COMP 282
Advanced Data Structures

Lecture 07

Graphs

Shortest Path

(Dijkstra's Algorithm)

Shortest Path

- Dijkstra’s general algorithm computes a value w (weight) for each node v in a graph. W is the smallest cost possible to get to v from a particular vertex 0.
- If 0 is your starting point (city) then the weight w at any node v is the minimum distance required to get from the starting point (0) to city v.
- We need to also know how to get to v so we will eventually have to keep a bit more information at each node than just w so that the proper path information can be recovered as well.

Dijktra’s general

```
shortestPath(theGraph, weight)
  {
    mark[origin] = 1; // this tags vertex 0 as belonging to vertexSet.
    N = number of vertices in theGraph;
    for (v = 0 through n-1) {
      weight[v] = matrix[origin][v];
    }
    for (steps 2 through n) { // n-1 iterations
      // find the smallest weight[v] such that v is
      // not in not in vertexSet (mark[v]==0)

      mark[v] = 1;

      for (all vertices u not in vertexSet [mark[u]==0]) {
        if (weight[u] > weight[v] + matrix[v][u]) {
          weight[u] = weight[v] + matrix[v][u];
        }
      }
    }
  }
```

Keeping track of path info

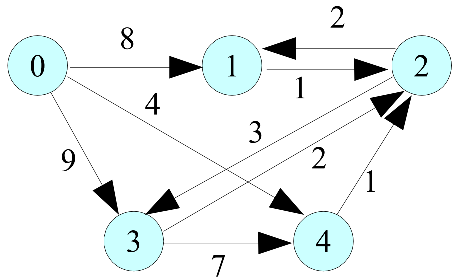
```
shortestPath(theGraph, weight)
    {
      mark[origin] = 1; // this tags vertex 0 as belonging to vertexSet.
      N = number of vertices in theGraph;
      previous[origin] = origin;
      for (v = 0 through n-1) {
        weight[v] = matrix[origin][v];
        previous[v] = origin; // we would get to v from the origin.
      }
      for (steps 2 through n) { // n-1 iterations
        // find the smallest weight[v] such that v is
        // not in not in vertexSet (mark[v]==0)

        mark[v] = 1;

        for (all vertices u not in vertexSet [mark[u]==0]) {
          if (weight[u] > weight[v] + matrix[v][u]) {
            weight[u] = weight[v] + matrix[v][u];
            previous[u] = v; // we would get to u from v
          }
        }
      }
    }
```

Generalized

```
shortestPath(origin, weight, previous)
  {
    int marked = 0;

    For (int i=0;i<numberOfNodes;i++) // estimate distances to all nodes as unreachable
      weight[i] = Infinity;

    weight[origin] = 0;   // by default it is zero units to reach the origin
    previous[origin] = origin;

    while (marked < numberOfNodes) { // n iterations
      v = smallestUnmarked(weight, mark);

      mark[v] = 1;

      for (int u=0;u<numberOfNodes;u++)
        if (mark[u]==0 && weight[u] > weight[v] + matrix[v][u]) {
          weight[u] = weight[v] + matrix[v][u];
          previous[u] = v; // we would get to u from v
        }
      }
    }
  }
```

Trace of Dijkstra's

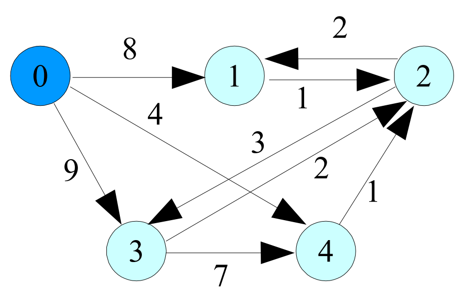
- First we set up the information so a start is possible:



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞

Trace: Step 1

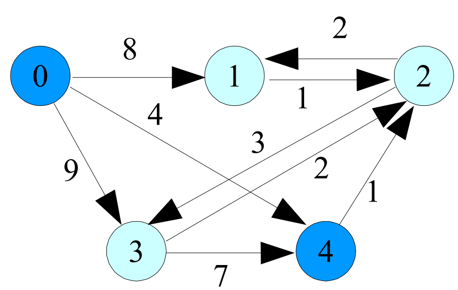
- Each step we select the vertex that has smallest distance estimate and is not marked.



	0	1	2	3	4
Start =	$0{:}_0$	∞	∞	∞	∞
0	$*{:}_0$	$8{:}_0$	∞	$9{:}_0$	$4{:}_0$

Trace: Step 2

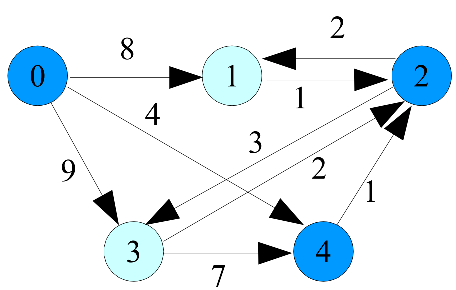
- On this pass the smallest is node 4 so it gets marked and any nodes that can be reached from node 4 get their distances updated if going through node 4 would yeild a shorter path.



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$

Trace: Step 3

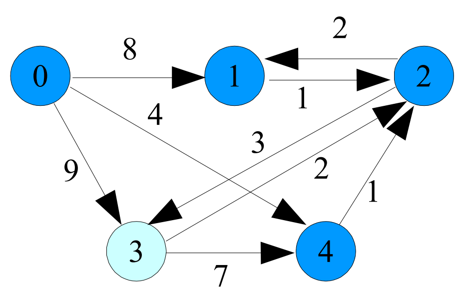
- node 2 is now smallest. If we go along the path “0, 4, 2, 1” it costs us 7 units of effort which is better than the previously though 8 that resulted from going along the path “0, 1” directly. So we update that estimate.



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$
2		$7:_2$	$*:_4$	$8:_2$	

Trace: Step 4

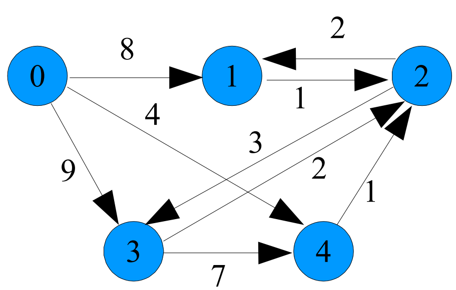
- If there is more than one minimum distance either one can be selected.
- Node 1 now has the minimum remaining estimate.



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$
2		$7:_2$	$*:_4$	$8:_2$	
1		$*:_2$		$8:_2$	

Trace: Results

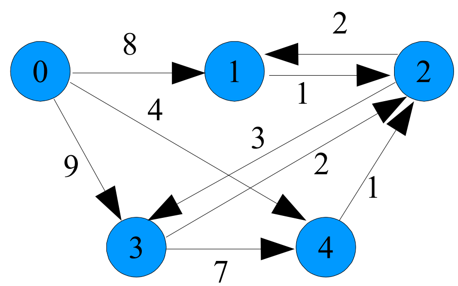
- Node 3 is last remaining unmarked node. So it gets marked and doesn't end of updating anything else



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$
2		$7:_2$	$*:_4$	$8:_2$	
1		$*:_2$		$8:_2$	
3				$*:_2$	

Path information

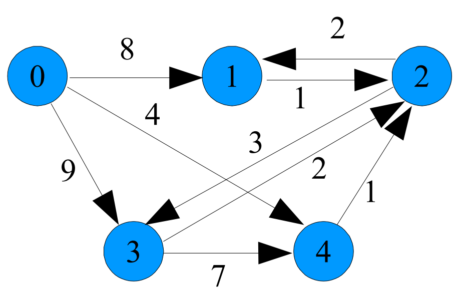
- The path information has been stored in an array or such and is shown by the subscripted values.
- The cell just above a star indicates the final results for that node:



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$
2		$7:_2$	$*:_4$	$8:_2$	
1		$*:_2$		$8:_2$	
3				$*:_2$	

Information

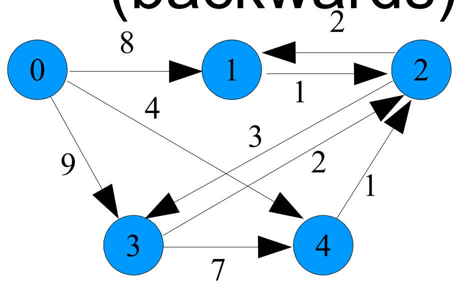
- For instance the shortest path to node 1 is 7 units in length. You would get to node 2 by going through node 4 first.



	0	1	2	3	4
Start =	$0:_0$	∞	∞	∞	∞
0	$*:_0$	$8:_0$	∞	$9:_0$	$4:_0$
4		$8:_0$	$5:_4$	$9:_0$	$*:_0$
2		$7:_2$	$*:_4$	$8:_2$	
1		$*:_2$		$8:_2$	
3				$*:_2$	

Path Recovery

- So what's the shortest path from node 0 to node 1?
  - To get to node 1 you come from 2
  - To get to node 2 you come from 4
  - To get to node 4 your come from 0. = “0, 4, 2, 1” (backwards)



	0	1	2	3	4
Start =	$0:_{0}$	∞	∞	∞	∞
0	$*:_{0}$	$8:_{0}$	∞	$9:_{0}$	$4:_{0}$
4		$8:_{0}$	$5:_{4}$	$9:_{0}$	$*:_{0}$
2		$7:_{2}$	$*:_{4}$	$8:_{2}$	
1		$*:_{2}$		$8:_{2}$	
3				$*:_{2}$	