Advanced Data Structures

COMP282
Functional Dependencies

Definitions

- Some definitions:
  - relation: any two dimensional table
  - attribute: a column in a relation
  - tuple: a single row of a relation

Functional Dependency (FD)

- Given:
  - r(R): a relation
  - A: a set of attributes in r(R)
  - B: a set of attributes in r(R)
- `A → B:` iff for any tuples t1 and t2 t1(B)=t2(B) where t1(A)=t2(A).
- A → B is a functional dependency (FD)

An example

- Take for example:

Caspian Sea	Asia-Europe	143244	760
Superior	North America	31700	350
Victoria	Africa	26828	250
Aral Sea	Asia	24904	280
Huron	North America	23000	206
Michigan	North America	22300	307
Tanganyika	Africa	12700	420

- Name $\rightarrow$ Length
- Continent $\nrightarrow$ Name

FD Test Algorithm

- Step 1) Sort the tuples of r(R) on the A attributes
- Step 2) Check that tuples with equal A values also have equal B values
- If step 2 fails then the functional dependency (FD) is not satisfied; otherwise $A \rightarrow B$

Example

- Sort the example table:

Victoria	Africa	26828	250
Tanganyika	Africa	12700	420
Aral Sea	Asia	24904	280
Caspian Sea	Asia-Europe	143244	760
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- Now we can clearly see that Continent $\nrightarrow$ Name

Candidate Keys

- Candidate keys:
  - attribute set K is a candidate key of r(R) iff $K \rightarrow A_j$ where A_j is any attribute set in r(R)
- Primary Key: One of the Candidate keys selected.

Inference Axioms

- if $Y \subseteq X$, then $X \rightarrow Y$
- if $X \rightarrow Y$, then $XW \rightarrow Y$ and/or $XW \rightarrow YW$
- if $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- if $X \rightarrow Y$ and $YW \rightarrow Z$, then $XW \rightarrow Z$
- if $X \rightarrow Z$ and $X \rightarrow Y$, then $X \rightarrow YZ$
- if $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Closure Set

- Given F, a set of Function Dependencies the closure set (denoted F^+) is the set of all FDs that are logically implied by F and the inference axioms
- F^+ can be very large.

Equivalence Set

- Given two sets of FDs F and G.
- F and G are equivalent iff F^+ G^+