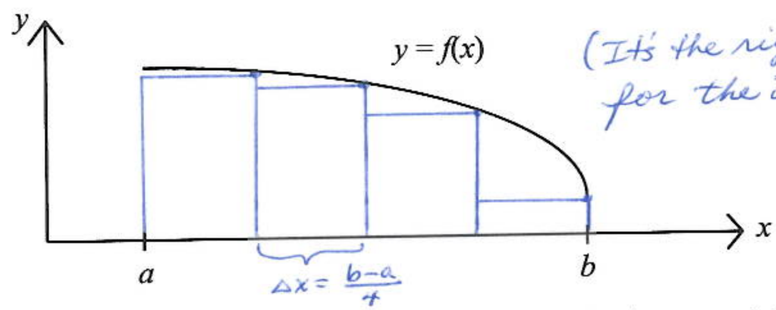
MATH 150A
SCHILLING

NAME KEY

PRACTICE TEST #3

1. (a) Draw what $\sum_{i=1}^{4} f(x_i)\Delta x$ represents geometrically on the graph below, where $\Delta x = \dfrac{b-a}{4}$.



(It's the right-hand sum for the interval shown)

(b) Next to each of the three sums below, indicate which is the largest, which is the smallest, and which is in between the other two for the function shown above.

$\sum_{i=1}^{4} f(x_i)\Delta x$: SMALLEST (RHS) $\quad \sum_{i=0}^{3} f(x_i)\Delta x$: LARGEST (LHS) $\quad \int_a^b f(x)dx$: IN BETWEEN (AREA UNDER CURVE)

2. A curtain is suspended between two vertical poles that are 120 inches apart (see below). The curtain reaches completely to the floor. The top of the curtain is $y = f(x)$ inches above the floor at a point x inches to the right of the left pole. Give a physical interpretation of the integral

$\int_0^{120} f(x)dx$ and give its units.

It's the area of the curtain; the units are inches × inches = square inches.

3. A frozen steak is removed from a 20° (Fahrenheit) freezer and left to thaw for two hours. The temperature (in Fahrenheit) of the center of the steak t minutes after its removal is given by the function $f(t) = 70 - 50e^{-0.01t}$.

Write an expression that represents the average temperature of the center of the steak during the thawing period.

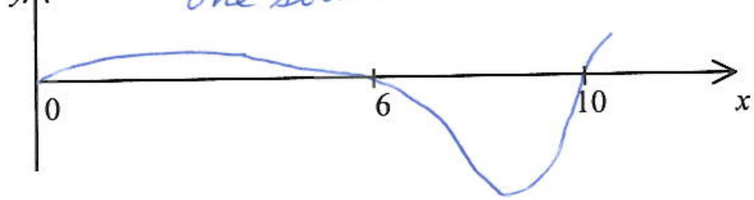
$$\frac{1}{120-0}\int_0^{120}\left(70-50e^{-0.01t}\right)dt$$

(since 2 hrs. = 120 minutes)

4. Draw a single function $y = f(x)$ on the axes below that <u>clearly</u> satisfies **both** of the following:

(i) $\int_0^6 f(x)dx > 0$ (ii) $\int_0^{10} f(x)dx < 0 \leftarrow$ Need more area <u>below</u> the x-axis than <u>above</u> it.

one solution:

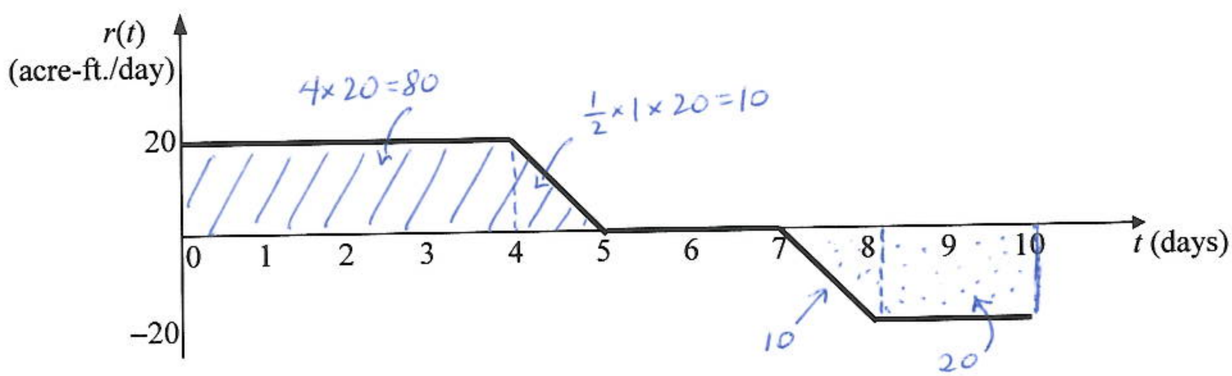


5. Evaluate the integral $\int_0^{\frac{\pi}{2}} \underbrace{(\sin x + e^x)}_{F'(x)} dx$ exactly using the Fundamental Theorem of Calculus.

$F(x) = -\cos x + e^x$ is an antiderivative, so we have

$$\int_0^{\pi/2} (\sin x + e^x)dx = F(\pi/2) - F(0) = \left[-\cos(\pi/2) + e^{\pi/2}\right] - \left[-\cos 0 + e^0\right]$$
$$= e^{\pi/2} - 0 - [-1+1] = \boxed{e^{\pi/2}}.$$

6. The graph below shows the flow rate $r(t)$ of water into (if positive) or out of (if negative) a reservoir, as a function of time:



At time $t = 0$, the reservoir contains 550 acre-ft of water. Determine:

(a) the maximum amount of water that the reservoir contains during the 10 day period shown.

Let $A(t)$ be the amount of water after t days. Then the maximum is $A(5) = A(0) + \underbrace{\int_0^5 r(t)dt}_{\text{shaded area}}$ (by the FTOC) $= 550 + (80+10)$ $= \boxed{640 \text{ acre-ft.}}$

(b) the amount of water in the reservoir after 10 days.

$$A(10) = A(0) + \underbrace{\int_0^{10} r(t)dt}_{\text{shaded area} - \text{dotted area}} = 550 + 90 - 30 = \boxed{610 \text{ acre-ft.}}$$

7. The right hand sum estimate of $\int_1^8 x^{1/3}dx$ with $n = 1000$ subdivisions is 11.2465.

(a) How far off from the exact value could this be? Since $x^{1/3}$ is an increasing fn, we can use the formula

$$\text{max error} = |f(b)-f(a)|\cdot\frac{b-a}{n} = |8^{1/3}-1^{1/3}|\cdot\frac{8-1}{1000} = (1)\cdot\frac{7}{1000} = \boxed{.007}$$

(b) What value of n would be needed to estimate the integral to within .0005?

Solve $|8^{1/3}-1^{1/3}|\cdot\frac{8-1}{n} = .0005$: $(1)\cdot\frac{7}{n} = .0005 \Rightarrow n = \frac{7}{.0005} = \boxed{14{,}000}$

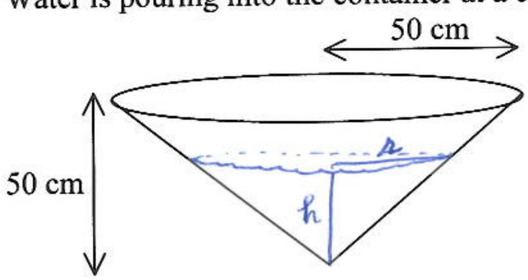
8. The electric potential P of a particle at a point on the positive x-axis is given by the formula $P = 2\pi(\sqrt{x^2+4}-x)$. A particle is moving to the left at 0.2 cm/sec. At what rate is the potential changing when the particle is at $x = 3$?

$$\frac{dP}{dt} = \frac{dP}{dx}\cdot\frac{dx}{dt} = 2\pi\left(\tfrac{1}{2}(x^2+4)^{-1/2}\cdot 2x - 1\right)\cdot\frac{dx}{dt}$$

$$\left.\frac{dP}{dt}\right|_{x=3} = 2\pi\left(\tfrac{1}{2}(3^2+4)^{-1/2}\cdot 6 - 1\right)\cdot(-0.2) = \boxed{.211 \text{ units per second.}}$$

since moving to the left

9. A conical container is 50 cm high and has a radius of 50 cm at the top (see figure below). Water is pouring into the container at a constant rate. When the water has filled the container to a depth of 20 cm, this depth is increasing at a rate of 5 cm/sec. Find the rate at which the water is pouring into the container. (Hint: Recall that the formula for the volume of a cone is $V = \pi r^2 h/3$.)



Letting h be the depth of the water and r the radius of the surface of the water as shown, similar triangles gives $\frac{h}{r} = \frac{50}{50}$, so $h = r$. Then the volume of the water is $V = \pi r^2 h/3 = \frac{\pi}{3}h^3$. Differentiating both sides with respect to t gives $\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} = \frac{\pi}{3}\cdot 3h^2\frac{dh}{dt} = \pi h^2\frac{dh}{dt}$.

When the depth is 20 cm we have

$$\left.\frac{dV}{dt}\right|_{h=20} = \pi(20)^2\cdot 5 = \boxed{2000\pi \text{ cm}^3/\text{sec.}}$$