MATH 340 PRACTICE PROBLEMS FOR TEST 2
SCHILLING

1. Ten employees were eligible for promotion at a certain firm. Four were men and six were women. Although all were equally qualified, only five could be promoted. Only one of the six women was promoted. Define an appropriate random variable and use it to determine the chance that no more than one woman would be promoted, assuming sex was not a factor in the promotion decisions.

2. Eight people are gathered at a small party, and the conversation turns to their birthdays. Assume that for each person, the probability of being born in any particular month is 1/12, independently of the birthdays of any of the other people.

(a) Define an appropriate random variable and use it to determine the chance that exactly three people were born in March.
(b) What is the chance that at least two people were born in March?

3. Suppose that the number of email messages Y that a certain user receives in a given week follows a Poisson distribution with parameter $\lambda = 35$.

(a) What is the expected number of messages received in a week?
(b) Suppose that the user spends 30 minutes per week logging on to check her email, and five minutes per message reading and answering her email. Let T be the total amount of time she spends on email in a week. Write T in terms of Y and find $E(T)$, $V(T)$ and $SD(T)$.

4. Suppose that the number of email messages Y that a certain user receives in a given week follows a Poisson distribution with parameter $\lambda = 35$.

(a) Give an expression for the probability that the user receives more than 30 messages in the week in question. <u>Do not evaluate the expression</u>.
(b) Let X be the number of messages received on a particular *day* of the week. Name the distribution of X and specify any parameter(s). Then find the chance that the user will receive at most one message on that day.

5. Suppose Y has probability function $p(y) = 1/4$ for $y = 1, 2, 3, 4$. Find the moment generating function of Y and use it to determine $E(Y)$.

6. Suppose that X is a continuous random variable with probability density function

$$f(x) = \frac{1}{36}(x^2 - 8x + 19) \text{ for } 1 \le x \le 7; \;\; f(x) = 0 \text{ elsewhere.}$$

Verify that $f(x)$ is a legitimate probability density function. For full credit you need to show <u>two</u> things.

7. The mean of the density given in #6 is $\mu = 4$. Find $E(X^2)$ and use it along with μ to determine $V(X)$.

8. The Trans-Alaska pipeline runs for 800 miles across Alaska from the Arctic Ocean to the Gulf of Alaska in the Pacific Ocean. For various reasons the pipeline is subject to numerous spills--as many as 40 or more in some years. Let X be the distance from the northernmost end of the pipeline, at the Arctic Ocean, to the location of the next spill that will occur. Assuming that this next spill occurs at a completely random location,

   (a) what is the distribution of X?
   (b) Use the pdf of X to determine the chance that the next spill will occur within the northernmost 200 miles of the pipeline.
   (c) Determine the probability that each of the next three spills all occur within the northernmost 200 miles of the pipeline.

9. Air samples from a large city are found to have 1-hour carbon monoxide concentrations that follow an exponential distribution with a mean of 3.6 ppm (parts per million).

(a) Find the probability that the concentration will exceed 9 ppm in a randomly selected hour.
(b) Determine the probability that the carbon monoxide concentration will exceed 12 ppm in a randomly selected hour, given that it exceeds 9 ppm.
(c) Without doing any calculations, give the probability that the carbon monoxide concentration will exceed 3 ppm in a randomly selected hour. Justify your answer verbally.

10. The army reports that the distribution of head circumference X among soldiers is approximately normal with mean 22.8 inches and standard deviation 1.1 inches.

(a) Find the probability that a randomly selected soldier has a head size greater than 24.5 inches.
(b) The army plans to provide custom-made helmets for soldiers whose head circumference falls in the top 1% of the distribution. What head circumferences fall into this range?