MATH 150A **Calculus I** **Fall 2011**

Review Sheet for Midterm Exam II

Problem 1. Determine whether the following statements are True or False. Justify your answer.

(a) The curve described by $3x^2 + y^2 = 12$ has a vertical tangent at $x = 2$.

(b) If $f'(c) = 0$, then f has a local maximum/minimum at $x = c$.

(c) If f has an absolute maximum at $x = c$, then $f'(c) = 0$.

(d) If f and g are increasing on an interval I, then $f + g$ is also increasing on I.

(e) If f is odd, then f' is also odd.

(f) If f and g are increasing on an interval I, then $f - g$ is also increasing on I.

(g) If f and g are increasing on an interval I, then fg is also increasing on I.

(h) There exists a function $f(x)$ such that $f(1) = -2$, $f(3) = 0$, and $f'(x) > 1$ for all x.

(i) If f and g are positive increasing functions on an interval I, then fg is also increasing on I.

(j) If f is increasing and $f(x) > 0$ on I, then $g(x) = 1/f(x)$ is decreasing on I.

(k) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(l) The tangent line to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

(m) The sum of the x- and y-intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

Problem 2. As a spherical raindrop evaporates, its volume changes at a rate proportional to its surface area S.

(a) If the constant of proportionality is K, find the rate of change of the radius r when $r = 4$.

(b) Show that the rate of change of the radius is always constant.

(c) Does part **(b)** mean that the rate of change of the volume is always constant? Why or why not?

Problem 3. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the pint $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/s . How fast is the x-coordinate of the point changing at that instant?

Problem 4. Consider the function

$$f(x) = \frac{2x^2 - 5x + 5}{x - 2}.$$

(a) Find its domain and range

(b) Find its x- and y-axis intercepts

(c) determine whether the graph is symmetric with respect to the y-axis or the origin

(d) Find its asymptotes (horizontal, vertical, and oblique)

(e) Find critical points, relative extrema and intervals where the function is increasing/decreasing

(f) Find inflection points and intervals where the function is concave up/down

(g) Skech the graph $y = f(x)$, including all the features in parts **(a)** – **(f)** above.

Problem 5. Repeat problem 4 for the function

$$f(x) = \frac{x^2}{x^2 + 9}.$$

Problem 6. Let $g(x) = \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}$

(a) Compute $g'(x)$.

(b) Rewrite $g'(x)$ in terms of $\sin x$ and $\cos x$

(c) Use your answer in part **(b)** to rewrite $g(x)$ in terms of $\sin x$ and $\cos x$

Problem 7. Suppose that your foot length L is related to your height h in incles by $L = \frac{4}{3}\sqrt{h}$. In one (non-leap) year, you have a growth spurt in which you grow from 64 to 69 inches.

(a) Assuming your height is changing at a constant rate throughout the year, find an expression for $\frac{dL}{dt}$.

(b) What was the fastest rate of growth that your foot experienced during this time? When did it occur?

Problem 8. Problem # 34 on pg. 188.

Problem 9. Problem # 68 on pg. 260.

Problem 10. Problem # 72 on pg. 261.