Homework Week 4 Solution

In all the circuits asking for the Thevenin or Norton Equivalent below, the equivalent should be of the circuit between terminals A and B.

1. What is the Thevenin equivalent of this circuit?

![Circuit Diagram]

Ans:

The simplest way to deal with this is by superposition.

With the voltage source on and the current source off (open circuit):

\[ V_{OC} = \frac{2(8/(1+8))}{1} = 1.7778V \]

\[ I_{SC} = 2/1 = 2A \]

With the current source on and the voltage source off (short circuit):

\[ V_{oc} = -0.5(1*8/(1+8)) \text{ (the current source times the parallel combination of the 1 and 8 ohm resistors)} \]

= -0.4444V

\[ I_{SC} = -0.5A \text{ (with the voltage source off and a short across A and B, the only current flowing is the current source out of the short circuit and back through the parallel combination of the 2 and 4 ohm resistors.)} \]

Therefore, the \( V_{OC} = 1.7778 - 0.4444 = 1.3333V = V_{TH} \)

\[ I_{SC} = 2A - 0.5A = 1.5A \]

\[ R_{TH} = 1.3333/1.5 = 0.89\Omega \]
2. What is the Thevenin equivalent of this circuit?

Ans: Note! Superposition will NOT work here because the second source is a dependent source.

$V_{oc}$ could best be found using nodal analysis, marking the nodes as shown. Notice $V_d$ at the junction of the dependent current source and the 10 ohm resistor.

Write the nodal equations as:

\[
\begin{align*}
\frac{V_a - V_b}{100} &= \frac{V_b - V_c}{100} \\
\frac{V_b - V_c}{100} &= \frac{V_c - V_d}{10}
\end{align*}
\]

We know that $V_d = -10V_g$ and $V_g = V_b$, therefore $V_d = -10V_b$

Also $V_a$ is obviously = 1V

Substituting this information into the above equations gives us the following system of equations:

\[
\begin{align*}
1 &= 2V_b - V_c \\
0 &= 99V_b + 11V_c
\end{align*}
\]

Solving for $V_c$ gives -0.8182V. This is the open circuit voltage and therefore also $V_{TH}$.

Next, place a short circuit across the A and B terminals.

Since the voltage across A and B = 0, the $V_g$ can be quickly calculated as the two 100 ohm resistors form a voltage divider. Since the resistors are equal, the voltage at $V_g$ is 0.5 times the source or 0.5V.

That determines the dependent voltage source voltage as -5V (or -$10V_g$).

The short circuit current, $I_{sc}$, is the sum of the currents flowing into the node formed by shorting A to B. This includes the current from the dependent source and the current from the independent source through the two 100 ohm resistors in series.

Current from dependent source:
Since the voltage at AB is 0, all of the voltage from the dependent source appears across the 10 ohm resistor. The current is:

\[ I_1 = \frac{-5V}{10} = -0.5 \ A \]

Note the minus sign!

Current from the independent voltage source is:

\[ I_2 = \frac{1V}{100 + 100} = 0.005 \ A \]

Their sum is \( I_{SC} \): \( I_{SC} = 0.005 - 0.5 = -0.495 \ A \)

\( R_{TH} \) is the open circuit voltage divided by the short circuit current:

\[ R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{-0.8182}{-0.495} = 1.65 \ \Omega \]

Notice that the minus signs cancel.

3. What is the Norton equivalent of this circuit?

Ans: Since there are only DEPENDENT sources in this circuit, we can determine immediately that \( I_N = 0 \). That is, unless some other independent source is connected to the circuit through the A and B terminals, the circuit will generate no currents or voltages.

To determine \( R_N \), we apply a test voltage across the A and B terminals. We connect a 1V source, with the positive terminal connected to the A terminal.

We then analyze the response of the circuit for the total current flowing into the circuit. This current, \( I_T \), is related to the Norton equivalent resistance by:

\[ R_n = \frac{V_{Test}}{I_T} = \frac{1}{I_T} \]
We can see the current, $I_C$, is made up of two currents: the current through the resistors and the current through the dependent current source.

The current through the resistors, $I_1$, is simply the test voltage divided by the series combination of the two resistors:

$$I_1 = \frac{1V}{(100 + 200)} = 0.00333 A$$

The current through the dependent source is controlled by $V_g$ which is across the 200 ohm resistor. The 100 ohm and 200 ohm resistors form a voltage divider and $V_g$ is given by:

$$V_g = \frac{1V(200)}{(100 + 200)} = 0.667V$$

Therefore the dependent source is $10V_g$ or 6.67A. This is also $I_2$ in our calculations. Therefore the total current through the circuit due to the test voltage is $I_1 + I_2 = 0.003333 + 6.67 = 6.67A$ (rounded up).

This gives $R_N$ as:

$$R_N = \frac{1}{6.67} = 0.15\Omega$$

4. What is the Norton equivalent of this circuit?

Ans: This circuit could be solved by superposition, provided the dependent source was left in place for both independent sources.

However, nodal analysis will be quicker. Marking the nodes as shown above, we use nodal analysis to give us $V_{OC}$, which is equal to $V_b$. 
The equations are:
\[ 0.6 = \frac{V_a}{80} + \frac{V_a - V_b}{10} \]
\[ V_b = 20 \]
\[ \frac{V_b - V_c}{100} = 0.1V_g + 0.6 + \frac{V_c - V_d}{50} \]
\[ V_g = V_a \]
\[ \frac{V_c - V_d}{50} = \frac{V_d}{10} \]

Solving for \( V_a \) gives \(-16.944\)V for \( V_{OC} \).

Placing a short across the A and B terminals makes \( V_a = 0 \) and \( I_{sc} \) would be given by
\[ I_{sc} = \frac{V_c}{50} \]

The nodal equations become (with \( V_a = 0 \)):
\[ 0.6 = \frac{V_a}{80} + \frac{V_a - V_b}{10} \]
\[ V_b = 20 \]
\[ \frac{V_b - V_c}{100} = 0.1V_g + 0.6 + \frac{V_c}{50} \]
\[ V_g = V_a \]

\( V_c \) is solved for as \(-90.37V\), making \( I_{sc} = -1.808A \). This is also \( I_N \) and would be drawn with the arrow pointing down.

Together with the \( V_{OC} \) obtained above, \( R_N = -16.944/-1.808 = 9.37\Omega \).

5. This is a model of a transistor amplifier. What \( R_L \) will receive the most power from it? What is that power in terms of \( V_s \)?

\[ \text{Ans: This is a maximum power problem so we need to find } V_{TH} \text{ and } R_{TH}. \]
\[ \text{Nodal analysis would be best here.} \]
We ignore $R_L$ for the moment since we are trying to find the Thevenin equivalent of the circuit to the left of the terminals.

We mark the nodes as shown above and write the nodal equations:

$$ V_a - V_b = \frac{V_b}{100} + \frac{V_b - V_c}{4700} $$

$$ V_b - V_c = \frac{4700}{4700} = 100I_b $$

We then add the other information that the circuit provides us to define the dependent current source in terms of the nodal voltages and add in any known independent sources:

$$ V_a = V_S $$

$$ I_b = \frac{V_b}{1000} $$

Substituting these values into the equations and simplifying gives us:

$$ V_S = 1.121V_b - 0.0213V_c $$

$$ 0 = 469V_b + V_c $$

These give $V_c$ which is also $V_{oc}$ as $-42.21V_s$.

To get $I_{oc}$ now, we place a short across the two terminals and analyze for $I$ through that short. We can write a single equation to define $V_b$ in terms of $V_S$ based on the first nodal equation above with $V_c$ now set to 0, since it is shorted to the reference node:

$$ V_a - V_b = \frac{V_b}{100} + \frac{V_b}{4700} $$

We also know the relations for the dependent current source and the knowns still hold. Solving for $V_b$ in the equation above gives us:

$$ V_b = \frac{V_S}{1.1213} $$

The short circuit current, $I_{sc}$, is made up of the current from $V_b$ through the 4700 ohm resistor AND the current from the dependent current source.

$$ I_{sc} = \frac{V_b}{4700} - \frac{100V_b}{1000} $$

$$ = \frac{V_S}{1.1213(4700)} - \frac{V_S}{1.1213(10)} $$

$$ = -0.0894V_S $$

With the $V_{oc}$ from above:

$$ V_{TH} = V_{oc} = -42.21V_S $$

$$ R_{TH} = \frac{V_{sc}}{I_{sc}} = \frac{-42.21V_S}{-0.0894V_S} = 472.3\Omega $$
Based on the maximum power theorem, $R_{TH}$ is also the load that will absorb the most power from the circuit and that power is:

$$P_{\text{max}} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(-42.21V_s)^2}{4(472.3)} = 0.943V_s^2$$

6. What $R_L$ will receive the most power from this bridge circuit? What is that power in terms of $V_s$?

This circuit can best be analyzed by “splitting” the $V_s$ into two copies of itself and ignoring the $R_L$ for the moment. We can analyze the two voltage dividers formed by replacing them with their Thevenin equivalent circuits as shown below:

In both cases, the Thevenin equivalent resistance is the parallel combination of the resistors in each voltage divider.
\[ R_{TH1} = \frac{100(200)}{100 + 200} = 66.67 \Omega \]
\[ R_{TH2} = \frac{100(200)}{100 + 200} = 66.67 \Omega \]

*The Thevenin equivalent voltages are the voltage divider voltages:*

\[ V_{TH1} = V_s \frac{200}{100 + 200} = 0.667V_s \]
\[ V_{TH2} = V_s \frac{100}{100 + 200} = 0.333V_s \]

*The total Thevenin equivalent voltage is the sum of the equivalent sources, but note the polarity of the voltages:*

\[ V_{TH} = V_{TH1} - V_{TH2} = 0.667V_s - 0.333V_s = 0.333V_s \]

*Note: we have chosen the polarity of the equivalent voltage arbitrarily. It would be the same value but a negative if we decided we wanted the plus terminal on the right. It does NOT affect the power, though.*

*The total equivalent resistance is the sum of the two equivalent resistances:*

\[ R_{TH} = R_{TH1} + R_{TH2} = 66.67 + 66.67 = 133.3 \Omega \]

*Using the maximum power theorem, we can immediately determine the \( R_L \) that will absorb the maximum power as 133.3 \( \Omega \).*

*That power is:*

\[ P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(0.333V_s)^2}{4(133.33)} = 2.0833E^4V_s^2 \]
7. Your boss brings you another black box. It has two terminals. She wants you to model it. You may not short out the two terminals. In fact, you may draw or input no more than 0.1A in or out of them.

In the lab you measure the open circuit voltage as 1.5V. You place a 100Ω resistor across the terminals and the voltage across the resistor is 1.3V. What is the Thevenin equivalent circuit of the box? What is the Norton equivalent circuit?

V_{TH} = 1.5V \quad R_{TH} = 15.38Ω

I_N = 0.098A (pointing up) \quad R_N = 15.38Ω

Ans: The open circuit voltage gives us the Thevenin equivalent voltage immediately: 1.5V. With the 100 ohm test load, the circuit looks like:

The current through the 100 ohm resistor also flows through R_{TH}. We know 1.3V are across the 100 ohm resistor and therefore 0.2V are across R_{TH}. This gives us R_{TH}:

I = \frac{1.3}{100} = 0.02 = \frac{0.2}{R_{TH}}

R_{TH} = \frac{100(0.2)}{1.3} = 15.38Ω

Together with the V_{TH} this leads to I_N:

I_N = \frac{V_{TH}}{R_{TH}} = \frac{1.5}{15.38} = 0.098A

And R_N = R_{TH}

8. You want to use power source A and power source B to deliver power to a load. Each source’s Thevenin equivalent is shown below. What is the Thevenin equivalent circuit of A and B in parallel?

V_{TH} = 12.06V \quad R_{TH} = 1.2Ω

V_{TH} = 12.1V \quad R_{TH} = 1.3Ω

V_{TH} = 12.0V \quad R_{TH} = 1.2Ω

V_{TH} = 12.06V \quad R_{TH} = 1.2Ω
Ans: By connecting the two sources in parallel, we see a current must flow from the higher 12.1V source, through the equivalent resistances to the 12.0V source. This current is given by the differences in the voltage between the sources divided by the total resistance:

\[ I = \frac{12.1 - 12.0}{(2 + 3)} = \frac{0.1}{5} = 0.02\,A \]

The open circuit voltage would be given by calculating the drop across the 3 ohm resistor due to this current and adding it to the 12.0V to give the voltage at the terminals:

\[ V_\infty = V_{TH} = 12.0 + 0.02\,A(3\Omega) = 12.06\,V \]

The Thevenin equivalent resistance would be given by turning off the 12.1V and 12.0V source leaving the 2 and 3 ohm resistors in parallel. This gives:

\[ R_{TH} = \frac{2(3)}{(2 + 3)} = \frac{6}{5} = 1.2\,\Omega \]

9. Try Problem 8 by converting each source to its Norton equivalent and paralleling them. Then convert back to Thevenin. Is this easier?

Ans: Converting each source in turn gives:

\[ I_{N1} = \frac{V_{TH1}}{R_{TH1}} = \frac{12.1}{2} = 6.05\,A \]

\[ I_{N2} = \frac{V_{TH2}}{R_{TH2}} = \frac{12.0}{3} = 4\,A \]

The Norton resistances would be the same as the Thevenin resistances.

Putting the sources in parallel makes the output current the sum of the two currents: 4+6.05=10.05A

The resistances are in parallel as well and give the same result as above: 1.2\,\Omega.

To convert back to Thevenin:

\[ V_{TH} = I_N R_N = 10.05(1.2) = 12.06\,V \]

\[ R_{TH} \text{ is the same as } R_N: 1.2\,\Omega \]

This way seems simpler, although it does involve the conversions.

10. You have an ideal source (R_{TH} = 0) of 100V in the lab. Design a voltage divider to reduce to voltage to 25V. The Thevenin equivalent circuit should look like this:
How much power does the voltage divider absorb?

Ans: The circuit would have to look as below:

![Circuit Diagram]

We solve this knowing the voltage divider equation:

\[ 25V = 100V \frac{R_2}{(R_1 + R_2)} \]

And the Thevenin equivalent resistance constraint gives the other equation we need:

\[ R_{TH} = 75 \Omega = \frac{R_1 R_2}{(R_1 + R_2)} \]

Re-writing the first equation:

\[ \frac{25V}{100V} = \frac{R_2}{(R_1 + R_2)} = 0.25 \]

And substituting into the second:

\[ 75 = 0.25 R_1 \]

This gives \( R_1 \) immediately as 300\( \Omega \). Putting this back into the first or second equation gives \( R_2 = 100 \Omega \).

The power absorbed by the divider circuit is simply the square of the voltage across the entire resistance, 100V divided by the resistance:

\[ P = \frac{100^2}{400} = 25W \]
11. Use any method(s) that you have learned so far to determine the Norton equivalent of this somewhat complex circuit:

Ans: This would be best solved as a mesh analysis, since, as we will see it becomes one super mesh:

Because of the dependent current source and the independent source, we have to make this one super mesh.

Starting on the lower left, the mesh equation is:

\[10I_1 + 10 + 10I_3 + 30I_3 + 2I_3 + 5I_2 = 0\]
\[10I_1 + 5I_2 + 42I_3 = -10\]

We then write the relations give to us for the dependent source:
\[ I_1 - I_2 = 0.02V_g \]
\[ V_g = 30I_3 \]
\[ I_1 - I_2 = 0.6I_3 \]
\[ I_3 - I_2 = 0.5A \]

This last equation together with the mesh equation gives us \( I_3 = -0.0396 \), and \( V_{oc} = 10I_3 \):

\[ V_{oc} = -0.396V \]

To get the short circuit current, we place a short across A and B and re-write the super mesh equation:

\[ 10I_1 + 10 + 30I_3 + 2I_3 + 5I_2 = 0 \]
\[ 10I_1 + 5I_2 + 32I_3 = -10 \]

The other relationships from above still hold and we solve for \( I_3 \) which is also \( I_{sc} = -0.0472A \)

This with the \( V_{oc} \) from above gives us \( I_N = -0.0472A \) (pointing to B) and \( R_N = V_{oc}/I_{sc} = 8.4\Omega \).