Practice Problems I
Math 250, Spring 2024 – Jacek Polewczak

Problem 1.
Find two vectors of length 10, each of which is perpendicular to both $4\mathbf{i}+3\mathbf{j}+6\mathbf{k}$ and $-2\mathbf{i}-3\mathbf{j}-2\mathbf{k}$.

Problem 2.
Find the smaller of the angles between the planes $3x-2y+5z=7$ and $4x-2y-3z=2$.

Problem 3.
Find the distance between the parallel planes $-3x+2y+z=9$ and $6x-4y-2z=19$.

Problem 4.
Find the equation of the plane, each of whose points are equidistant from $(-2,1,4)$ and $(6,1,-2)$.

Problem 5.
Find equation of the plane through $(5,1,6)$, $(-1,-2,-3)$, and $(4,-2,1)$.

Problem 6.
Find the equation of the plane through $(6,2,-1)$ and perpendicular to the line of intersection of the planes $4x-3y+2z+5=0$ and $3x+2y-z+11=0$.

Problem 7.
Write both the parametric equation of the line through the given point and parallel to the given vector

$$(-1,3,2), \qquad \mathbf{v} = <4,2,-1> .$$

Problem 8.
(a) Find the distance from $Q(2,-1,3)$ to the line $x=1+2t$, $y=-1+3t$, $z=-6t$.

(b) Find the distance between two nointersecting skew lines: $x=1+2t$, $y=-2+3t$, $z=-4t$ and $x=3t$, $y=1+t$, $z=-5t$.

Problem 9.
Show that if an object moves subject only to a central force (that is $\mathbf{r}''(t)=c\mathbf{r}(t)$, where c is a constant), then the object moves in a plane.

Hint: Show that $\mathbf{r}(t)\times\mathbf{r}'(t)$ is a constant vector for all $t\geq 0$.

Problem 10.
Find the natural domain for the function

$$f(x,y)=\frac{\sqrt{\frac{x}{y}}}{\sqrt{1-x^2-y^2}}$$

Problem 11.
Describe the largest set S on which the following functions are continuous.

(a) $f(x,y)=\ln(1-x^2-y^2)$ (b) $f(x,y)=\begin{cases}\dfrac{\sin(xy)}{xy} & xy\neq 0\\ 1 & xy=0\end{cases}$ (c) $f(x,y)=\dfrac{x^3-2x^4y-y^5-4}{x^2+y^2-9}$

Problem 12.
If $f(x,y)=\dfrac{xy^2}{x^2+y^4}$, analyze the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

Problem 13.
Use $\epsilon-\delta$ proof to show that

$$\lim_{(x,y)\to(0,0)}\frac{3x^2y}{x^2+y^2}=0.$$

Problem 14.
Find the equation of the tangent plane at $(2, -1)$ for $f(x, y) = \dfrac{x^2}{y}$.

Problem 15.
Find the directional derivative of f at the given point P in the direction of $\mathbf{a}$.

(a) $f(x, y) = \exp(-xy)$; $P = (-1, 1)$; $\mathbf{a} = -\mathbf{i} + \sqrt{3}\mathbf{j}$

(b) $f(x, y, z) = x^2 + y^2 + z^2$; $P = (1, -1, 2)$; $\mathbf{a} = \sqrt{2}\mathbf{i} - \mathbf{j} - \mathbf{k}$

Problem 16.
Use the method of Lagrange's multipliers to find the minimum of $f(x, y) = x^2 + 4xy + y^2$, subject to the constraint $x - y - 6 = 0$.

Problem 17.
Use the method of Lagrange's multipliers to find the least distance between the origin and the plane $x + 3y - 2z = 4$.

Problem 18.
Find a point on the surface $z = 2x^2 + 3y^2$ where the the tangent plane is parallel to the plane $8x - 3y - z = 0$.

Problem 19.
Find all critical points; indicate whether each such point gives a local minimum, a local maximum, or whether it is a saddle point.

$$f(x, y) = x^2 + a^2 - 2ax\cos y; \quad S = \{(x, y) : -\infty < x < \infty, -\pi < y < \pi\}$$

Problem 20.
Use the Second-Partials Test to find the shortest distance from the origin to the plane $x + 2y + 3z = 12$.

Problem 21.
Use the Second-Partials Test to find the shape of the rectangular box of volume V_0 for which the sum of the edge lengths is least.