Homework # 1

Linear Algebra I

Due: Fri. Sept. 14, 2018

Goals: (1) Review of linear algebra concepts, (2) Practice LaTeXtypesetting

Directions:

Solve the problems listed below.

Start by downloading the file `math382_hw1_yx.tex` and replace `yx` with your initials (last name initial first, `y`, first name initial last, `x`). Write the solution to the given problems using LaTeX syntax and process it to obtain the document `math382_hw1_yx.pdf` like this. Use canvas to submit your work.

You can work with others and discuss the problems, but each student must write his/her own, independent solution. If you are unsure what i mean by this, please ask!

Problem 1. Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, and $C \in \mathbb{R}^{p\times q}$. Show that $A(BC) = (AB)C$. *Hint:* Show that each element of $A(BC)$ is equal to the corresponding element of $(AB)C$, that is $[A(BC)]_{ij} = [(AB)C]_{ij}$, for $i = 1, \ldots, m, j = 1, \ldots, q$ (use sigma notation).

Problem 2. Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$. Show $(AB)^\top = B^\top A^\top$. *Hint:* Show that each element of $(AB)^\top$ is equal to the corresponding element of $B^\top A^\top$, that is $(AB)^\top_{ij} = (B^\top A^\top)_{ij}$, for $i = 1, \ldots, m, j = 1, \ldots, p$ (use sigma notation).

Problem 3. Consider the system $A\mathbf{x} = \mathbf{b}$ with:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 0 & \alpha \end{pmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

For what values of α does the system have

(a) no solution?

(b) one solution?

(c) infinitely many solutions?

In each case explain why.

Problem 4. Show that if $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ are solutions of the system $A\mathbf{x} = \mathbf{b}$ with $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^{m\times 1}$, so is $\mathbf{z} = \alpha\mathbf{x} + (1-\alpha)\mathbf{y}$ for any $\alpha \in \mathbb{R}$.

Problem 5. Show that for any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\|\mathbf{x}+\mathbf{y}\|_2 \leq \|\mathbf{x}\|_2 + \|\mathbf{y}\|_2$.