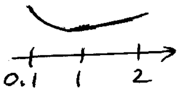
MATH 150A

SELECTED HW SOLUTIONS, SEC. 4.3

4. $f'(x) = 1 - \frac{1}{x} = 0$ at $x = 1$, $f''(x) = \frac{1}{x^2} > 0$ so $x = 1$ gives a l. m

Thus the graph looks like  , so $x = 1$ is a

a g. min and $x = 0.1$ and $x = 2$ are l. max's. Comparin

$f(0.1) = 2.40$ and $f(2) = 1.31$ shows that $x = 0.1$ is the g. max

8. a) $\frac{dT}{dD} = CD - D^2 = 0 \Leftrightarrow D = 0$ and C. The second deriva

test gives $\frac{d^2T}{dD^2} = C - 2D$. at $D = 0$ this is > 0 and at D

this is < 0, thus $D = C$ maximizes T.

b) $\frac{d}{dD}\left(\frac{dT}{dD}\right) = \frac{d^2T}{dD^2} = C - 2D = 0 \Leftrightarrow D = C/2$. This gives a m

by the 1st derivative test.

9. $v = aRr^2 - ar^3 \Rightarrow \frac{dv}{dr} = 2aRr - 3ar^2 = ar(2R - 3r) =$

$\Leftrightarrow r = 0,\ r = \frac{2}{3}R$. The 2nd derivative test gives

$\frac{d^2v}{dr^2} = 2aR - 6ar = 2a(R - 3r)$, which is > 0 at $r = 0$ an

< 0 at $r = \frac{2}{3}R$. Thus $r = \frac{2}{3}R$ maximizes v.

13. $f'(r) = 2Ar^{-3} - 3Br^{-4} = 0 \Leftrightarrow \frac{2Ar - 3B}{r^4} = 0 \Leftrightarrow r = \frac{3B}{2A}$.

Clearly if $r < \frac{3B}{2A}$ then $f'(r) =$ is < 0, while if $R >$

then $f'(r)$ is > 0. Thus we have minimized $f(r)$, by t

first derivative test.

18. $\frac{dE}{dx} = \frac{k\cdot(x^2+r_0^2)^{3/2} - kx\cdot\frac{3}{2}(x^2+r_0^2)^{\frac{1}{2}}\cdot 2x}{(x^2+r_0^2)^3} = 0 \Leftrightarrow$ numerator

$\Leftrightarrow (x^2+r_0^2)^{3/2} - 3x^2(x^2+r_0^2)^{1/2} = 0 \Leftrightarrow x^2 + r_0^2 - 3x^2 = 0$

$\Leftrightarrow r_0^2 = 2x^2 \Leftrightarrow x = \pm\frac{r_0}{\sqrt{2}}$. The first derivative test g

	$-r_0/\sqrt{2}$		0		$r_0/\sqrt{2}$		x
$\frac{dE}{dx}$	0		$+$		0	$-$	

, thus $r_0/\sqrt{2}$ gives a max and $-r_0/\sqrt{2}$ gives a min.

MATH 150A

SELECTED HW SOLUTIONS, SEC. 4.5

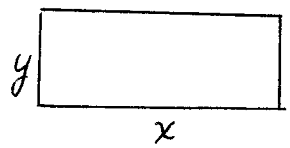
5. a) $V = x^2 h$

c) Use $A = 2x^2 + 4xh \Rightarrow h = \dfrac{A-2x^2}{4x} \Rightarrow V = \frac{A}{4}x - \frac{1}{2}x^3$ upon substituting h into $V = x^2 h$. Then

$$\frac{dV}{dx} = \frac{A}{4} - \frac{3}{2}x^2 = 0 \Leftrightarrow \ldots \; x = \pm\sqrt{\frac{A}{6}}.$$

Only $x = +\sqrt{\frac{A}{6}}$ can be a length. Substituting into V gives ... $V = \left(\frac{A}{6}\right)^{3/2}$. To check this is a max, note $\frac{d^2V}{dx^2} = 0 - \frac{3}{2}\cdot 2x < 0$.

8.



Suppose the fencing is on the top side.

The cost is then $C = 25(x+2y) + 10x$. We can eliminate the independent variable y by using the area constraint $xy = 3000 \Rightarrow y = 3000/x$

$$\Rightarrow C = 25(x + 6000x^{-1}) + 10x = 35x + 150{,}000x^{-1}.$$

Then $\frac{dC}{dx} = 35 - 150{,}000x^{-2}$. $\frac{dC}{dx}$ is undefined at $x = 0$ but this is not a length. $\frac{dC}{dx} = 0$ at $x = \sqrt{\frac{150{,}000}{35}}$ ≈ 65.5 ft. The cost is then $35(65.5) + 150{,}000/65.5 \approx \$45$

Note $\frac{d^2C}{dx^2} = 300{,}000x^{-3} > 0$ so we have minimized C.