COMP 282

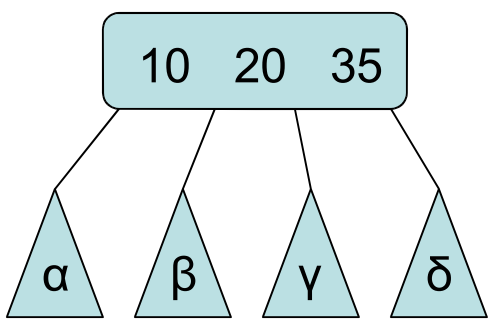
Lecture 17

Red-Black Trees

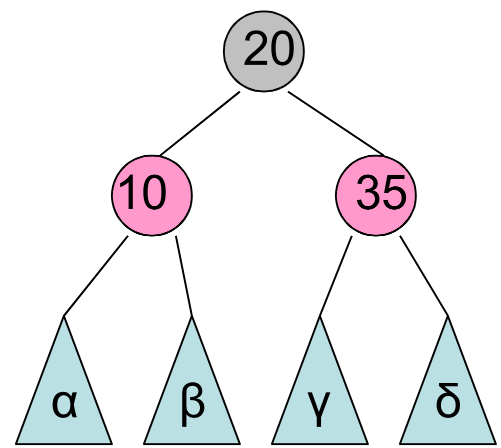
Red-Black Trees

- Motivation:
  - The good: 2-3 and 2-3-4 trees produce balanced trees of height bounded by log(n)
  - The bad: without extreme efforts in OO design 2-3 and 2-3-4 trees typically waste a lot of memory on unused items and children references.
    - 2-nodes are really 4-nodes with 2/3 of memory for that node being unused.

- Red-Black trees are conceptually identical to 2-3-4 trees.



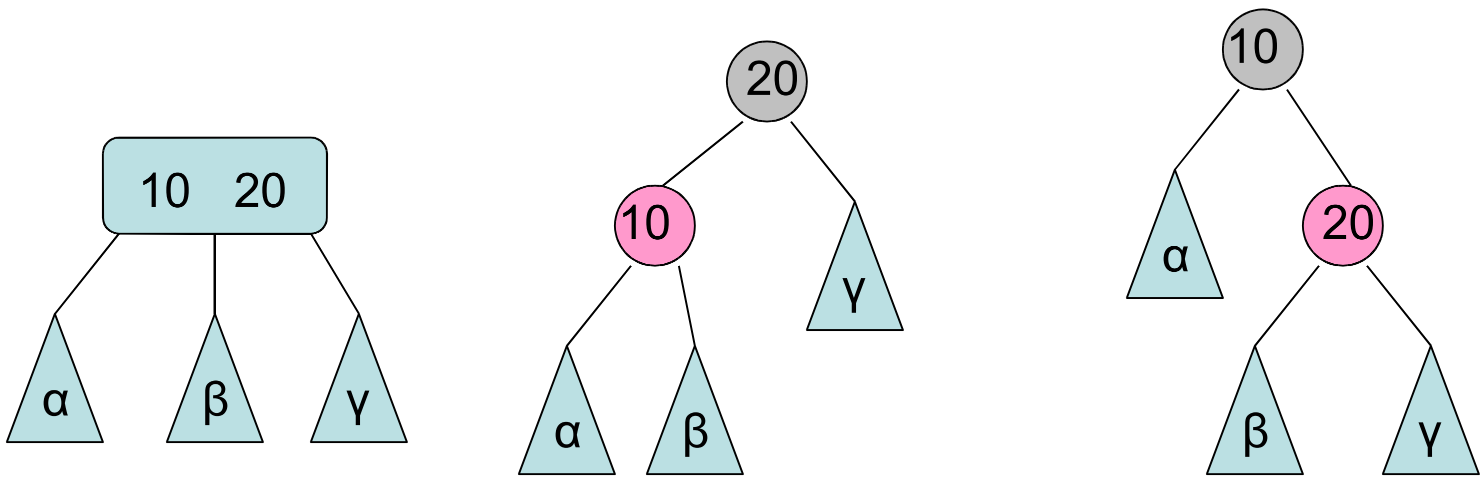
Here is a 4-node



"4-node" Represented as 2-nodes in a Red-Black Tree

- All nodes in a Red-Black tree are 2-nodes. Just like BSTs.
- 2-nodes are simply represented as they would be in a BST.
- 3-nodes and 4-nodes are represented by separate 2-nodes (one node for each item) and these nodes are “bound” to each other using “red” links

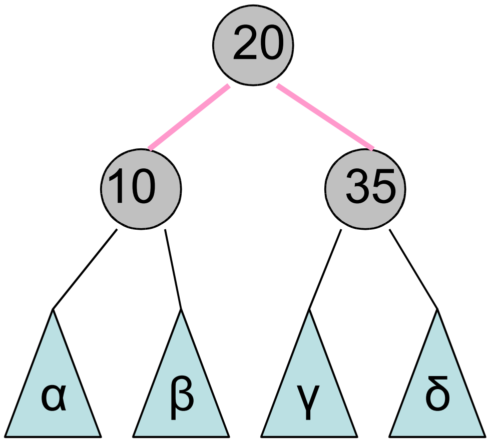
- Examples of 3-nodes…



Either way
is fine.

Implementation

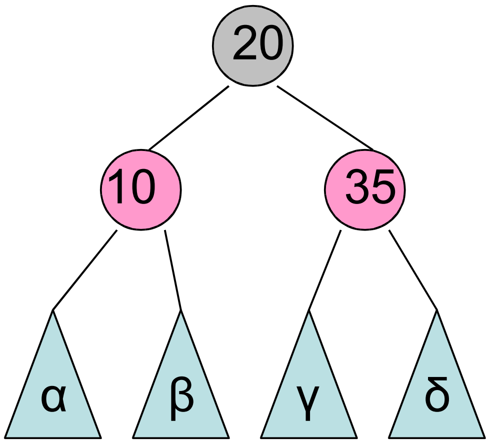
- Implementation of red "links"
- "links" or children references do not have the ability to contain information about their "color"
- The book would like to represent 4-nodes like the example given here:



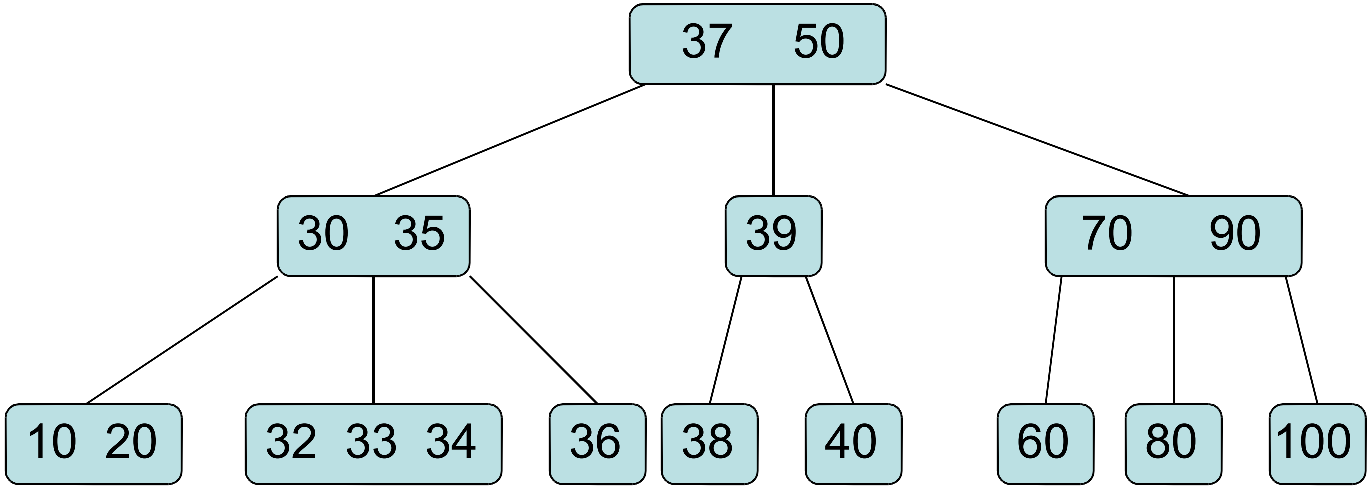
- But that isn't practical.

- Instead we represent red “links” by coloring the node the link references red.
- This can be done by simply adding a boolean to the BST Node class:

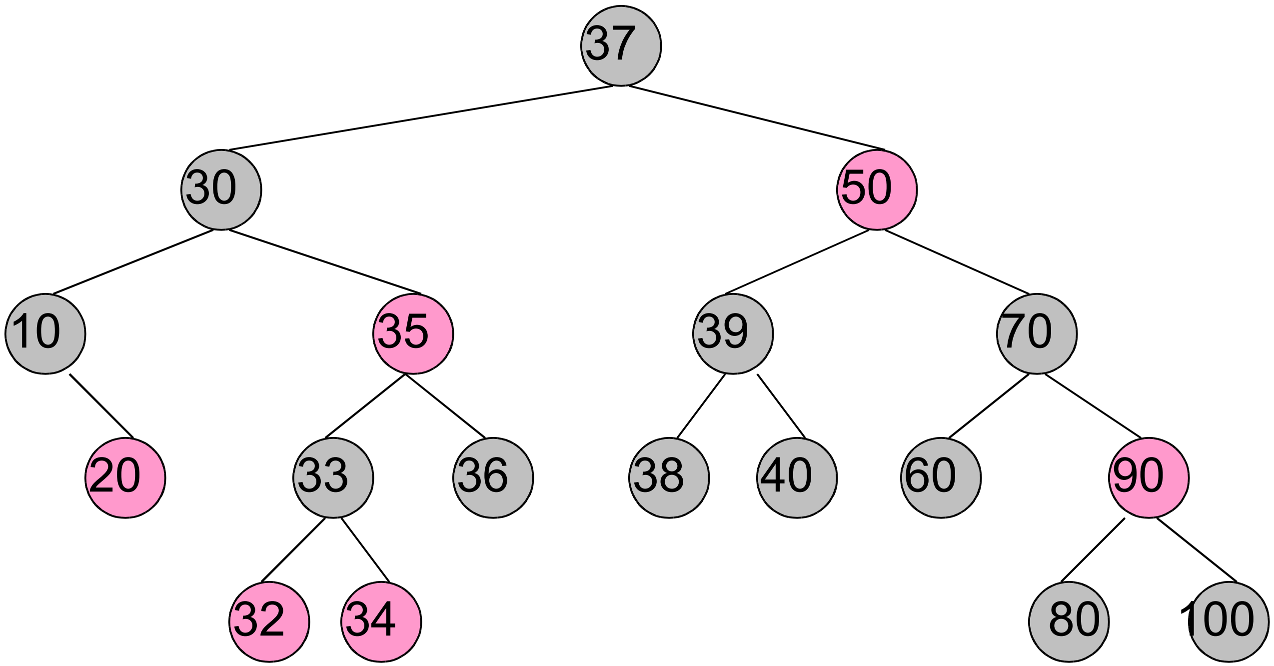
```
class Node
  {
        Object stored;
        Node left, right;
        boolean coloredRed;
  }
```



- If you take any 2-3-4 tree you can illustrate it as a Red-Black Tree…
- Figure 12-20 from the book as a 2-3-4 tree

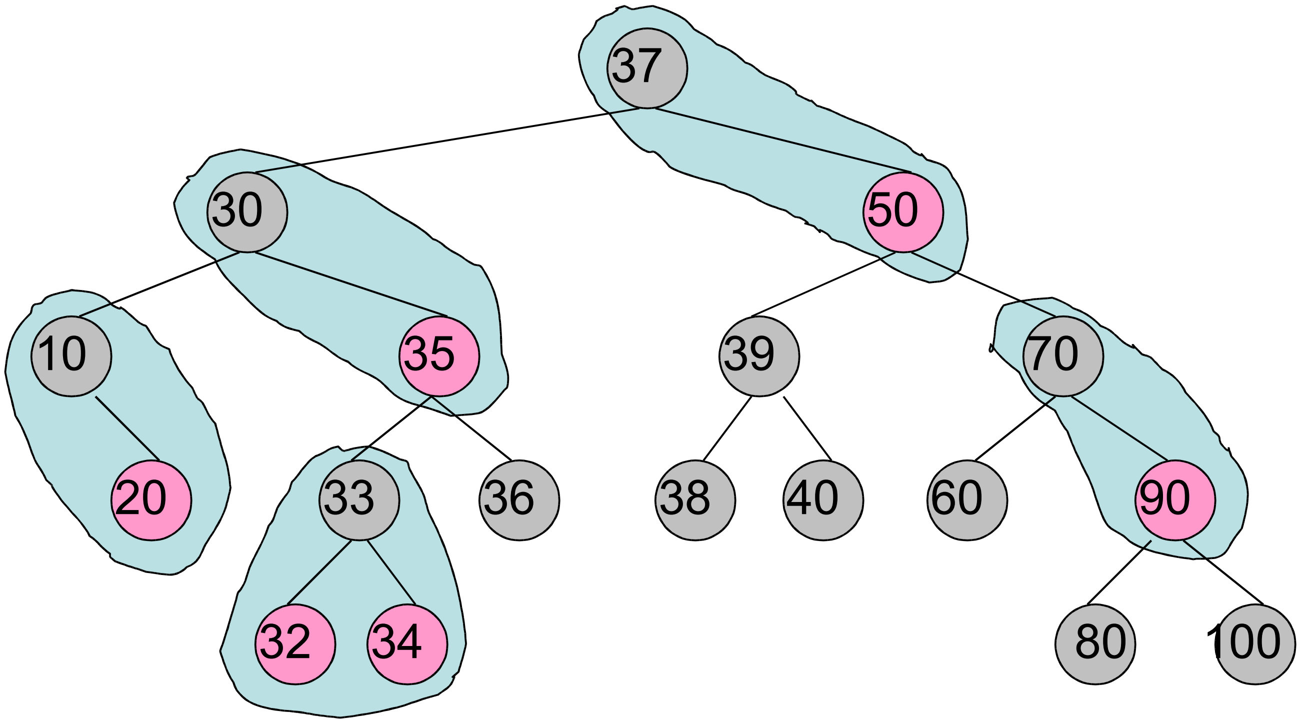


- Same tree stored as a Red-Black tree.



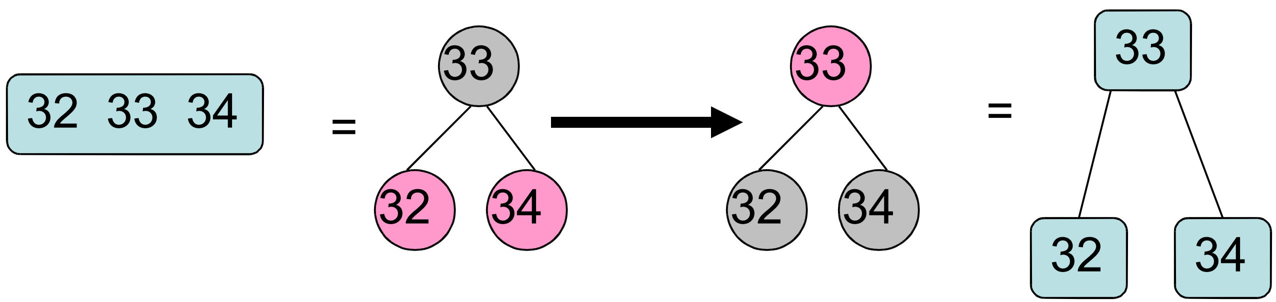
- There are 15 other valid variations.

- Why is it conceptually the same tree?



- We could then treat a Red-Black tree exactly like a 2-3-4 tree. We can tell if a node is really a 4-node by checking if both its left and right children are colored red.
- A node is a 3-node if its not a 4-node and has one red child.

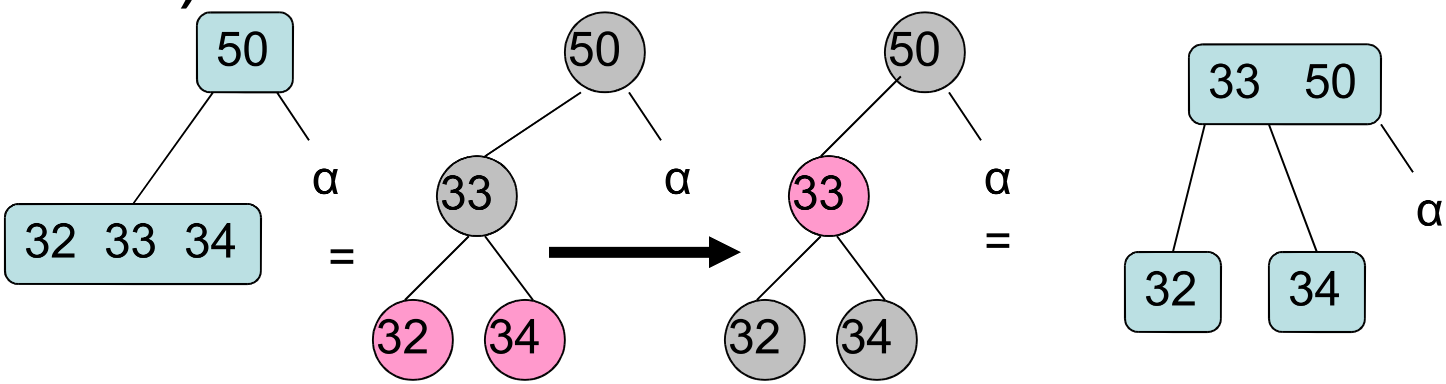
- During insertion 4-nodes get “split” by simply recoloring the nodes…



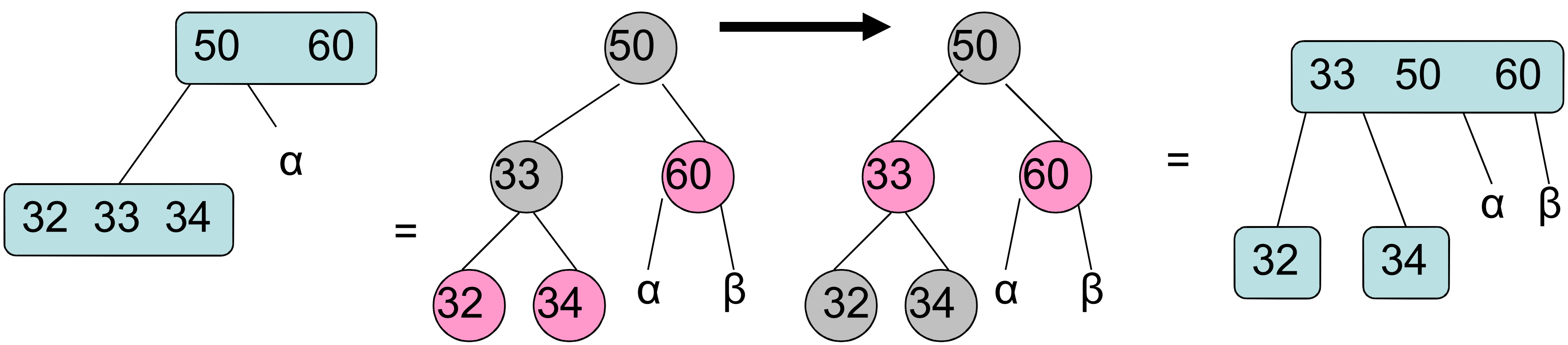
- Simply splitting a 4-node works to correct the root because the root node can simply be recolored black.

Splitting 4-nodes and adopting the middle.

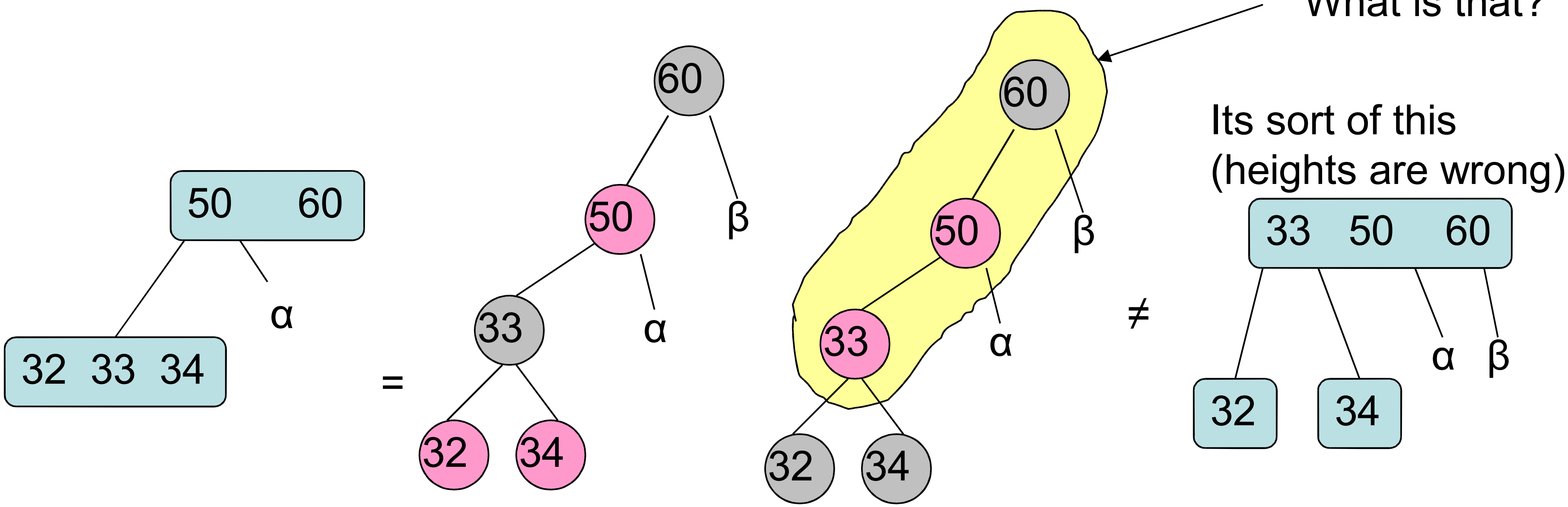
- Non-root nodes need to have the middle item "pushed" up to and included with the parent: (We can't just recolor the top node black.)



- Works great for 2-nodes as was shown.
- Also good for “easy” 3-nodes:

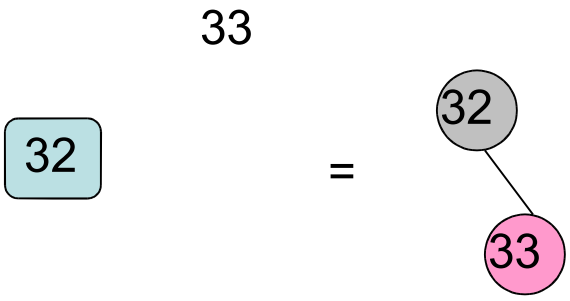


- Some three nodes are harder:



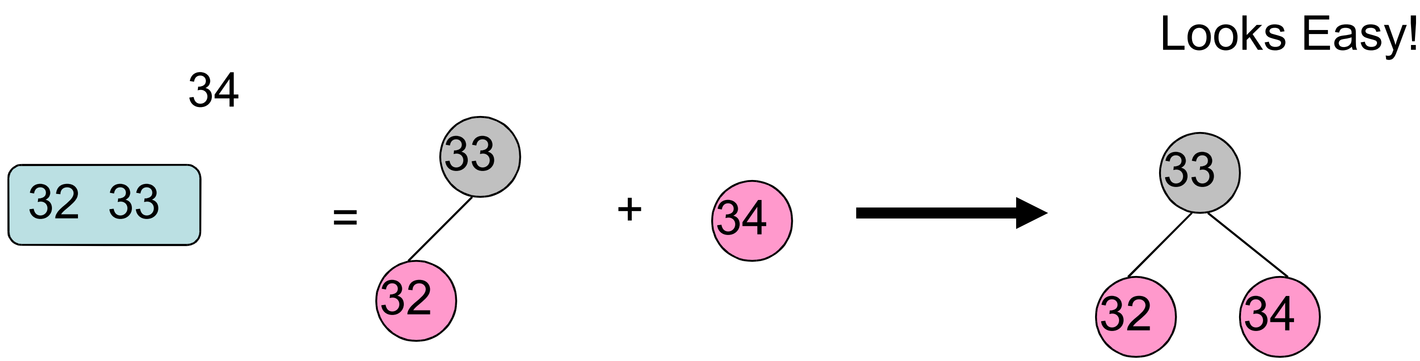
But that’s not a truly valid 4-node.
(Red nodes cannot have red children.)

- Adding an item to a 2-node makes it a three node…

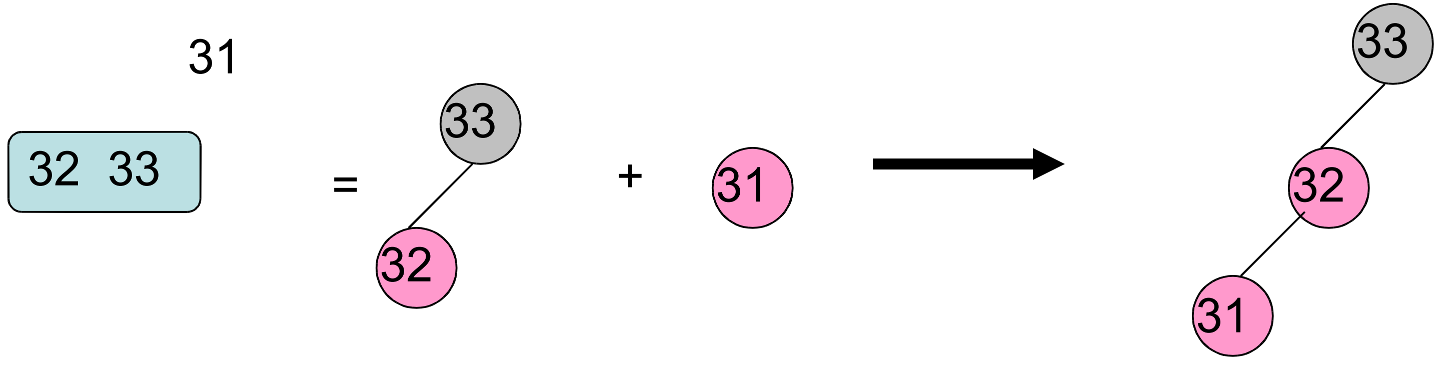


- It’s the same as adding a node to a BST tree. The node is added as a leaf.
- We start its color as red though.

- Adding a value to a 3-node to make it a 4-node isn't quite a simple.

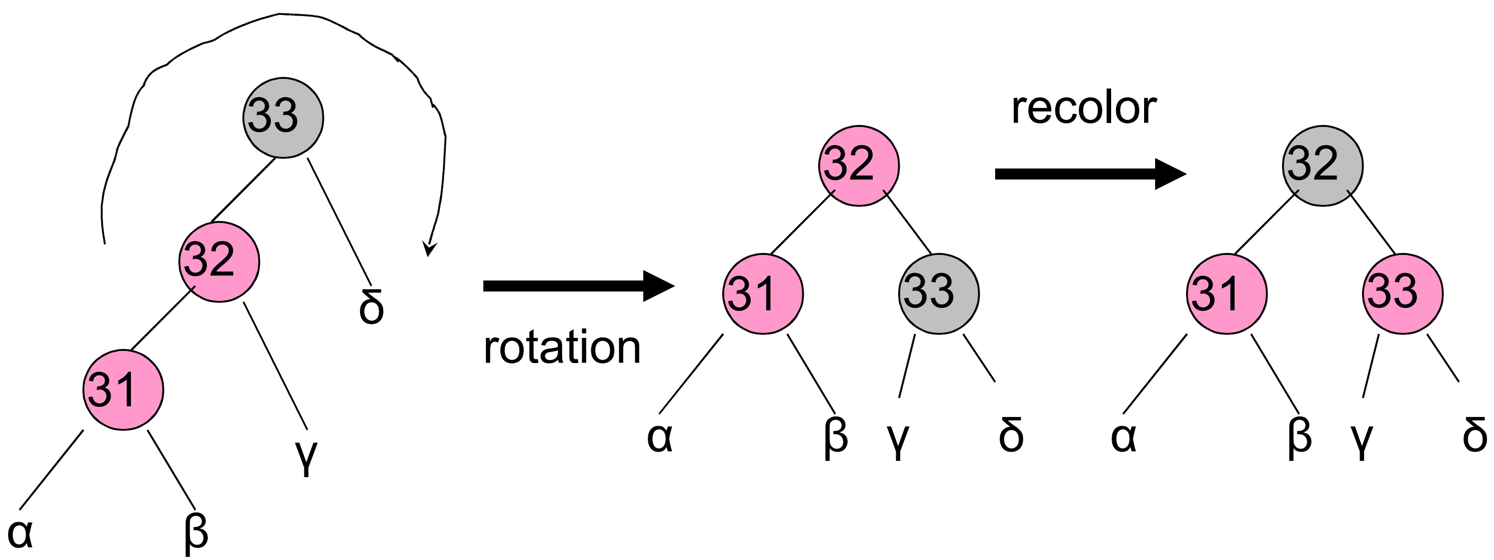


- But its not always that easy…

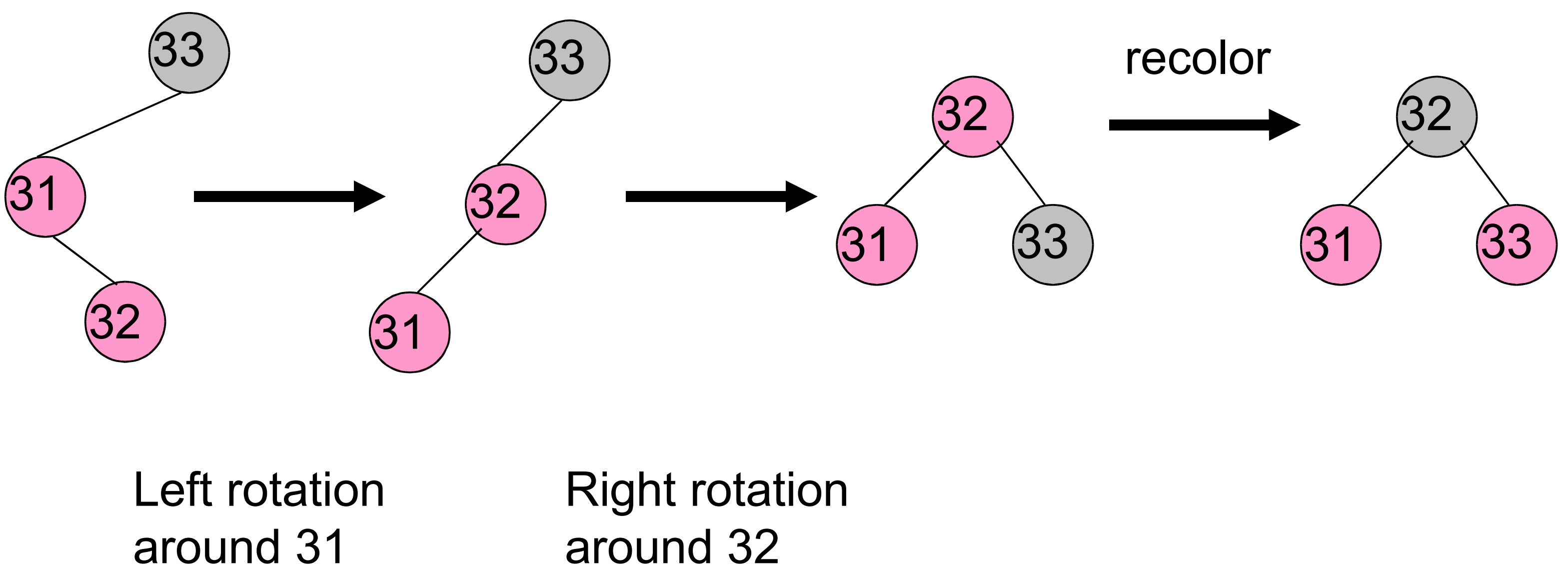


- What the heck kinda node is that?
- What we need to do is rotate this subtree.

- Right rotations:



- Here's a possible configuration that wants to be a four node but can't be right rotated to correction in one step:



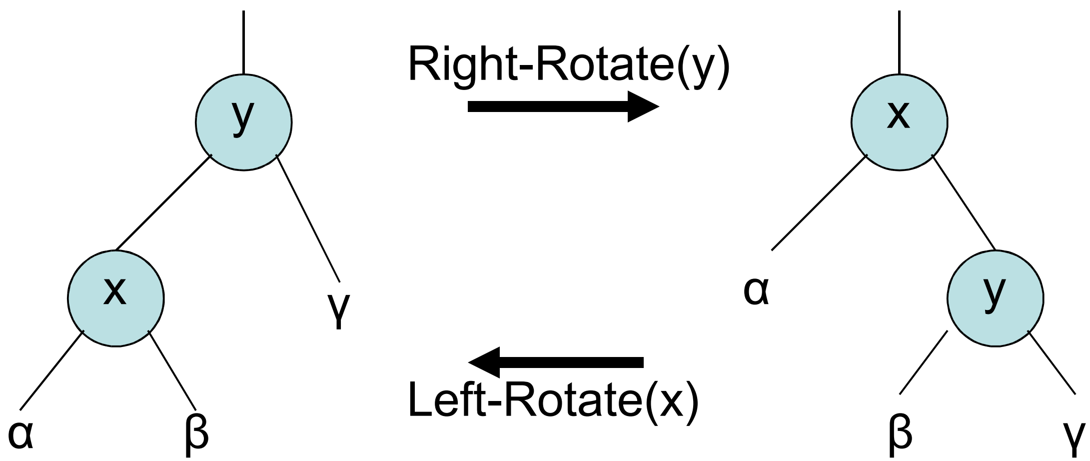
- Red-Black tree as a more formal definition (Warning: this is not in the text):
- We can abstract the idea of Red-Black trees away from conceptually thinking of them as 2-3-4 trees.
- This is done making a formal set of rules for the creation and maintenance of Red-Black Trees.

The Rules

- Every node is either red or black
- Every null reference is considered to be a link to a black node
- Red nodes must have ONLY black children
- Every simply path from the root to any leaf contains the same number of black nodes as any other such simple path.

Operations

- The balancing of Red-Black trees is done by maintaining the rules through the use of Left and right rotations and recoloring.



- Introduction to Algorithms (Cormen, Leiserson & Rivest) present a much more detailed analysis of Red-Black trees from this perspective.
- It is sufficient to implement a Red-Black tree by treating it exactly the same as a 2-3-4 tree taking into account the colors of node in determining whether you are about to step into a 4-node (for insertion) or a 2-node (for deletion).
- You also need to make sure that no red node has red children.