

Another Surprise in Bologna

I submitted a paper on "Surprize Snippets" to a computing conference in Bologna, Italy, After it was accepted I realized that a solution to one of my snippets was related to a discovery hundreds of years ago that had taken place also in Bologna. That was before computers roamed the earth.

My snippet, called "Maximum Surprize" is a very common algorithm for finding the maximum of a number of many values (such as 100 random real numbers). Following is pseudoCode for the algorithm; the paper provides code in Java. The maximum value is output each time that it is updated.

```
Set max = aRandomReal
Loop many times
  Set val = aRandomReal
  If (max < val)
    Set max = val
  Output max
EndIf
EndLoop
```

The significant question is:
How often does max get updated?
the answer is surprising.

The most common answer is 50
(among most students and many faculty).
The real answer is closer to 5.
Think about it.

One way to verify this, and gain insight,
is to simulate many runs (say thousands)
observe them and compute an average.

Another way to verify this is to realize that the probability of getting a larger value decreases, as shown by the series sum:

$$1/2 + 1/3 + 1/4 + \dots 1/100$$

This is very close to the Harmonic series (actually it is off by exactly 1).

This series was studied by Pietro Mengoli of Bologna, in the mid 1600s yet! He proved (at the age of 19 or so) that this harmonic series does not converge, but just keeps growing ever so slowly. The first 100 terms sum to around 5; the first million terms sum only to 13.

It is rather counter-intuitive that in finding the maximum of a hundred (random) values the maximum value gets updated only 5 times, on average. But it is severely counter-intuitive that finding the max of a million values updates the maximum only 13 times! (Actually there are only 12 updates; programmers are often off by one).

It is somehow fitting that these two concepts meet in Bologna after about 360 years being unrelated.

Unfortunately I could not be in Bologna to celebrate this "confluent coincidence"; I was undergoing an operation to remove a cancerous growth on my kidney.

I have since found over two dozen proofs of the divergence of this Harmonic series on the internet. Isn't the web wonderful?

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