Living in a World of Risk

One thing that distinguishes probability and statistics from other areas of mathematics is that the former often have direct and immediate relevance to everyday life. An especially important domain of application is the evaluation of risk. The ability to reason well both probabilistically and statistically allows one to intelligently assess both individual and societal risks.

Don't ~~Drink~~ Phone and Drive?

"A new study has found that using a car phone is like driving drunk" blared the national news media recently. This startling announcement made cell phone users and manufacturers across the country sit up and take notice. Could it possibly be true? In an age when everything we do and eat turns out, it seems, to be dangerous, many people increasingly react to such reports with skepticism, denial, apathy or even anger. What's next, they declare? The formation of *Mothers Against Phoning Drivers*? Public campaigns encouraging drivers to bring along a "designated phoner"? Laws requiring that cell phones be kept in the back seat or the trunk?

What led to the news reports was a statistical analysis by physician and epidemiologist Donald A. Redelmeier and professor of statistics Robert J. Tibshirani, published in the *New England Journal of Medicine* (Feb. 17, 1997). This study looked at the risk of a motor vehicle collision for drivers using cell phones. Redelmeier and Tibshirani (R&T) give a superb account of their study in the Spring 1997 issue of *Chance* magazine. Interestingly, friends cautioned them about carrying out the investigation since it might "threaten the enormous financial interests of private industry." R&T note that the cellular phone companies in North America have significantly greater daily revenues than Microsoft.

Designing a study that accurately estimates the risk of cell phone use is a difficult proposition, due to the many outside factors that could influence the results. Previous studies, not so widely reported, had in fact been done—but all had serious flaws. One early "case-control study" found that the overall frequency of traffic collisions during the one year study period was actually slightly *lower* among the mobile telephone subscribers than among the public at large (11% vs. 12%). The problem is that car phone users (the "cases") tended to be young urban professionals, a group that has low collision rates, while the control sample was drawn from the general driving population.

How could a study avoid this bias? One way is to use each driver as his or her *own* control. Another past investigation had done just that by comparing the percentage of drivers who had a collision in the year *before* the purchase of a cellular telephone to the rate for the year *following* purchase. This study also turned out indicating car phones are safe, with a significantly lower collision rate (6.6%) after purchase than before (8.2%). However, some individuals may have purchased a mobile phone *in response* to having had an accident. The drop in rates could also simply reflect the fact that a person's driving ability improves over time.

In an attempt to avoid these possible biasing factors, R&T elected to use a "case cross-over design" for their study. They selected 699 cellular phone owners in Toronto, Canada who had had an accident involving significant damage between July 1994 and August 1995 and were willing and able to provide accurate telephone billing records. For each one, R&T determined whether they had made a cellular telephone call during the 10-minute period immediately prior to the collision (the "hazard interval"), and also whether they had made a call during a comparable time period the day before the collision (the "control interval"). Here is what they found:

		Call during control interval?	
		Yes	**No**
Call during	**Yes**	13	157
hazard interval?	**No**	24	505

What do these data reveal regarding whether cell phone use raises or lowers accident risk? The cells in which the response is the same to both questions in fact play no role in the analysis. However we would expect the other two entries to be roughly equal if the accident risk when one is using a cell phone were the same as when one is not. Instead, $157/24 = 6.54$ times as many accidents occurred when the phone was in use during the hazard interval only as the other way around. However, the

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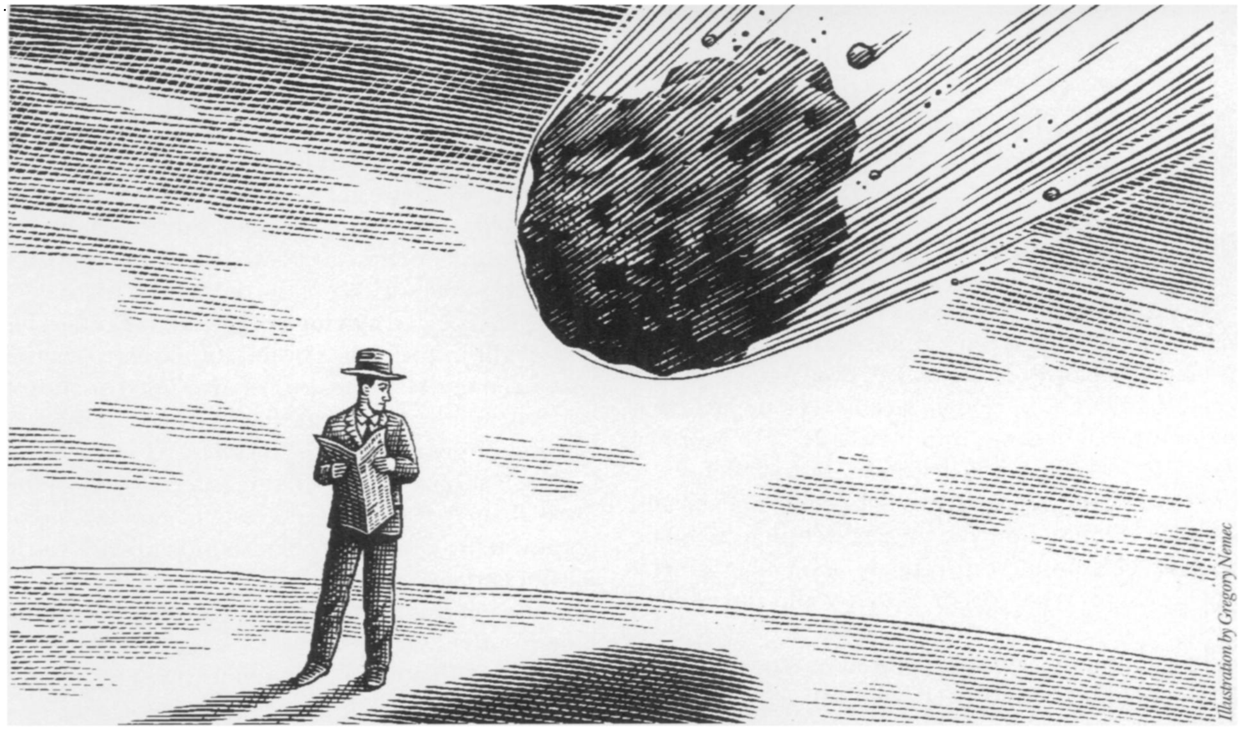

validity of this ***relative risk*** of 6.54 assumes that those in the study were *driving* during the control interval. Using their survey, R&T estimate that 65% actually were, thereby obtaining an adjusted relative risk of 65% × 6.54 = 4.3.

R&T perform numerous additional analyses to check the stability of this result. They compare the accident day to other control days, such as the day exactly one week before the accident. They also look at the data for experienced cell phone users alone. Throughout these analyses R&T consistently find a relative risk of around 4 for using a phone while driving. This means that a driver on the phone is *four times as likely to have an accident* during the call as if (s)he were not using the phone. (One exception is for users of hands-free phones, where the relative risk is, surprisingly, actually *higher*, at 5.9.)

R&T's study suggests that the dangers of using a cell phone while driving should be taken quite seriously. As seriously as getting behind the wheel after consuming alcohol? Driving with a blood alcohol level at the legal limit *does* in fact yield a similar relative risk. However, a drunk driver's alcohol content is often significantly above the amount required to be driving under the influence. Also, the effects of alcohol are likely to last for a much longer time than a phone call. Fortunately cellular phone calls tend to be brief and infrequent, which accounts for the lack of a dramatic increase in accidents at a time when the use of cellular phones is increasing rapidly.

Watch Out for That Tree

Motor vehicle accidents are the leading cause of accidental death in the United States, killing more people than all other types of accidents combined. But any time the human body is traveling at high speed, the chance of injury or death is present. This past winter saw the tragic deaths of two famous individuals in skiing accidents, Michael Kennedy and Sonny Bono. Remarkably, both were political celebrities, both died from skiing into a tree within less than a week of each other, and both accidents occurred at nearly the identical time of day.

What are the chances of such an unusual event? A total of 36 skiers died skiing in the United States in the winter of 1996–7, a very small number considering that ten million people ski. We need to have some idea of how many "famous" people there are in the United States, as well as the amount that they ski. My almanac lists about 1600 "Notable Personalities", but this represents only those in the entertainment industry. So perhaps there are somewhere between 2500 and 5000 "celebrities" altogether. Such individuals tend to be much wealthier than the general population, and their participation in skiing is undoubtedly much greater than the population rate of 10 million ÷ 260 million (the approximate U.S. population) ≈ 4%.

Let's postulate, then, that 20% of celebrities ski, and that they do so as often on average as other skiers. Using the conservative estimate of 5000 celebrities, we can calculate the chance that two of the 1000 skiing celebrities will die skiing in a given year from the binomial distribution. If the chance of death for any one of them is 36/10,000,000, the likelihood that exactly two would die in the same year is

$$\binom{1000}{2}.0000036^2(1-.0000036)^{998} = .0000065.$$

That means such an occurrence would occur only about once every 150,000 years. Restricting consideration to political celebrities only, and/or taking into account the other coincidental factors, would raise these odds to truly astounding levels. You may wish to try your hand at such a calculation.

How can we explain such an amazing coincidence? Primarily by realizing that the number of *potential* coincidences that pass by constantly, both in our personal lives and in the world at large, is extraordinarily large. Using a very simple model makes the point: Suppose that in any given year, there are many potential coincidences that would receive national attention, let's say 100,000 or so, each having probability .0000065 of occurring. Then the expected number of such "amazing" events would be 100,000 × .0000065 = 0.65 per year. From this perspective it is not at all unlikely that one would occur in a given year, and fully half a dozen or so would occur per decade.

Danger from Above?

The tragic explosion and crash of TWA Flight 800 has defied all efforts to identify its cause. Investigators have considered mechanical failure, terrorism, and even a collision with a stray missile, but answers remain lacking. Is it possible that the plane was brought down by a meteorite? *The New York Times* (Dec. 12, 1997) recently discussed this possibility.

Before reading on, get a piece of paper and a pencil and think for a few minutes how you might attempt to estimate the probability of a commercial airplane being hit by a meteorite while in flight. What sort of factors would you need to consider?

OK, all done? Which of the following did you come up with:

an estimate of the rate at which meteorites of sufficient size to destroy an airplane strike the earth per unit time (this is the hard one)

the total surface area of the earth (easy to compute as $4\pi(4000 \text{ mi.})^2 \approx 5.6 \times 10^{15}$ sq. ft.)

the surface area of the typical commercial airplane when viewed from above

the average number of hours that each plane is in flight

the geographical distribution of both meteorites and airplane traffic (the pock-marked surface of the moon and some planets suggests that meteorite impacts probably occur everywhere rather evenly, in which case the distribution of airplane traffic is irrelevant—do you see why?)

It is difficult to get accurate information on some of the above factors, especially the first. This is a typical situation in the area of risk estimation, and explains why different experts often arrive at widely differing estimates.

There is a critical issue to address before going further. Keep in mind that we are considering the TWA Flight 800 explosion *after* the fact. This is not the same as thinking about the chance that the next flight you take will be struck. If *any* commercial flight in history had been destroyed by a meteorite, would we not—assuming the meteorite left no evidence of its culpability—be in exactly the same situation of wondering about the cause as for the TWA flight? Thus, what seems most relevant is not the probability that a *specific* flight will be intercepted by a meteorite, but rather the chance that *some* flight in the entire history of modern aviation would have been hit. The latter probability, based on the total number of commercial airplane flights to date, is many orders of magnitude greater than the former.

The National Transportation Safety Board commissioned Dr. William A. Cassidy, a professor of geology and planetary science at the University of Pittsburgh, to study the question. He finessed several of the factors above by counting all known incidents in this century of houses being hit by meteorites (apparently just within the U.S.), finding fourteen in all, and then estimating the total roof area. This allowed him to determine how many meteorites on average hit a given area in a specified period of time. Finally, he estimated the "target area" of all planes in the air at any given moment, from data provided by the NTSB. Dr. Cassidy concludes that the expected frequency of meteorites hitting planes in flight is one such event every 59,000 to 77,000 years.

A radically different result was obtained by Professors Charles Hailey and David Helfand of Columbia University. They claim that approximately 3,000 meteorites a day with the requisite mass to cause a crash strike Earth. Assuming an average flight time of two hours, they find a 1-in-10 chance that a commercial flight would have been knocked from the sky by meteoric impact in the last 30 years.

Whose estimate seems most reasonable to you? As an exercise, show that the rate of meteoric impacts Hailey and Helfand gave is not consistent with Cassidy's data involving houses. (You will need to make an educated guess of the total roof area of houses in the United States on average during this century.) Then you may wish to make your own calculation of the risk exposure of airlines to meteorites. □