Orbital Mechanics

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Objectives:

- Students should become familiar with the basic concepts of celestial mechanics.
- Students should learn to interpret the common format of satellite tracking data.
- Students should be able to predict satellite positions from tracking data.

Topics:

- Review of mathematical tools neccesary:
  - Trigonometry
  - Vector Calculus
  - Differential Equations
  - Linear Algebra

- Elementary Celestial Mechanics
  - Newton's Laws and Newton's Law of Gravity
  - Derivation of Kepler's Laws for point masses
  - Orbital Elements
  - Kepler's Equation

- Interpreting orbital tracking data
  - NORAD two line elements

  - Solving Kepler's Equation numerically
  - Transforming orbital elements into Cartesian coordinates

- Earth is not a point mass!
  - The effects of the lack of spherical symmetry - perturbations
  - Describing perturbations in terms of Orbital elements

- Utilize software to describe satellites orbits (orbsim)

- Final Projects

Orbital Elements:

In an ideal world (point masses) satellites move on elliptical orbits around earth. To accurately describe the location of a satellite and to predictits future motion we could use Cartesian coordinates, time, and the three velocity components of the satellite. However, we can equally describe its motion by identifying the elliptical orbit and the location of the satellite on this orbit. To do this we need to identify the plane of the ellipse, the direction of the semi-major axis on this plane, and the location of the satellite in polar coordinates. This is the common way in which tracking data are kept. And these are called orbital elements. The problem starts with defining a frame of reference. We need to remember that everything moves, the center of the earth and points on the surface of the earth. To start we use a Cartesian coordinate system with origin at the earth center, z axis along the earth axis of rotation, and x-axis in the direction of the vernal equinox aat some given time.As the time is referred to as *Epoch* and commonly is the time since the beginning of January 1, 1950.

Identification of the plane of the ellipse:

We fix the equatorial plane. For a given orbit, we define the *node* as the line of intersection of the plane orbit and the equatorial plane. This line is uniquely identified by its angle with the x-axis: Ω. This is called the *right ascension of the node.* The plane can now be uniquely identified by its *angle of inclination* i.

Identification of the ellipse itself

The direction of the semi-major axis of the ellipse is given by its angle to the *node* ω *argument of the perigee.* The second number we need is the eccentricity e of the ellipse.

Identification of the location of the satellite

The location of the satellite is now given by the angle of the ray connecting the satellite to the focus with the semi-major axis. This angle is called the *true anomaly.* Alternatively, one could use the angle between the semi-major axis and the line connecting the satellite and the center of the ellipse. This angle is called the *eccentric anomaly* E. Either angle can be used to give the postion in cartesian coordinates centered at the focus. However, none of these angles are commonly used in satellite tracking data. The common measurement is the *mean anomaly* ν which is directly proportional to the time travelled since the perigee. *Kepler's Equation* relates the mean and the eccentric anomaly:

$$\nu = E - e \sin E.$$

Example:

NORAD keeps track of all the satellites currently in orbit. The followin is the data downloaded from NORAD this morning for the satellite **Alouette** launched in 1962.

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ALOUETTE 1 (S-27)

1 00424U 62049A   03336.46151225  .00000147  00000-0  16145-3 0  8814

2 00424  80.4642 114.4933 0022502 219.0014 140.9510 13.68552980 54869
```

The particular information is given in the following tables. The first row includes:

Column	Description
01	Line Number
03–07	Satellite Number
08	Classification
10–11	Launch year
12–14	Launch number
15–17	Launch piece
19–20	Epoch year
21–32	Epoch day and fraction of a day
34–43	First time derivative of mean motion
45–52	Second time derivative of the mean motion
54–61	B^* drag term
63	Ephemeris type
65–68	Element number
69	Check sum

The second row is more interesting:

Column	Description
01	Line number
03–07	Satellite number
09–16	Inclination i in degrees
18–25	Right ascension of the ascending node Ω
27–33	Eccentricity e
35–42	Argument of the perigee ω
44–51	Mean anomaly M (or ν)
53–63	Mean motion (revolutions per day)
64-68	Revolution number at epoch
69	Check sum

From this information students could compute the cartesian coordinates of this satellite, by first solving Kepler's equation, and then computing cartesian coordinates in a system centered at the focus and with axes along the axes of the ellipse. Three rotations will then transform this into the cartesian coordinate system.

Final Projects

The final projects for students were different for the two times the class was run. The first time all students used tracking software to investigate MOLNYA orbits, the second time the projects were different and more tailored to the students backgrounds. Engineering students wrote a program that identified all GPS satellites that are visible from any location at any given time. Other projects involved the theoretical investigation of perturbations, mathematical investigation of discrete dynamical systems, and others.

MOLNYA orbits

Communications satellites are usually in geosynchronous orbit around the earth. This makes them appear stationary from any given location on earth. Unfortunately, geosynchronous orbits lie in or very near to the equatorial plane. This makes it difficult or even impossible to have a line sight of them from higher latitudes. The Soviet Union used communications satellites which have elliptical orbits whose plane osculates do the perturbation effects of the earth. This made these satellites nearly stationary over far northern regions. Students explored this highly perturbed motion.