Different types of reduced row-echelon forms for $\mathbf{n \times m}$ matrices with $\mathbf{1 \le n, m \le 3}$

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Two $n \times m$ matrices in reduced row-echelon form are of the same type if they contain the same number of leading 1's in the same positions.

1×1 matrices

$$[0], \quad [1].$$

1×2 matrices

$$\begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & k \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad \text{where } k \text{ is an arbitrary constant.}$$

2×1 matrices

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

2×2 matrices

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{where } k \text{ is an arbitrary constant}$$

1×3 matrices

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & a & b \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & c \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},$$

where a, b, and c are arbitrary constants.

3×1 matrices

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

3×2 matrices

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & k \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{where } k \text{ is an arbitrary constant.}$$

2×3 matrices

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & c \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & e \end{bmatrix}, \begin{bmatrix} 1 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where a, b, c, d, e, and f are arbitrary constants.

3×3 matrices

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & e \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & f & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where a, b, c, d, e, and f are arbitrary constants.