# Workshop Statistics: Discovery with Data, Second Edition

## Topic 21: Tests of Significance I: Proportions

### Activity 21-5: Calling Heads or Tails (*cont.*)

(a) Answers will vary.
(b) statistic since it's computed from sample data
(c) Let θ=proportion of population (all statistics students?) who would respond "heads"
(d) Ho: θ=.5 (half of the population responds "heads")
Ha: θ≠.5 (the population proportion that responds heads differs from .5)
(e)

(f)-(h) Answers will vary.

(i) The sample proportions of successes would change (1-above □) and the test statistic would change in sign, but the p-value would remain the same.

### Activity 21-6: Calling Heads or Tails (*cont.*)

Ho: θ=.7 (the proportion responding "heads" in the population is as claimed)
Ha: θ≠.7 (the population proportion differs from 70%)
Answers will vary.

### Activity 21-7: Flat Tires (*cont.*)

For the one-sided test to be significant at the .10 level, we need our test statistic $z$ to exceed about 1.28 (finding the z-value corresponding to the probability below closest to .90 in Table II). So need (.3-.25)/sqrt(.25(.75)/$n$) > 1.28 or □>1.28 sqrt(.25(.75))/(.05) = 10.83, so $n$>117.19 which rounds up to 118. We would need at least 118 responses in the sample for the test statistic to exceed 1.28 when □=.30. This gives us a p-value < .10.

## Activity 21-8: Baseball "Big Bang" (*cont.*)

(a) Let $\theta$=proportion of all major league games that contain a "big bang" inning
Ho: $\theta$=.5 (50% of all games have a big bang)
Ha: $\theta \neq$.5 (the population proportion differs from .5)
□=419/968 = .433
Since.968(1-.5) and (968)(1- .5) both exceed 10, the sample size condition is met. However, it is not clear if this can be considered a simple random sample of all major league games. We will apply the significance test for one population proportion with caution.
z=(.433-.5)/sqrt(.5(.5)/968) = -4.17
p-value = 2Pr(Z>|-4.17|) < 2(.0002) = .0004 from Table II
If the population proportion of games containing a "big bang" was .5, we'd observe a sample result as least as extreme as .433 in less than .04% of samples from this population. With such a small p-value (<.02) we reject the null hypothesis and conclude that the proportion of games containing a big bang differs significantly from .5.
(b) □=651/968 = .673
Ho: $\theta$=.75 (the population proportion is .75)
Ha: $\theta \neq$.75 (the population proportion differs from .75)
Since (968)(.75) and (968)(1-.75) exceed 10, the sample size condition is still met.
z=(.673-.75)/sqrt(.75(1-.75)/968) = -5.53
p-value=2Pr(Z>|-5.53|)<.0004 < .08. We can reject Ho at the .08 level. The data provide strong evidence that the proportion of "big bang" games is significantly different from grandpa's claim.

## Activity 21-9: Racquet Spinning (*cont.*)

(a) Let $\theta$=long-run proportion of spins in which the racquet would land "up"
Ho: $\theta$=.5 (the racquet is equally likely to land "up" as "down")
Ha: $\theta \neq$ .5 (the racquet will not land "up" 50% of the time)
Since 100(.5) and 100(1-.5) both exceed 10 and we consider these 100 spins a random sample from all possible spins, the technical conditions are met.
z=(.46-.5)/sqrt(.5(.5)/100) = -.8
p-value = 2Pr(Z>|-.8|) = 2(1-.7881) = .4238
Since the p-value is large, we fail to reject the null hypothesis. It is not surprising to get 46/100 landing heads when the long-run proportion of landing heads is .5. We do not have convincing evidence that the racquet would not land "up" 50% of the time.
(b) No, since our p-value > .05

(c) No. We don't know for sure that θ=.5, we just don't have strong evidence otherwise from this sample.
(d) .4238
(e) We would use □=.54, the z-value would be +.8, but the p-value would be the same. We would not have convincing evidence that the racquet would not land "down" 50% of the time.
(f) z* for 95% confidence: 1.96
.45 ± 1.96 sqrt(.46(.54)/100) = (.36, .56)
(g) This interval does contain the value .5.
(h) Both the interval and the test are indicating that .5 is a plausible value for θ.

## Activity 21-10: Therapeutic Touch (*cont.*)

(a) Let θ=long-run proportion of trials in which the practitioners correctly distinguish which hand the experimenter was holding her hand over.
Ho: θ=.5 (equally likely to be right or wrong)
Ha: θ>.5 (practitioners distinguish correctly more often than incorrectly)
(b) □=123/280 = .439
z=(.439-.5)/sqrt(.5(.5)/280) = -2.04
p-value = Pr(Z>-2.04) = .9793
Note 280(.5) > 10
(c) The p-value > .5 since the sample result was already below the hypothesized value, opposite from the direction hypothesized in Ha.
(d) No, because it is still conceiveable that θ>.5 but we found a sample proportion less than .5 simply due to sampling variability. It is also possible that θ<.5.
(e) The practitioners do not appear to have the ability to distinguish which hand the experimenter was covering. This was not a random sample but if anything we might have expected them to perform better than those who did not volunteer.

## Activity 21-11: Magazine Advertisements (*cont.*)

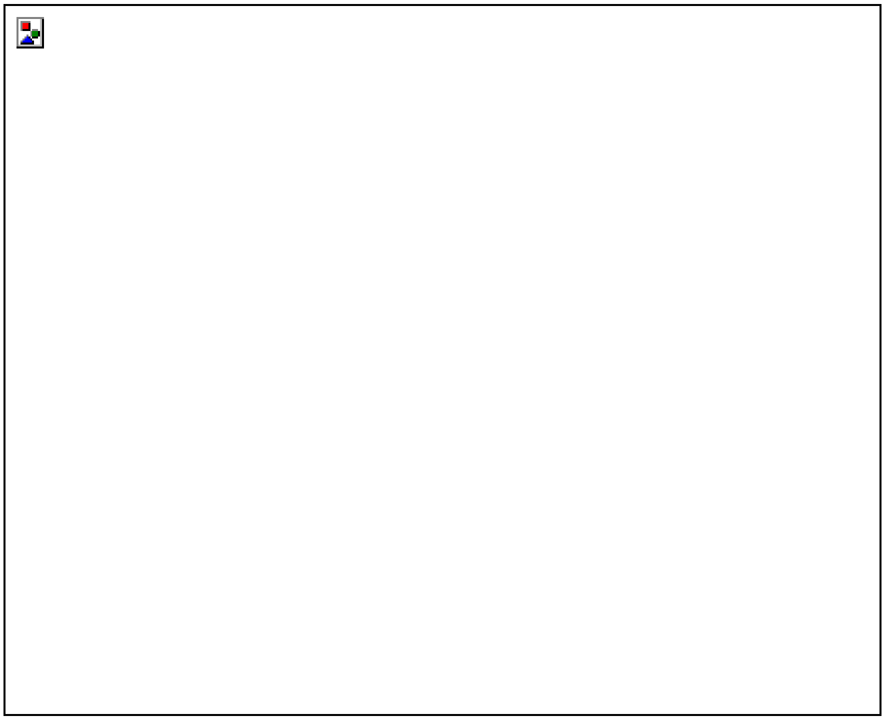
(a) Let θ=proportion of pages with advertisements from all of *Sports Illustrated*'s pages.
(b) Ho: θ=.30: null hypothesis
(c) Ha: θ≠ .30
□=54/116 = .4655
z=(.4655-.3)/sqrt(.3(1-.3)/116) = 3.89
p-value = 2Pr(Z>3.89) < 2(1-.9998) = .0004
These data are very strong evidence against the subscriber's conjecture.
(d) It is very unlikely (p-value<.0004) that we would see a sample proportion as large as .4655 if the subscriber had been correct. It appears that the subscriber underestimated the proportion of pages with advertisements.
(e) 116(.30) and 116(1-.30) > 10 so that condition is met. This one issue is not strictly a random sample from all *Sports Illustrated* pages. We would have to investigate whether we believed this issue was representative in terms of propensity of advertising.

## Activity 21-12: Volunteer Work

(a) Let θ=proportion of people in the United States who claim to have done some volunteer work during 1996
Ho: θ=.5 (50% of the population claims to have done volunteer work)
Ho: θ≠.5 (the population proportion differs from 50%)
□= .488
2719(.5) and 2719(1-.5) > 10. We are not told whether this sample was selected randomly so we have to assume that in order to conduct this test.
z=(.488-.5)/sqrt(.5(.5)/2719) = -1.25
p-value = 2Pr(Z>|-1.25|)) = 2(1-.8944)=.211
With such a large p-value, we fail to reject Ho. This sample result does not provide evidence that the proportion of the population who claim to have done volunteer work in 1996 differs from 50%.
(b) To be significant at the .05 level, need the two-sided p-value < .05. So need z<-1.96 (finding the z-value corresponding to the probability below closest to .025 in Table II).
(.488-.5)/sqrt(.5(.5)/*n*) < -1.96
□> 1.96sqrt(.5(.5))/(.012) = 81.67
*n*> 6670 (rounding up)
z=(.488-.5)/sqrt(.5(.5)/6670) = -1.96, p-value = 2Pr(Z>|-1.96|) = .05

## Activity 21-13: Hiring Discrimination

(a) Parameter, θ=long-run proportion of hired teachers who are black
(b) Ho: θ=.154
Ha: θ< .154
405(.154) and 405(1-.154) > 10 so that technical condition is met. We have to assume that the 405 teachers are a simple random sample of school teachers hired by the school district in order to conduct this test.
□=15/405 = .037



z=(.037-.154)/sqrt(.154(.845)/405) = -6.53

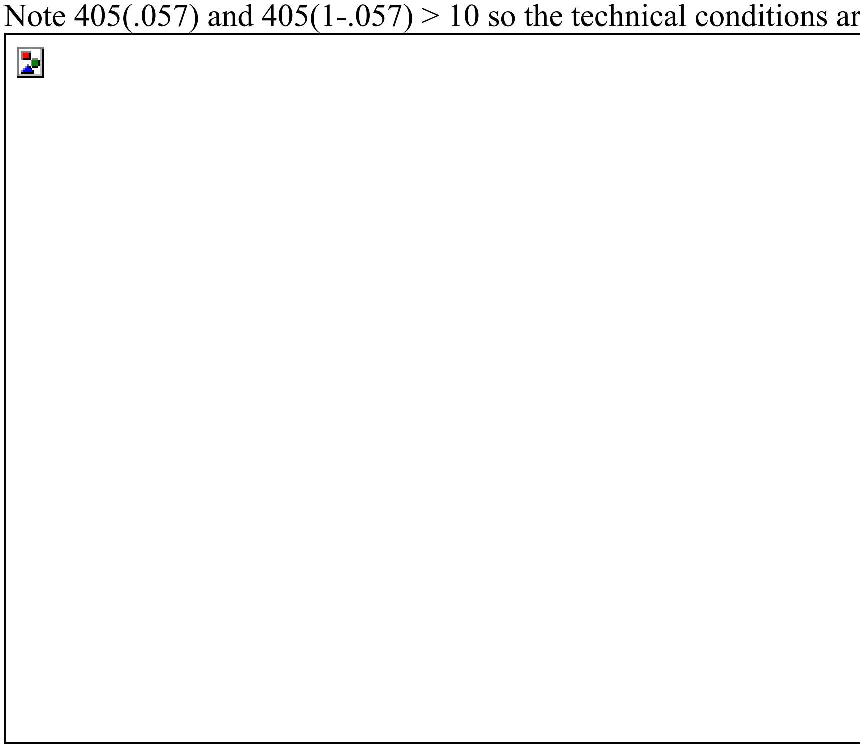
p-value = Pr(Z<-6.53)<.0002 (from Table II) or $3.5\text{x}10^{-11}$ from TI

This is very strong evidence against the null hypothesis.

(c) Ho: θ=.057

Ha: θ< .057

Note 405(.057) and 405(1-.057) > 10 so the technical conditions are still met.



z=(.037-.057)/sqrt(.057(1-.057)/405) = -1.74

p-value = Pr(Z<-1.74) = .0409

We would fail to reject Ho at the .01 level but would reject at the .05 level!
(d) There is very strong evidence that the hiring rate of blacks is below that of the entire county but may be similar to the proportion of blacks in the county excluding the city of St. Louis.

## Activity 21-14: Television Magic (*cont.*)

(a) Let θ=proportion of all *Nick-at-Nite* viewers who choose Samantha.
Ho: θ=.5 (half the viewers choose Samantha)
Ha: θ≠.5 (it is not equally likely for the viewers to pick Samantha as Jeannie)
Since 614,000(.5) > 10 this technical condition is met. However, there is voluntary response bias (viewers chose to phone in) that may mean these results are not representative of all *Nick-at-Nite* viewers. We should interpret these results with much caution.

= 810,000/(614,000+810,000) = .5688
z=(.5688-.5)/sqrt(.5(.5)/1,424,000) = 164.20
p-value is essentially zero.
Since p-value < .001, the sample result is significant at the .001 level. We have very strong evidence against the null hypothesis. We would pretty much never see a sample result like this if they were equally chosen.

## Activity 21-15: Marriage Ages (*cont.*)

Let θ=proportion of couples in this population in which the bride is younger than the groom
Ho: θ=.5 (half of the couples in the population have the bride younger)
Ha: θ>.5 (the bride is younger than the groom in more than half of all marriages in the

county)
Of the 100 couples, 67 had the bride younger and 27 had the groom younger
□=27/(67+27) = .287
Note: 94(.5) and 94(1-.5) > 10. While technically a random sample of all married couples in the county there is no reason to think that the sample from June and July of 1993 is not representative of the population in terms of age differences.
z=(.287-.5)/sqrt(.5(.5)/100) = -4.26
p-value = Pr(Z<-.426) = essentially zero
We have strong evidence that more than half of the couples in this county have the bride younger than the groom.

## Activity 21-16: Veterans' Marital Problems (*cont.*)

(a) Let θ=proportion of all Vietnam veterans who are divorced
Ho: θ=.27 (the population proportion of divorces for Vietnam veterans is the same as for all American men aged 30-44 in 1985)
Ha: θ>.27 (a higher proportion of Vietnam veterans are divorced)
□=777/2101 = .3698
Note 777(.27) and .2101(1-.27) so as long as the veterans for this study were randomly selected from among all veterans the technical conditions are met.
z=(.3698-.27)/sqrt(.27(.73)/2101) = 10.3
p-value=Pr(Z>10.3) = essentially zero.
We have strong evidence that there is a higher divorce rate among Vietnam veterans than the population of males in general.