Review Sheet for Midterm Exam I

Problem 1. Determine whether the following statements are True or False. Justify your answer.

(a) If $\lim_{x\to 1^+} f(x) = 2.1$, and $\lim_{x\to 1^-} f(x) = 1.9$, then $\lim_{x\to 1} f(x) = 2$.

(b) The equation $x^3 - 10x^2 + 5 = 0$ has at least one solution in the interval $(0, 2)$.

(c) If $f(1) = -1$ and $f(3) = 1$, then there exists a number c in the interval $(1, 3)$ such that $f(c) = 0$.

(d) The equation $4x^3 - 6x^2 - 5x + 3 = 0$ has at least one solution in the interval $(0, 2)$.

(e) If the line $x = 1$ is a vertical asymptote of the graph $y = f(x)$, then $f(x)$ is not defined at $x = 1$.

(f) The function $f(x) = \frac{x^2-2x+1}{x-1}$ has a removable singularity at $x = 1$.

(g) If $f(x)$ is continuous at $x = a$, then $f(x)$ is also differentiable at $x = a$.

(h) If $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists, then $f(x)$ is continuous at $x = a$.

(i) If $s(t)$ describes the path of a particle, then its derivative, $s'(t)$, represents the average velocity of the particle.

(j) If $\lim_{x\to 3} f(x)\, g(x)$ exists, then it must be $f(3)\, g(3)$

(k) $\lim_{x\to 2} \left(\frac{4}{x-2} - \frac{2x}{x-2}\right) = \lim_{x\to 2} \frac{4}{x-2} - \lim_{x\to 2} \frac{2x}{x-2}$

(l) If f and g are differentiable functions, then $\frac{d}{dx}\left[f(x)\, g(x)\right] = f'(x)\, g'(x)$

(m) If f and g are differentiable functions, then $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\, g'(x)$

(n) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

(o) If f is differentiable, then $\frac{d}{dx} f(\sqrt{x}) = \frac{f'(\sqrt{x})}{2\sqrt{x}}$

Problem 2. Find the limits

(a) $\lim_{x\to 4^+} \frac{4-x}{|4-x|}$

(b) $\lim_{x\to 1^+} \frac{x^2-9}{x^2+2x-3}$

(c) $\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2}$

(d) $\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{1}{x^2-3x+2}\right)$

(e) $\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}$

(f) $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$

Problem 3. Use the definition of continuity and the properties of limits to show that $f(t) = \frac{2t-3t^2}{1+t^3}$ is continuous at $t = 1$

Problem 4. Find the values of a and b so that the function is continuous everywhere

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if} \quad x < 2 \\ ax^2 + bx + 3 & \text{if} \quad 2 \leq x < 3 \\ 2x - a + b & \text{if} \quad x \geq 3 \end{cases}$$

Problem 5. Find an equation for the tangent line to the curve of $f(x) = \sqrt{1 + 4\sin x}$ at the point $(0, 1)$

Problem 6. Find the derivative of the following functions

(a) $f(x) = \tan^2(\sin x)$

(b) $f(x) = \frac{(y-1)^4}{(y^2+2y)^5}$

(c) $f(x) = \sqrt{\frac{x-1}{x+1}}$

(d) $f(x) = (x^2 + 1)\sqrt[3]{x^2 + 2}$

(e) $f(x) = \left[x + (x + \sin^2 x)^3\right]^4$

(f) $f(x) = x \sin \frac{1}{x}$

Problem 7. Problem # 76 on pg. 163 of the text.

Problem 8. Problem # 85 on pg. 163 of the text.

Problem 9. Problem # 88 on pg. 163 of the text.

Problem 10. Find all the points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

Problem 11. This is not midterm material, but you can try it! Find the value of x

$$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$$