving" the Formulas

How are my students supposed to remember which formula, πr² or πd, to use when? Once more, we embark on a discovery process. Using a compass, we construct circles of radius 2 cm, 4 cm, 6 cm, and 8 cm. With the correct formulas in hand, we find the circumference and area in terms of π for all four circles and combine the results in a chart (see fig. 4). As a group, we conclude that when we start with a radius of 2, the circumference and the area are the same number. When we dou-
ble the radius to 4, we double the circumference but multiply the area by 4, not by 2. When we multiply the original radius of 2 by 3, we multiply the circumference by 3 but multiply the area by 9. When we multiply the original radius of 2 by 4, we multiply the circumference by 4 but multiply the area by 16. Most classes first notice that the area grows much faster than the circumference. After all, we started out with a radius of 2, and the circumference and area were the same number, 4π. By the time we multiplied the radius by 4, the area had practically exploded.

With some classes, I do not take this lesson any further, but other classes explore the fact that when we multiply the radius by 2, we multiply the area by 2^2; when we multiply the radius by 3, we multiply the area by 3^2; and when we multiply the radius by 4, we multiply the area by 4^2. For some groups, an interesting extension is to explore the results when we multiply the radius by a fraction, such as 2/3. Students are often pleased to discover that the area is multiplied by (2/3)^2, or 4/9.

We have used the formulas to get these results, but can we believe them? After all, my students are taking those formulas on faith. They will not be able to prove the formulas until they take calculus, which is a few years in the future. Our next step is to make the results look reasonable. My students take out their compasses again and try to fit circles of radius 2 inside the bigger circles. The students find that they cannot fit too many whole circles inside the bigger circles, but with a little imagination, they begin to be convinced (see fig. 5).

Next we consider the prices that pizza restaurants charge for pizzas of different sizes. When Pizza Hut doubles the price, does it double the radius? No, just an extra inch or two in radius may double the price. Is the restaurant taking unfair advantage of us? No, because the area, or the amount of pizza, grows faster than the radius; therefore, it grows faster than the circumference.

Once my students believe that the area grows faster than the circumference, we start thinking about the formulas. When πr tells us to multiply the diameter by π, we watch the result of the formula grow at the same rate; that is, when we double the diameter, the result of the formula doubles. The formula that asks us to multiply the radius times itself, πr^2, behaves differently. The bigger the radius is, the faster the result grows. We have already discovered that the area grows faster than the circumference; therefore, the formula for area must be the one that involves multiplying the radius times itself, πr^2. Because circumference grows slowly, we can first find the diameter by doubling the radius, then (Continued on page 457)
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multiply the diameter by \( \pi \). With that understanding, students can see that area \( \approx \pi r^2 \) and that circumference = \( \pi d \). Any time they forget, they can think the formulas through again quickly. Approaching this idea from another angle, students remember that the way we found \( \pi \) from our circles was by measuring the circumference and the diameter of the circles. If

\[
\pi = \frac{\text{circumference}}{\text{diameter}},
\]

then circumference = \( \pi \times \text{diameter} \). We certainly did not measure the area of the circle to find our approximation of \( \pi \).

The number \( \pi \) is a constant, as opposed to a variable, such as \( x \) or \( y \). In other words, \( \pi \) never changes. I return to this comparison between constants and variables later in the year when we are dealing with variables. Some of my students think that \( \pi \) is infinitely large because it can be written using so many digits to the right of the decimal point. They have to be reminded that \( \pi \) is really not very big at all, only slightly more than 3. In class, we use a variety of approximations for \( \pi \), ranging from "just plain 3"; to 3.1, 3.14, and 3.14159; to that wonderful fraction \( 22/7 \).

**Conclusion**

IN A SENSE, \( \pi \) IS A MAGICAL NUMBER, allowing us to deal with circles in a way that makes seemingly difficult problems fairly easy. The only real difficulty is remembering how to use \( \pi \) so that it is a reliable help. By the time that we have finished the course, most of my students can find the area and circumference of a circle consistently because they have learned to think about circles and \( \pi \) intelligently.

**Annotated Bibliography**


A rather unorthodox history of mathematics with some strong opinions about the roles of various cultures but fascinating information about the history of the number \( \pi \).


A little volume with a great deal of information presented in a format that my students enjoy reading. Superimposed on the text are the first one million digits of \( \pi \), a treatment that impresses students greatly.


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