SCHILLING
440 A

HOMEWORK SOLUTIONS, SECS. 9.4–5

9.32 $f(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta}, \; y>0 \Rightarrow L(\theta) = \prod_{i=1}^{n} f(y_i;\theta) = \underbrace{\left(\frac{2}{\theta}\right)^n e^{-\frac{1}{\theta}\sum y_i^2}}_{g(\sum y_i^2,\, \theta)} \underbrace{\prod_{i=1}^{n} y_i}_{h(y_1,\dots,y_n)}$

9.33 $f(y) = \frac{1}{\alpha} m y^{m-1} e^{-y^m/\alpha}, \; y>0 \Rightarrow L(\alpha) = \prod_{i=1}^{n} f(y_i;\theta) = \underbrace{\left(\frac{m}{\alpha}\right)^n e^{-\frac{1}{\alpha}\sum y_i^m}}_{g(\sum y_i^m,\, \alpha)} \underbrace{\prod_{i=1}^{n} y_i^{m-1}}_{h(y_1,\dots,y_n)}$

9.35 $f(y) = \alpha y^{\alpha-1}/\theta^\alpha \Rightarrow L(\alpha) = \prod_{i=1}^{n} f(y_i;\theta) = \frac{\alpha^n}{\theta^{n\alpha}} \prod_{i=1}^{n} y_i^{\alpha-1} = \underbrace{\frac{\alpha^n}{\theta^{n\alpha}} \left(\prod y_i\right)^{\alpha-1}}_{g(\prod y_i,\, \alpha)} \cdot \underbrace{1}_{h(y_1,\dots,y_n)}$

9.43 Since θ affects the <u>range</u> of possible values of y, this problem is like the example $Y_1,\dots,Y_n \sim \mathcal{U}(0,\theta)$ (D.U.C.S. missiles)
$L(\theta) = \prod_{i=1}^{n} f(y_i;\theta) = \prod_{i=1}^{n} e^{-(y_i-\theta)} = e^{n\theta - \sum y_i}$ if all $y_i \geq \theta$.

I.e., $L(\theta) = e^{n\theta - \sum y_i} I(\theta \leq \text{all } y_i) = e^{n\theta - \sum y_i} I(\theta \leq \min Y_i)$
where $I(A) = 1$ if A is true, 0 if A is false. Then we have
$L(\theta) = \underbrace{e^{n\theta} I(\theta \leq \min Y_i)}_{g(\min Y_i,\, \theta)} \underbrace{e^{-\sum y_i}}_{h(y_1,\dots,y_n)}$

9.50 $\sum Y_i^2$ is sufficient by #9.32, and is a minimal sufficient sta because it is a single quantity. Thus any function of it that is unbiased will be a MVUE of θ.
We have $E\left(\sum_{i=1}^{n} Y_i^2\right) = \sum_{i=1}^{n} E Y_i^2 = n E Y^2 = n \int y^2 f(y)\,dy$

$= n \int_0^\infty \frac{2y^3}{\theta} e^{-y^2/\theta} dy = n\theta \int_0^\infty u e^{-u} du = n\theta \cdot 1$ (from integral tables or integ. by parts)
Let $u = y^2/\theta \Rightarrow du = \frac{2y}{\theta} d\theta$

$= n\theta$. Thus $E\left(\frac{\sum Y_i^2}{n}\right) = \frac{n\theta}{n} = \theta$, so $\frac{\sum Y_i^2}{n}$ is a MVUE of θ.