Practice Problems II
Math 250, Spring 2024 – Jacek Polewczak

Problem 1.
Evaluate the indicated double integrals.

$$\text{(a)} \quad \iint_R xy\sqrt{1+x^2}\, dA; \quad R = \{(x,y) : 0 \le x \le \sqrt{3},\, 1 \le y \le 2\}$$

$$\text{(b)} \quad \int_{-2}^{2}\int_{-1}^{1} |x^2y^3|\, dydx,$$

$$\text{(c)} \quad \iint_S x\, dA; \quad S \text{ is the region between } y = x \text{ and } y = x^3. \quad (\textit{Note that } S \textit{ has two parts.})$$

Problem 2.
Find the volumes of the indicated solids by an iterated integration.
(a) The tetrahedron bounded by the coordinate planes and the plane $3x + 4y + z - 12 = 0$.
(b) The solid bounded by the parabolic cylinder $x^2 = 4y$ and the planes $z = 0$ and $5y + 9z - 45 = 0$.

Problem 3.
S is the smaller region bounded by $\theta = \pi/6$ and $r = 4\sin\theta$. Find the area of the region S by calculating $\iint_S r\, drd\theta$.

Problem 4.
Evaluate the following double integral by using polar coordinates

$$\iint_S \sqrt{4 - x^2 - y^2}\, dA,$$

where S is the first quadrant sector of the circle $x^2 + y^2 = 4$ between $y = 0$ and $y = x$.

Problem 5.
Find the volume of the solid lying under the graph of $z = f(x,y) = x^3 + 4y$ and above the region R in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

Problem 6.
Evaluate $\iint_R (2x - y)\, dA$, when R is the region bounded by the parabola $x = y^2$ and the line $x - y = 2$.

Problem 7.
Evaluate

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dxdy.$$

Problem 8.
Evaluate

$$\int_0^1 \int_x^1 \sin(y^2)\, dydx.$$

Problem 9.
Evaluate $\iint_R (2x + 3y)\, dA$, where R is the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Problem 10.
Use a double integral to find the area enclosed by one loop of the three-leaved rose $r = \sin(3\theta)$.

Problem 11.
Evaluate the integral

$$\int_{-\infty}^{\infty} \exp(-x^2)\, dx.$$

Problem 12.
Evaluate $\iiint_E yz\, dV$, where E is the solid tetrahedron bounded by the four planes
$x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.