Homework 2

1. Simplify this circuit to find the total power absorbed by all resistors.

Ans: Use series and parallel combinations to simplify circuit.

\[ R_{1}+R_{2}= 47+ 47 =94 \Omega \text{ and this resistance is in parallel with } R_{4}: \]

\[ R_{eq} = \frac{94 \bullet 100}{(94 + 100)} = 48.45 \Omega \]

This is in series with \( R_{1} \) so the total resistance across the 6V source is \( 10+48.45=58.45 \Omega \).

\[ P_{\text{TOTAL}} = \frac{6^{2}}{58.45} = 0.616 \text{W} \]

2. Simplify this circuit to find what fraction of \( V_{s} \) appears across the 180 ohm resistor as \( V_{o} \).

Ans: The circuit can be analyzed as a voltage divider with upper part formed by the sum of \( R_{1} \) and \( R_{2} \) in series = \( R_{o} = 33+47=80 \Omega \). The other leg is formed by the combination of \( R_{3}, R_{4}, R_{5} \) and \( R_{6} \), which can be done as follows:

\( R_{4} \) and \( R_{5} \) in parallel: \( 470 \times 470/(470+470)=470/2 = 235 \Omega \). This resistance is in series with \( R_{3} \), so they add: \( 235+47=282 \Omega \). This resistance is in parallel with \( R_{6} \), so the equivalent resistance for the lower leg of the divider, \( R_{o}=282 \times 180/(282+180)=109.9 \Omega \)

Now, \( R_{o} \) and \( R_{6} \) form a voltage divider. The fraction of \( V_{s} \) across \( R_{o} \) and, hence, across \( R_{6} \) (since it is the resistance between the same two nodes as \( R_{6} \)), is:

\[ 109.9/(109.9 +80)=0.579V. \]
3. You have a current sensor that has an internal resistance of 10Ω. It can handle a maximum current of 1A, but you want to measure currents up to 100A. What value of Rs would allow this? What is the maximum power it may have to absorb. (By the way, this is often done and Rs is called a “shunt”.)

Ans: Use current division: The maximum current through the sensor is:

\[
I_{\text{SENSOR}} = 100A \left(1 - \frac{R_S}{10 + R_S}\right)
\]

Solving for Rs when \(I_{\text{SENSOR}} = 1A\):

\[
1 = \frac{100(R_S)}{10 + R_S}
\]

\[
10 + R_S = 100R_S
\]

\[
10 = 99R_S
\]

\[
0.101\Omega = R_S
\]

If 1A is flowing through the sensor, then 99A must be flowing through Rs when the total current is 100A.

\[
P_{R_S} = I_{R_S}^2 R_S
\]

\[
= 99^2 (0.101) = 989.9W
\]

Note: The shunt would get VERY hot.

4. You have a voltage sensor that measures up to 10V, but you need to measure the voltage between a wire and ground that can reach up to 1000V (1KV). The sensor has an internal resistance of 1000Ω. What value of Rm would you use? What is the maximum power it may have to absorb? (Again, this is often done. Rm is referred to as a “multiplier”.)

Ans: This is a voltage divider problem.
\[ 10V = 1000V \left( \frac{1000}{R_m + 1000} \right) \]

\[ .01 = \frac{1000}{R_m + 1000} \]

\[ .01R_m + 10 = 1000 \]

\[ .01R_m = 990 \]

\[ R_m = \frac{990}{.01} = 99000\Omega \text{ or } 99K\Omega \]

\[ P_{R_m} = \frac{V_{R_m}^2}{R_m} = \frac{(1000 - 10)^2}{99000} = 9.9W \]

This resistor would also get somewhat hot.

5. You need to reduce the 1000V in the problem above to 100V for a lab experiment. What circuit would you use and with what values? Whatever circuit you use may not absorb more than 1 watt.

Ans: This calls for a voltage divider of 0.1, but also the constraint that the total resistance can absorb no more than 1W. We will use this constraint first to calculate the total resistance of the divider, \( R_T \).

Remember the entire 1000V appears across this resistance.

\[ P_{\text{divider}} = \frac{V^2}{R_T} \]

\[ 1W = \frac{1000^2}{R_T} \]

\[ R_T = \frac{1000000}{1} = 1000000\Omega \text{ or } 1\text{megohm (1 M\Omega)} \]

This gives us the total resistance for the formula for the voltage divider:

\[ V_{\text{divider}} = V \left( \frac{R_b}{(R_a + R_b)} \right) = V \left( \frac{R_b}{R_T} \right) \]

\[ 100 = 1000 \left( \frac{R_b}{1M\Omega} \right) \]

\[ .1(1M\Omega) = R_b = 100000\Omega \text{ or } 100K\Omega \]

This means \( R_a \) is 1000000-100000=900000\Omega or 900K\Omega
6. In the lab, you have more problems. The experiment calls for a 520 ohm resistor in some part of the circuit, but all you have are boxes of 470 and 100 ohm resistors. How can you make a 520 ohm resistor with a combination of these values?

Ans: Try two 100 ohm resistors in parallel (50 ohms) and that in series with a 470: 470+50=520.

7. Below is a schematic of a circuit someone else designed. The designer got sick before finishing his design (It happens!). How many watts does R4 absorb? Resistors come in sizes able to absorb 1/8, 1/4, and 1/2 watt. What size would you use? The next day they tell you Vs is now 5V. Can you tell them how much R4 has to absorb without doing all the nodal analysis again? How much must it absorb? Can you use the same size resistor (in wattage) as before? If not, what size?

Ans: Using nodal analysis with the nodes marked as shown:

The node equations are:

\[ V_a = 3 \]
\[ \frac{V_a - V_b}{3.3} = \frac{V_b - V_c}{10} + \frac{V_c}{4.7} \]
\[ V_b - V_c = \frac{V_c}{5} \]
Which yield $V_c = 0.9259V$. This means $R_4$ will absorb $V_c^2/5$ watts or $0.171W$. A 1/4watt resistor should handle this nicely. When they change the specification to a voltage source of 5V, you can use the property of linearity to determine the new value of $V_c$:

$$V_{c(5V)} = V_{c(3V)} \left( \frac{5}{3} \right) = 0.9259(1.667) = 1.543V$$

This leads to a new power for $R_4$ of: $(1.543)^2/5=0.476W$. The resistor would have to be changed to at least a 1/2watt. However, good engineering practice dictates it should be twice that or 1W.

8. This is the circuit model of an actual transistor amplifier (really!). The gain of the amplifier is $\frac{V_o}{V_s}$. What is the gain of the amplifier? Note the direction of $I_b$ and the dependent current source.

Ans: Using nodal analysis and naming the center node voltage $V_b$:

$$\frac{V_b}{100} = I_b + 100I_b = 101I_b$$

$$I_b = \frac{V_s - V_b}{1000}$$

$$\frac{V_b}{100} = 101 \left( \frac{V_s - V_b}{1000} \right)$$

$$V_b = \frac{101}{10} (V_s - V_b)$$

$$V_b = 10.1V_s - 10.1V_b$$

$$11V_b = 10.1V_s$$

$$V_b = 0.91V_s$$

Substituting this into the first equation to get $I_b$: 

$$\frac{V_b}{100} = 101I_b$$

$$I_b = \frac{V_b}{101}$$
0.91V_s = 101I_b
0.91V_s = 10100
9.01E^{-3}V_s = I_b

And this goes into the equation for Vo in terms of the dependent current source:
\[ V_o = -100I_b \times (4700) \]
\[ = -42.35V_s \]

So the gain of the circuit is -42.35. The minus sign indicates it will invert the signal.

9. What is Vc in this circuit?

Ans: Using nodal analysis and the nodes as marked:

We can write the following node equations:
10mA = \frac{V_a}{50} + \frac{V_b}{100} + \frac{V_b - V_c}{50}

\frac{V_b - V_c}{50} = \frac{V_c}{82}

Noting that \( V_a \) and \( V_b \) form a super node. The voltage source gives us one more piece of information:

\[ V_a = V_b + 0.5 \]

This can be substituted into the first equation to eliminate \( V_a \):

\[ 10mA = \frac{V_b + 0.5}{50} + \frac{V_b}{100} + \frac{V_b - V_c}{50} \]

which gives:

\[ 50(0.01) = 0.5 + 2.5V_b - V_c \]

\[ 0.5 = 0.5 + 2.5V_b - V_c \]

\[ V_c = 2.5V_b \]

\[ V_b = \frac{V_c}{2.5} \]

This can be substituted into the second equation to give:

\[ \frac{V_c}{2.5} \left( \frac{1}{50} \right) - \frac{V_c}{50} = \frac{V_c}{82} \]

Notice what is happening. We will have a strange expression:

\[ -0.012V_c = 0.013V_c \]

This can ONLY be true if \( V_c = 0 \).

10. What is \( V_o \)?
Ans: The one volt voltage source determines exclusively what happens to the right of it. This makes the two right resistors a voltage divider and Vo is given by:

\[ V_o = -1V \left( \frac{3}{(3+3)} \right) = -0.5V \]

Notice the signs.

11. This is the model for a circuit using a FET. What is Vo in terms of Vs?

Ans: The voltage source, Vs, and the two resistors should be familiar by now as a voltage divider circuit. \( V_g \) in terms of Vs is:

\[ V_g = -V_s \left( \frac{2000}{(1000 + 2000)} \right) = -\frac{2}{3} V_s \]

Notice the sign, since Vs is upside down.

This makes the output dependent current source:

\[ -\frac{2}{3} V_s \]

Noting the direction of the dependent current source and the polarity indicated for Vo:

\[ V_o = -10000\left( -\frac{2}{3} V_s \right) = 666.7V_s \]

A very high gain circuit.
12. What is $V_o$?

**Ans:** Using nodal analysis on the three nodes from left to right $V_a$, $V_b$ and $V_c$, the nodal equations are:

- $2 = \frac{V_a}{2} + \frac{V_a - V_b}{1}$
- $\frac{V_a - V_b}{1} = \frac{V_b}{4} + \frac{V_b - V_c}{2}$
- $\frac{V_b - V_c}{2} = \frac{V_c}{3}$

These simplify into the system of three equations:

- $2 = 1.5V_a - V_b + 0V_c$
- $0 = -V_a + 1.75V_b - 0.5V_c$
- $0 = 0V_a + 0.5V_b - 0.8333V_c$

*Using Cramer’s Rule to solve for $V_c$, which is also $V_o$, gives: $V_o = 1.0212V$*