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Occurrences of Wet and Dry Spells in Taiwan Described by the Simple Markov Chain Probability Model and Lograrithmic Series
馬可夫連鎖機率模式和對數級數對臺灣乾濕期發生之解釋

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中文摘要

本文以簡易馬可夫連鎖機率模式和對數模式探討台灣八個主要測候站乾溼期發生之頻率，以每日降水量達0.1公厘或以上者爲溼日，並以卡方值檢定此二模式所計算得之乾溼期頻率是否與觀測值相符合。研究結果顯示台灣北部及東部東北季風之迎風地區，馬可夫機率模式符合溼期發生之頻率，但此模式不適用於解釋台灣西部溼期發生之頻率，在此地區，尤其是雨量特別集中於夏季之測站，對數模式比馬可夫模式更能解釋溼期之發生，馬可夫和對數模式皆不能解釋台灣西部乾期發生之頻率，對數模式比馬可夫模式更適用於台灣北部及東北部冬季迎風地區之乾期頻率。台北雖然是夏雨區，但冬季雨量亦不少，馬可夫和對數模式皆不能解釋乾溼期之頻率。

It has been found that in many parts of the world the persistence of wet and dry spells follows either the simple Markov chain probability model or the logarithmic series. Gabriel and Neumann (1962) has found that the simple Markov chain model fits Tel Aviv data of daily rainfall occurrences. Caskey (1963) recognized that theoretical probabilities derived from a simple Markov chain model agreed with the empirical values of probabilities of precipitation occurrences in intervals of various lengths at Denver. Weiss (1964) has pointed out that a simple Markov chain model is shown to fit sequences of wet and dry days in records of various lengths for Kansas City. Fitspatrick and Krishnan (1967) has concluded that the Markov model seems to be a practical statistical tool for assessing the long-term incidence of runs of wet or dry weather in central Australia. Hershfield (1970) presented examples of the simple Markov chain probabilities of dry spells in Minnesota and Washington, D.C., area. On the other hand, Williams (1952) has found that the frequency distribution of sequences of fine days at Harpenden conforms very closely to a logarithmic series. Cooke

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(1953) has indicated that dry-spell data at Moncton, New Brunswick, fit geometric progression while wet-spell data fit the logarithmic curves. Srinivasan (1964) applied a logarithmic model to the occurrence of monsoon rain spells in India.

Markov Chain Model

A simple Markov chain probability model requires the computation of two variables, i.e., P_o and P_1. P_o refers to the conditional probability of a wet day, given a previous dry day, and P_1 is the conditional probability of a wet day, given a previous wet day. According to this probability model, a given day should fall in one of the following four categories, i.e., W|W, D|W, W|D, D|D; where W and D denote wet and dry days, respectively. The first letter represents today; the second letter yesterday. A wet day is defined as one with rainfall equal to or exceeding 0.1 mm. Thus

$$P_1 = \frac{N(W|W)}{N(W|W) + N(D|W)},$$

$$P_o = \frac{N(W|D)}{N(W|D) + N(D|D)},$$

where N denotes the total number of days. The probability of a wet spell of length n is

$$(1 - P_1)P_1^{n-1},$$

and of a dry spell of length n is

$$P_o(1 - P_o)^{n-1}$$

The cumulative distribution through n is, for wet sequences

$$1 - P_1^{n-1}$$

and for dry sequences

$$1 - (1 - P_o)^n .$$

The probability for wet sequences greater than n is

$$P_1^n ,$$

and for dry sequences

$$(1 - P_o)^n .$$

Annual and monthly probabilities of wet and dry spells of various lengths for the years 1959 to 1969 at Keelung, Taipei, Taichung, Kaoshiung, Taitung, Hwalien, and Yilan, and for the years 1959 to

TABLE 1.--ESTIMATES OF P_1 AND P_o, 1959-1969

Station	Preceding day	Actual day		Estimate of probability	
		Wet	Wet & Dry	P_1 (W/W)	P_o (W/D)
Keelung	Wet	1552	2142	0.7245	
	Dry	592	1876		0.3155
Taipei	Wet	1256	1870	0.6716	
	Dry	613	2148		0.2853
Taichung	Wet	710	1188	0.5976	
	Dry	477	2830		0.1685
Kaoshiung	Wet	580	985	0.5888	
	Dry	406	3033		0.1338
Hengchun	Wet	706	1184	0.5962	
	Dry	478	2103		0.2272
Taitung	Wet	890	1534	0.5801	
	Dry	644	2484		0.2592
Hwalien	Wet	1307	1960	0.6668	
	Dry	651	2058		0.3163
Yilan	Wet	1574	2201	0.7151	
	Dry	628	1817		0.3456

1967 at Hengchun were calculated. Table 1 shows the procedure for estimating the conditional probabilities of wet days on an annual basis by the simple Markov chain model for the eight Taiwan stations. Both P_1 and P_o show distinct spatial variations with values higher in the northern and eastern Taiwan than in the western and southwestern Taiwan. P_1 is very hign at all stations, varying from nearly 0.73 at Keelung to approximately 0.59 at Kaoshiung. At all stations, P_o is much lower than P_1; it varies from 0.33 at Keelung to only about o.13 at Kaoshiung. Therefore, the conditional probability that a wet day follows a wet day is much higher than that a wet day follows a dry day.

Both P_1 and P_o show distinct seasonal variations (Table 2). Higher values are observed in winter than in summer at Keelung, Yilan, and Hwalien, where the peak values of both P_1 and P_o reach maxima in February. It is known that both the Mongolian anticyclone, which is frequently preceded by a polar front, and the Taiwan low contribute a great amount of rainfall to Taiwan in February. In contrast,

TABLE 2.--MONTHLY VARIATIONS IN P_1 AND P_o

Month		1	2	3	4	5	6	7	8	9	10	11	12
Keelung	P_1	0.756	0.808	0.749	0.647	0.685	0.709	0.547	0.659	0.735	0.667	0.803	0.769
	P_o	0.365	0.408	0.307	0.344	0.444	0.351	0.175	0.193	0.294	0.307	0.393	0.440
Taipei	P_1	0.656	0.766	0.688	0.623	0.622	0.729	0.658	0.699	0.700	0.599	0.629	0.623
	P_o	0.288	0.285	0.271	0.216	0.330	0.333	0.265	0.259	0.265	0.314	0.284	0.324
Taichung	P_1	0.438	0.654	0.552	0.451	0.689	0.734	0.521	0.675	0.636	0.400	0.485	0.390
	P_o	0.153	0.174	0.195	0.205	0.196	0.299	0.327	0.314	0.164	0.044	0.064	0.090
Kaoshiung	P_1	0.250	0.381	0.381	0.392	0.605	0.740	0.609	0.719	0.592	0.471	0.400	0.235
	P_o	0.084	0.089	0.090	0.111	0.138	0.287	0.330	0.346	0.235	0.086	0.052	0.043
Hengchun	P_1	0.415	0.296	0.397	0.436	0.586	0.781	0.701	0.737	0.673	0.489	0.487	0.275
	P_o	0.133	0.195	0.163	0.149	0.239	0.313	0.409	0.435	0.430	0.233	0.219	0.145
Taitung	P_1	0.505	0.602	0.540	0.566	0.577	0.699	0.607	0.672	0.654	0.518	0.477	0.414
	P_o	0.190	0.295	0.319	0.324	0.376	0.255	0.210	0.217	0.339	0.225	0.238	0.201
Hwalien	P_1	0.621	0.733	0.665	0.708	0.704	0.711	0.559	0.667	0.662	0.629	0.659	0.581
	P_o	0.433	0.477	0.370	0.479	0.430	0.306	0.157	0.209	0.296	0.259	0.397	0.250
Yilan	P_1	0.697	0.770	0.659	0.644	0.726	0.746	0.594	0.693	0.738	0.721	0.777	0.726
	P_o	0.386	0.430	0.396	0.399	0.444	0.343	0.155	0.219	0.272	0.216	0.545	0.426

values of both P_1 and P_0 are higher in summer than in winter at Taipei Taichung, Kaoshiung, Hengchun, and Taitung. All stations show peak values of P_1 and P_0 in May and June, obviously associated with plum rain fronts.

The cumulative conditional wet-spell probabilities derived from the simple Markov chain model can be compared to the estimated unconditional probabilities of wet spells of various lengths at the eight Taiwan stations (Figure 1). The estimated unconditional

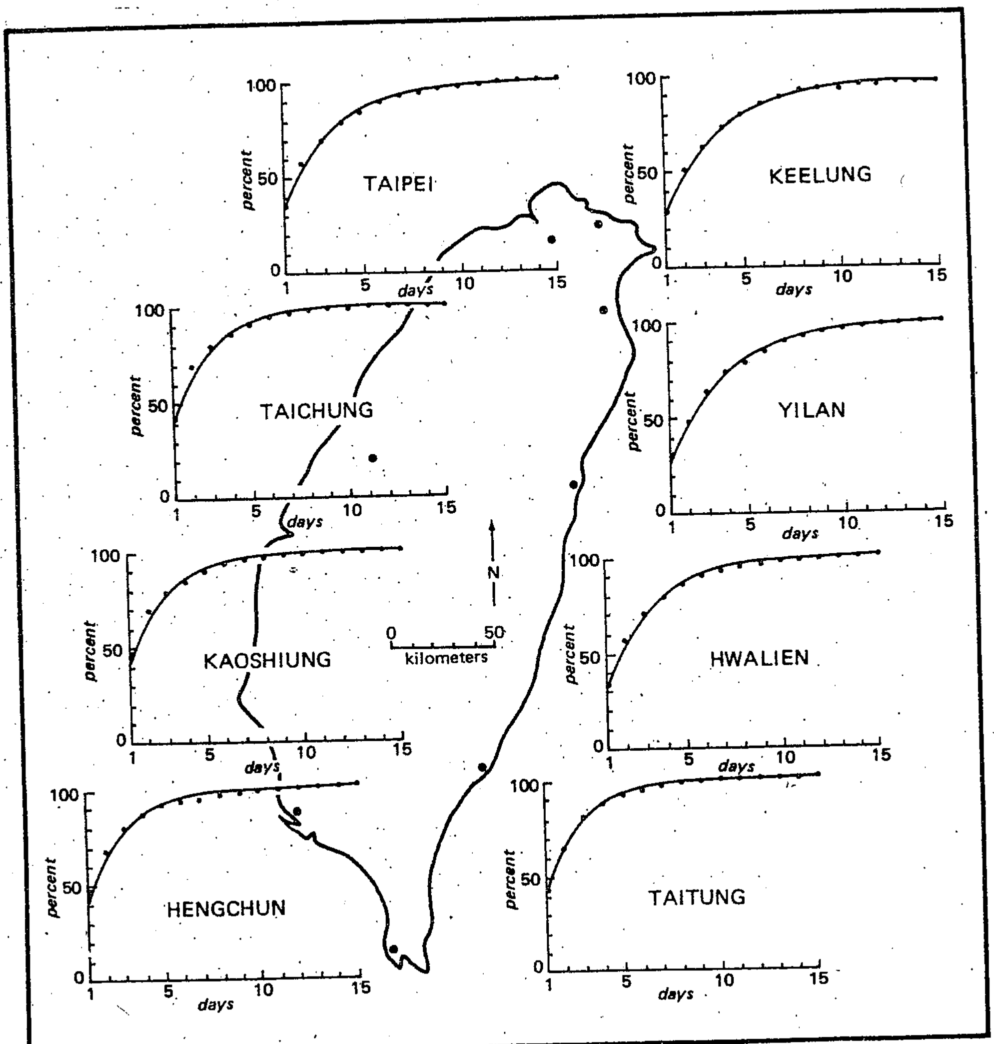

Figure I. Cumulative frequencies of wet spells derived from the simple Markov chain model (solid lines) and from observations (dots).

probability is taken as the ratio of the number of wet spells of a given length to the total number of wet spells. In general, the conditional probabilities fit well with the unconditional probabilities for wet spells for stations located in the north and east sectors of Taiwan. However, they deviate greatly from each other for dry spells at Taichung and Kaoshiung (Figure 2).

Chi-square values are calculated to test the goodness of fit of daily rainfall data to the simple Markov chain model. Spells with

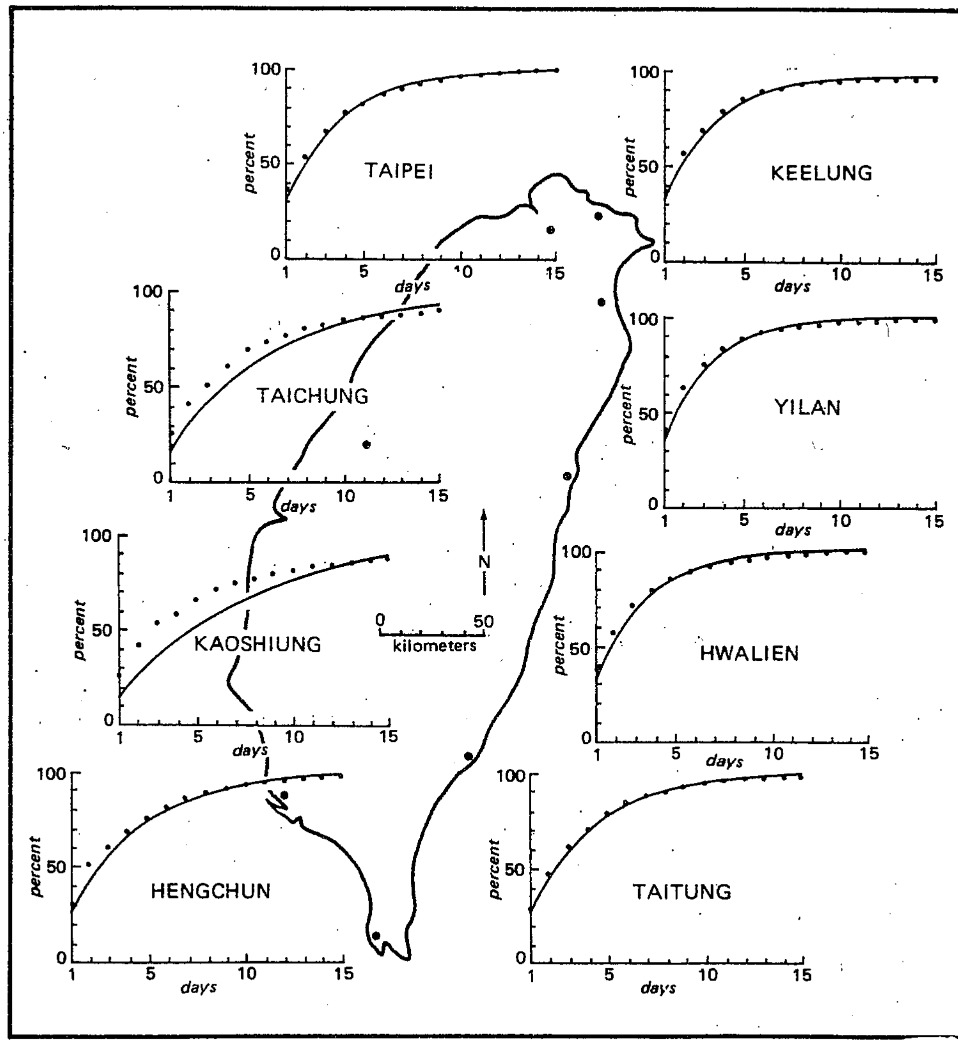

Figure 2. Cumulative frequencies of dry spells derived from the simple Markov chain model (solid lines) and from observations (dots).

expected frequencies of fewer than 5 are combined together to from a single cell listed at the bottom of each station. Chi-square values indicate that the theoretical frequencies of wet spells, derived from the simple Markov chain model, agree with the observed frequencies at the 0.1% level for Keelung, Yilan, Hwalien, and Taitung (Figure 1 and Table 3). These stations are located in the windward side of Taiwan with respect to the northeast monsoon. Winter synoptic flow types, such as the polar front, the Mongolian anticyclone, and the Taiwan low, contribute significant amounts of rainfall to these stations. The model fails to fit wet-spell sequences for Taipei, Kaoshiung, and Hengchun at the 0.1% level and for Taichung at the 5% level. These

TABLE 3. -- PREDICTED AND OBSERVED FREQUENCIES OF WET SPELLS OF VARIOUS LENGTHS AT EIGHT STATIONS ON TAIWAN, 1959-1969

Length	Keelung		Taipei		Taichung		Kaoshiung		Hengchun		Taitung		Hwalien		Yilan	
(days)	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS
1	163	180	201	224	192	204	168	203	193	230	270	278	217	252	179	206
2	118	126	134	126	114	127	99	85	115	97	156	146	145	130	128	105
3	86	73	91	72	68	47	58	40	68	56	91	103	97	86	91	104
4	62	61	61	61	41	30	34	21	41	33	53	42	64	55	65	54
5	45	39	41	39	24	25	20	22	24	24	30	27	43	38	47	34
6	33	33	27	33	15	16	12	15	35	37	18	12	29	29	33	36
7	24	18	18	18	9	8	7	9			10	16	19	15	24	27
8	17	17	12	17	5	7	8	13			6	7	13	12	17	15
9	12	10	8	10	7	12					7	11	8	8	12	9
10	9	5	5	5									6	6	9	11
11	7	7	12	31									13	21	21	27
12	5	5														
13	19	12														
Chi-square	11.29		41.17*		16.26**		24.47*		14.52**		13.10		16.13		19.31**	

*Significant difference at the 0.1% level
**Significant difference at the 5% level

stations are located on the western coast of Taiwan, the windward side with respect to the southwest monsoon. Rainfall is concentrated in summer with typhoons and monsoon troughs as major rainfall sources. The model approximates dry-spell sequences at the 5% level for

Keelung and Taitung but fails to fit those for other stations at the same significant level (Figure 2 and Table 4).

TABLE 4. -- PREDICTED AND OBSERVED FREQUENCIES OF DRY SPELLS OF VARIOUS LENGTHS AT EIGHT STATIONS ON TAIWAN, 1959-1969

Length	Keelung		Taipei		Taichung		Kaoshiung		Hengchun		Taitung		Hwalien		Yilan	
(days)	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS	PRI	OBS
1	186	219	175	217	81	129	55	106	109	142	167	184	207	255	216	259
2	127	119	125	102	67	73	47	66	84	92	124	125	141	123	141	139
3	87	72	89	84	56	47	41	42	65	44	92	87	97	91	92	76
4	59	62	64	55	46	49	35	26	50	43	68	59	66	50	61	48
5	41	37	46	34	39	41	31	26	39	31	50	53	45	39	40	28
6	28	24	33	31	32	19	27	25	30	33	37	44	31	23	26	25
7	19	11	23	26	27	20	23	12	23	20	28	16	21	20	17	8
8	13	13	17	9	22	12	20	11	18	13	20	17	14	9	11	8
9	9	6	12	17	18	8	17	9	14	8	15	17	10	11	8	9
10	19	25	8	15	15	10	15	5	11	12	11	10	7	9	14	25
11			6	5	13	8	13	9	8	9	32	34	15	24		
12			15	18	63	63	11	5	28	31						
13							72	66								
Chi-square	16.32		32.05*		51.51*		83.01*		25.35**		10.78		28.45**		32.13*	

*Significant difference at the 0.1% level
**Significant difference at the 5% level

Logarithmic Series

A wet or dry spell of length n by a logarithmic series can be written as:

$$ax^n/n.$$

The two constants, a and x, can be calculated once the total number of wet or dry days (N) and of spells of various lengths (S) are known:

$$S = a \log_e (1 + N/a),$$

$$N = a\, x/(1 - x).$$

However, the solution of these two simultaneous equations is complicated and indirect. Fisher, et al (1943) and Williams (1947) presented simple graphic and mathematic methods to derived values of a and x.

As with the simple Markov chain model, chi-square values are also calculated to test the agreement between the observed and expected frequencies of wet and dry sequences of various lengths derived from the logarithmic series. It is shown that the logarithmic series fits sequences of wet spells at the 0.1% level for Taichung, Kaoshiung Hengchun, and Hwalien. It is notable that the former three stations are located on the western coast of Taiwan, where the simple Markov chain model fails to approximate the occurrences of wet-spell sequences. The logarithmic series conforms the sequences of dry spells at the 0.1% level for Keelung, Yilan, and Hwalien but fails to conform those for the other stations at the same significant level (Table 5).

TABLE 5. -- AGREEMENTS BETWEEN PREDICTED AND OBSERVED FREQUENCIES OF WET AND DRY SPELLS AT THE 0.1% AND 5% LEVELS

Stations	Markov Chain		Logarithmic Series	
	Wet	Dry	Wet	Dry
Keelung	Yes	Yes	No	Yes
Yilan	No**	No	No	Yes
Hwalien	Yes	No**	Yes	Yes
Taitung	Yes	Yes	No	No
Taipei	No	No	No	No
Taichung	No**	No	Yes	No
Kaoshiung	No	No	Yes	No
Hengchun	No**	No**	Yes	No

Yes indicates agreement and No indicates disagreement.
** indicates significant difference at the 5% level but insignificant difference at the 0.1% level.

Conclusion

Green (1965) has pointed out that the statistical model may be helpfully suggestive regarding the mechanism of rainfall occurrences at places where it seems to apply. In Taiwan, the fit of the simple Markov chain model to wet-spell sequences at the stations deriving rainfall mainly from winter synoptic flow types, stands in contrast to data from the stations deriving rainfall mainly from summer synoptic flow types, where the logarithmic series is a better statistical tool to approximate wet-spell frequencies. In terms of dry-spell sequences, the logarithmic series appears more powerful than the simple Markov chain model in approximating the observed data for the stations located in the northeast sector of Taiwan, the windward side with respect to the northeast monsoon. Both the simple Markov chain model and logarithmic series fail to conform the dry-spell sequences for the western part of Taiwan where rainfall is concentrated in summer. At Taipei, a transitional station between the northeast and southwest monsoonal rainfall regimes, neither the simple Markov chain model nor the logarithmic model approximates the sequences of wet or dry spells.

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