1. What is $v_c(t)$ if the voltage source is turned on at $t=0$?

**Ans:** To begin any solution involving energy storage elements, we look at the initial conditions. We assume the switch has been open for a long time and there is no charge on the $5\text{F}$ capacitor. Therefore, $v_c(0) = 0$.

Once the switch is closed, we can write the nodal equation at the node between the resistor and the capacitor:

$$\frac{4 - v_c}{2} = 5 \frac{dv_c}{dt}$$

$$\Rightarrow A = \frac{v_c}{10} + \frac{dv_c}{dt}$$

This gives the characteristic equation and homogeneous solution:

$$0 = \frac{1}{10} + s$$

$$s = -\frac{1}{10}$$

$$v_h = Ae^{-t}$$

Dealing with the particular solution, the particular solution is in the same form as the forcing function, which is a constant in this case.

$$v_p = B$$

Substituting this equation for $v_c$ in the original equation gives us:

$$A = \frac{B}{10} + \frac{d}{dt}B$$

$$A = \frac{B}{10}$$

$$B = 4$$

Knowing that the total solution is the sum of the particular and homogeneous solutions:

$$v_c = 4 + Ae^{-\frac{t}{10}}$$

Applying the initial conditions of $v_c$: 
\[ v_c(0) = 0 = 4 + Ae^{-0/10} \]
\[ 0 = 4 + A \]
\[ A = -4 \]

Which gives the final solution as:
\[ v_c(t) = 4(1 - e^{-t/10}) \]

2. At what time will the voltage across the capacitor in Problem 1 reach 3V?
   Ans: Setting the equation from Problem 1 equal to 3 gives:
   \[ v_c(t) = 3 = 4(1 - e^{-t/10}) \]
   Solving for t gives:
   13.86s

3. What is \( v_c(t) \)? What is the time constant for this circuit?
   (Hint: Thevenize the resistors and voltage source.)
   Ans: Taking the hint, the Thevenin equivalent of the voltage source and resistors with the capacitor is:

The circuit can be analyzed as in Problem one. Note the initial condition is that \( v_c=0 \) because the 2 ohm resistor is across the capacitor. It would have discharged the capacitor.

The resulting equation (which also immediately gives the time constant) is:
\[ v_c(t) = 2(1 - e^{-t/2}) \]
\[ \tau = 2s \]

4. What is \( v_c(t) \)? What is the time constant? Be careful, the switch is opened at t=0.
Ans: With the switch open, this is a simple discharge circuit. The 1 ohm resistor is no longer a part of the circuit and the capacitor discharges through the 2 ohm resistor only. The key is the initial condition. If the switch has been closed for a long time, the capacitor does not affect the circuit. The 1 and 2 ohm resistors form a voltage divider with 2 volts across the 2 ohm resistor and, hence, across the capacitor.

We can write the nodal equation:

\[
0 = \frac{v_c}{2} + 3 \frac{dv_c}{dt}
\]

\[
0 = \frac{v_c}{6} + \frac{dv_c}{dt}
\]

A homogeneous differential equation with a characteristic equation of:

\[
0 = \frac{1}{6} + s
\]

\[
s = -\frac{1}{6}
\]

Which leads to the solution:

\[
v_c(t) = Ae^{-\frac{t}{6}}
\]

Applying the initial condition that \(v_c(0) = 2\) gives:

\[
v_c(t) = 2e^{-\frac{t}{6}}
\]

\[
\tau = 6s
\]

Which immediately gives the time constant as shown.

5. At t=0 the voltage source is turned off. What is \(v_o(t)\)?
Ans: To analyze the initial conditions here, we assume the capacitor has stabilized and its voltage is no longer changing. Therefore the current through it is zero, which means no current is flowing through the 3 ohm resistor. On the other side of the capacitor the two 4 ohm resistors form a voltage divider and 5 volts is present. So the voltage across the capacitor is 5 volts. With the voltage source turned off, the two 4 ohm resistors are now in parallel to form 2 ohms. This, with the 3 ohm resistor, makes 5 ohms total across the capacitor.

Based on an analysis similar to problem 4, we can write the voltage for the capacitor, $v_c(t)$, as:

$$v_c(t) = 5e^{-t}$$

Therefore the current flowing in the circuit is:

$$i(t) = \frac{v_c(t)}{5} = \frac{5e^{-t}}{5} = e^{-t}$$

Since the current is flowing counter-clockwise in the circuit (the left side of the capacitor was 5v and the right side was 0 at t=0), the output is:

$$v_o(t) = -3e^{-t}$$

6. If the source is turned on at t=0, what is $v_o(t)$?

Ans: Here the circuit is off before t=0 so the capacitor is discharged, i.e., $v_c(0)=0$.

The best approach here would then be to Thevenize the voltage source and resistors, determine the response of $v_c$ and then determine $v_o$ from that.

Thevenizing the source and resistors gives: $V_{th}=8V$ and $R_{th}=1.6\,\Omega$.

Using a similar analysis to problem 1 gives:

$$v_c(t) = 8(1 - e^{-\frac{t}{16}})$$

Looking back at the original circuit, we see that the two 4 ohm resistors form a voltage divider with $v_c$ as the input and $v_o$ as the output. Since the resistors are equal, the output voltage is half the input and:

$$v_o(t) = 4(1 - e^{-\frac{t}{16}})$$

Note: It is important to remember here that once the response of the capacitor was determined, the response of the remainder of the circuit could be determined as if the voltage across the
capacitor was a voltage source. This is because we have already taken the response of the rest of the circuit into account.