MATH 150A
SCHILLING

NAME KEY

TEST #2 (100 PTS.)
(SHOW ALL WORK)

1. A chemical reaction produces chlorine gas as a byproduct. The total output of chlorine gas (in grams) in the first t minutes is given by the function $f(t) = 10 - 10e^{-0.1t}$. Write a clear and natural sentence that interprets the expression $f'(30) = 0.15$.

Thirty minutes after the reaction starts, chlorine ga is being produced at a rate of 0.15 grams per minute

2. Find the derivative of each of the following functions. Simplify your answers where appropriate:

(a) $f(x) = x^7 - \dfrac{x^3}{5} + 2x^{-4}$

$$f'(x) = 7x^6 - \frac{3x^2}{5} + 2(-4x^{-5}) = \boxed{7x^6 - \frac{3}{5}x^2 - 8x^{-5}}$$

(b) $f(x) = \underset{u}{x}\,\underset{v}{\ln x} - x + 3$

$$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 + 0 = \ln x + 1 - 1 = \boxed{\ln x}$$

(c) $R(p) = \dfrac{e^p}{1+e^p}\begin{matrix} u \\ v \end{matrix}$

$$R'(p) = \frac{\overset{u'}{e^p} \cdot \overset{v}{(1+e^p)} - \overset{u}{e^p} \cdot \overset{v'}{e^p}}{\underset{u^2}{(1+e^p)^2}} = \boxed{\frac{e^p}{(1+e^p)^2}}$$

(d) $y = 5e^{t^2} + \ln(\cos(\pi t))$

$$\frac{dy}{dt} = 5e^{t^2} \cdot 2t + \frac{1}{\cos(\pi t)} \cdot (-\sin(\pi t)) \cdot \pi$$

$$= \boxed{10te^{t^2} - \pi \tan(\pi t)}$$

(e) $z = \arctan(e^x)$

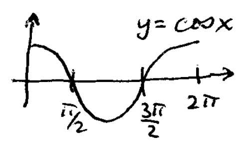
$$\frac{dz}{dx} = \underbrace{\frac{1}{1+(e^x)^2}}_{f'(g(x))} \cdot \underbrace{e^x}_{g'(x)} = \boxed{\frac{e^x}{1+e^{2x}}}$$

(15) 3. Use differentiation to determine where in the interval $[0, 2\pi]$ the function $f(x) = 4x + 3\cos x$ is concave up.

$$f'(x) = 4 - 3\sin x$$

$$f''(x) = -3\cos x$$

$$f''(x) > 0 \Leftrightarrow \cos x < 0$$



$$\therefore \boxed{\pi/2 < x < 3\pi/2}$$

(15) 4. A skydiver is falling through the air at a rate of $32t$ ft/sec, t seconds after jumping out of an airplane. Suppose that the ground area that the skydiver can view through her goggles at any given moment is $A = 20y + 100$ ft^2, where y is her elevation in ft. How fast is this visible area changing 20 seconds after the skydiver leaves the plane?

We have $\frac{dy}{dt} = -32t$. We want $\left.\frac{dA}{dt}\right|_{t=20}$.

Use

$$\left.\frac{dA}{dt}\right|_{t=20} = \left.\frac{dA}{dy} \cdot \frac{dy}{dt}\right|_{t=20}$$

$$= 20 \cdot (-32 \cdot 20)$$

$$= \boxed{-12{,}800 \text{ ft}^2/\text{sec.}}$$