MATH 250 **Calculus III** **Spring 2008**

Review Sheet for 2nd Midterm Exam

HW#7: Turn in problems 1 (a), (c), (h), and (j); 5; 6; and 9. **Due:** Thurs. April 3

1. True or False. Determine whether the following statements are true or false and provide an explanation for your answer.

(a) If a function $f(x, y)$ satisfies $f(0, 0) = \frac{1}{2}$ and

$$\lim_{(x,y)\to(0,0)} f(x, y) = \frac{1}{2}$$

along the paths $L_1 : y = x$ and $L_2 : y = x^2$, then the function is continuous at $(0, 0)$.

(b) The function $f(x, y) = \frac{\cos x}{x^2+y^2+2}$ is continuous everywhere in $\mathbb{R}^2$.

(c) If the maximum value of the function $f(x, y)$ along the line $L : ax + by = c$ occurs at the point (x_0, y_0), then $\nabla f(x_0, y_0)$ is perpendicular to L.

(d) The function $f(x, y) = \frac{x^4+2y^2+3}{x^2+y^2-1}$ is continuous everywhere in $\mathbb{R}^2$.

(e) The function

$$f(x, y) = \begin{cases} \frac{x^2+\sin^2 y}{x^2+2y^2} & \text{if} \quad (x, y) \neq (0, 0) \\ 0 & \text{if} \quad (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$.

(f) The function $u(x, t) = e^{(x-t)^2}$ satisfies the partial differential equation $u_t + u_x = 0$

(g) If $f(x, y)$ has two local maxima, then it must have a local minimum.

(h) If $f(x, y) = \sin x + \sin y$, then $-\sqrt{2} \leq D_{\mathbf{u}} f(x, y) \leq \sqrt{2}$.

(i) If $f(x, y)$ has a local minumim at (a, b) and f is differentiable at (a, b), the $\nabla f(a, b) = \mathbf{0}$.

(j) If $f_x(a, b)$ and $f_y(a, b)$ both exist, then f is differentiable at (a, b).

2. Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{2x - y}{x + 3y}$$

doesn't exist.

3. (a) Find the equation of the level curves

$$f(x, y) = k,$$

of the function $f(x, y)$ whose gradient at the point (x, y) is equal to $\langle 2x, 2y\rangle$.

(b) Sketch the level curves.

(c) For what values of k do the level curves $f(x, y) = k$ exist?

4. Let C be the curve of intersection of the plane $y = -3$ and the surface $z = x^3 + y^3 - 9xy + 27$. Find the equations of the tangent line to C at $(0, -3, 0)$.

5. The surface of a volcanic mountain range is given by

$$z = f(x, y) = \begin{cases} 40 - x^2y^2 & \text{if } x^2y^2 < 40, \\ 0 & \text{if } x^2y^2 \geq 40. \end{cases}$$

If lava flows from an orifice located at the point $(1, 2, 36)$, then, assuming it travels down the range always flowing in the direction of steepest descent, find the coordinates $(x, y, 0)$ of the point where the lava first reaches the ground level.

6. Let

$$w(r, s) = f(x(r, s), y(r, s), z(r, s)),$$

where

$$x(r, s) = \frac{r}{s}, \quad y(r, s) = s - r, \quad z(r, s) = s^2.$$

Use the chain rule to estimate the value of $w(2.1, 0.9)$ given that

$$f(2, -1, 1) = 5 \quad \text{and} \quad \nabla f(2, -1, 1) = (1, 2, 3).$$

7. Find the directional derivative of $f(x, y) = x^2 - 2y^2$ at $(3, 3)$ in the direction of $\mathbf{u} = \langle 2, -1 \rangle$.

8. Find the equation of the tangent plane to the surface $z = e^x \cos y$ at $(1, 0, e)$.

9. Consider the function $f(x, y, z) = xy + 2x^2 - z^3$.

(a) Find its directional derivative at $(2, -3, -1)$ in the direction $\mathbf{u} = \langle 1, -2, 2 \rangle$

(b) In what direction does $f(x, y, z)$ increase most rapidly at $(2, -3, -1)$?

(c) Use the chain rule to find $\frac{df}{dt}$ if $x = 2t + 1$ and $y = 3t - 7$.

10. At which points is the tangent plane to the surface $5x^2 - 6xy + 5y^2 - 8x + 8y + z^2 - 3z = 0$ parallel to the xy-plane?

11. Determine the tangent plane and the normal line to the surface $xy + yz + xz - 1 = 0$ at $(3, -1, 2)$.

12. Find all the relative extrema and saddle points for $f(x, y) = x^2 - 4xy + y^3 + 4y$.

13. Find all the relative extrema and saddle points for $f(x, y) = \frac{1}{x} - \frac{1}{y} + xy$.

14. Find the maximum value of $f(x, y) = y^2 - 3x^2 + 2x$ if $x^2 + y^2 = 9$.

15. Find the largest and smallest values of $f(x, y) = 2x^2 + y^2 - y$ on the circle $x^2 + y^2 = 1$.

16. The function $f(x, y)$ at $(1, 2)$ has a directional derivative equal to 3 in the direction towards $(2, 2)$ and equal to -1 in the direction toward $(1, 1)$. What is its directional derivative in the direction toward $(2, 3)$?