**Practice Problems Ia**
**Math 250, Spring 2026**
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**Problem 1.**
Find two vectors of length 10, each of which is perpendicular to both $4\mathbf{i}+3\mathbf{j}+6\mathbf{k}$ and $-2\mathbf{i}-3\mathbf{j}-2\mathbf{k}$.

**Solution**

Let $\mathbf{v} =< a, b, c >$ be a vector perpendicular to $< 4, 3, 6 >$ and $< -2, -3, -2 >$. Then

$$< a, b, c > \cdot < 4, 3, 6 >= 0 \quad \text{and} \quad < a, b, c > \cdot < -2, -3, -2 >= 0.$$

Therefore, $4a + 3b + 6c = 0$ and $-2a - 3b - 2c = 0$. This pair of equations for $a$, $b$, and $c$ has infinitely many solutions. For example, with $c = 3$, we can solve the equations for $a$ and $b$. The result is $a = -6$ and $b = 2$. Thus, vector $< -6, 2, 3 >$, which has length 7, is perpendicular to $< 4, 3, 6 >$ and $< -2, -3, -2 >$. Then the required vectors are:

$$\left(\frac{10}{7}\right) < -6, 2, 3 > \quad \text{and} \quad \left(\frac{-10}{7}\right) < -6, 2, 3 > .$$

**Note**: *Any choice of $c \neq 0$ in $4a + 3b + 6c = 0$ and in $-2a - 3b - 2c = 0$ works. On the other hand, $c = 0$ leads to the equations $4a + 3b = 0$ and $2a + 3b = 0$ without solutions.*

**Problem 2.**
Find the smaller of the angles between the planes $3x - 2y + 5z = 7$ and $4x - 2y - 3z = 2$.

**Solution**
Normals to the planes are $< 3, -2, 5 >$ and $< 4, -2, -3 >$, so the cosine of the smaller angle is

$$\cos\theta = \frac{|12 + 4 - 15|}{(38)^{1/2}(29)^{1/2}} = \frac{1}{\sqrt{1102}}.$$

Thus, the angle $\theta = 1.540668$ radians or $\theta = 88.27°$. Note that with

$$\cos\theta = \frac{12 + 4 - 15}{(38)^{1/2}(29)^{1/2}} = -\frac{1}{\sqrt{1102}},$$

$\theta = 1.600925$ radians or $\theta = 91.73°$.

**Problem 3.**
Find the distance between the parallel planes $-3x + 2y + z = 9$ and $6x - 4y - 2z = 19$.

**Solution**
Suppose $P(x, y, z)$ is a point in the plane $ax + by + cz + d = 0$. The distance from any point $Q(x_0, y_0, z_0)$ to the plane equals the length of the orthogonal projection of the vector $\overrightarrow{PQ}$ onto a vector $\mathbf{n} =< a, b, c >$ normal to the plane, which is

$$\frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{|ax_0 + bx_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

The point $(0, 0, 9)$ belongs to the plane $-3x+2y+z = 9$. The distance between the parallel planes $-3x+2y+z = 9$ and $6x - 4y - 2z = 19$ is

$$\frac{|6(0) - 4(0) - 2(9) - 19|}{(36 + 16 + 4)^{1/2}} = \frac{37}{\sqrt{56}} \approx 4.9443.$$

**Problem 4.**
Find the equation of the plane, each of whose points are equidistant from $(-2, 1, 4)$ and $(6, 1, -2)$.

**Solution**
The line segment between the points is perpendicular to the plane. Its midpoint, $(2, 1, 1)$, belongs to the plane we are looking for. Then, vector $< 6 - (-2), 1 - 1, -2 - 4 >=< 8, 0, -6 >$ is perpendicular to the plane. Therefore, $8(x - 2) + 0(y - 1) - 6(z - 1) = 0$ or $4x - 3z = 5$ is the equation of the plane.

**Problem 5.**
Find equation of the plane through $(5, 1, 6)$, $(-1, -2, -3)$, and $(4, -2, 1)$.

**Solution**
Two vectors in the plane are

$$< 4 - (-1), -2 - (-2), 1 - (-3) > = < 5, 0, 4 >$$

and

$$< 5 - 4, 1 - (-2), 6 - 1 > = < 1, 3, 5 >$$

A vector normal to the plane is $< 5, 0, 4 > \times < 1, 3, 5 > = < -12, -21, 15 > = -3 < 4, 7, -5 >$. Therefore, an equation of the lane is $4(x+1) + 7(y+2) - 5(x+3) = 0$ or $4x + 7y - 5z = -3$.

**Problem 6.**
Find the equation of the plane through $(6, 2, -1)$ and perpendicular to the line of intersection of the planes $4x - 3y + 2z + 5 = 0$ and $3x + 2y - z + 11 = 0$.

**Solution**
The cross product of vectors normal to those planes is parallel to the line of intersection of the planes. Thus, a normal vector to the plane we seek is $< 4, -3, 2 > \times < 3, 2, -1 > = < -1, 10, 17 >$. Equation of the plane is $-1(x-6) + 10(y-2) + 17(z+1) = 0$ or $x - 10y - 17z = 3$.

**Problem 7.**
Write both the parametric equation of the line through the given point and parallel to the given vector

$$(-1, 3, 2), \qquad \mathbf{v} = < 4, 2, -1 > .$$

**Solution**
Parametric equation: $\quad x = -1 + 4t, y = 3 + 2t, z = 2 - t.$

**Problem 8.**
(a) Find the distance from $Q(2, -1, 3)$ to the line $x = 1 + 2t$, $y = -1 + 3t$, $z = -6t$.

(b) Find the distance between two nointersecting skew lines: $x = 1 + 2t$, $y = -2 + 3t$, $z = -4t$ and $x = 3t$, $y = 1 + t$, $z = -5t$.

**Solution**
(a) $P(1, -1, 0)$ is on the line and so are vectors $\mathbf{PQ} = < 1, 0, 3 >$ and $\mathbf{a} = < 2, 3, -6 >$. Then

$$d = \frac{|\mathbf{PQ} \times \mathbf{a}|}{|\mathbf{a}|} = \frac{| < -9, 12, 3 > |}{\sqrt{4 + 9 + 36}} = \frac{3\sqrt{26}}{7} \approx 2.1853. \quad \text{(see page 847 of the textbook)}$$

(b) $P(1, -2, 0)$ and $Q(0, 1, 0)$ are on the lines $x = 1 + 2t$, $y = -2 + 3t$, $z = -4t$ and $x = 3t$, $y = 1 + t$, $z = -5t$, respectively. Now, the length of the projection vector of $\mathbf{PQ} = < -1, 3, 0 >$ on vector

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = < 2, 3, -4 > \times < 3, 1, -5 > = < -11, -2, -7 >$$

is the distance between these skew lines. Indeed, vector $\mathbf{n}$ is perpendicular to both lines. Therefore,

$$d = \frac{|\mathbf{PQ} \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{|\mathbf{PQ} \cdot (\mathbf{a} \times \mathbf{b})|}{|\mathbf{a} \times \mathbf{b}|} = \frac{| < -1, 3, 0 > \cdot < -11, -2, -7 > |}{| < -11, -2, -7 > |} = \frac{5}{\sqrt{174}} \approx 0.3790.$$