INCOMPLETE INFORMATION, HIGH-LOW BIDDING AND PUBLIC INFORMATION IN FIRST PRICE AUCTIONS*

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High-low bidding refers to auctions of a series of objects where for a subset of the auctioned objects one of two buyers bids high on some objects and low on others while the other buyer does the reverse. This paper shows that high-low bidding does not imply collusive behavior among buyers. In particular, through a formal modeling of a noncooperative game of information acquisition and bidding decisions, it shows that high-low bidding can be obtained. The paper also demonstrates that if the cost of information to a seller is less than the equilibrium joint expected information cost of the buyers, then it pays the seller to provide public information. Finally, if the provision of public information is not feasible but a seller can know whether buyers have acquired information, then it pays the seller to make known to the buyers and to pursue a policy that just prior to the start of an auction the seller will announce which buyer has acquired information.

1. Introduction

This paper investigates three policy questions. First, does the high-low bidding phenomenon imply collusive behavior among buyers in an auction? To the contrary, it provides a noncooperative game model which results in high-low bidding. The result is obtained through a formal modeling of information acquisition activities. Second, should a seller provide public information on auctioned objects? It states the condition under which it pays a seller to provide public information. Third, if the provision of public information is not a feasible alternative, is there an alternative which leads to a higher payoff to a seller? It demonstrates an institutional alternative which leads to a higher payoff to a seller.

The layout for the rest of the paper is as follows. §§2 and 3 provide a brief introduction to the literature related to the three policy questions and summarize the contribution of this paper to the literature. §4 spells out a model of first price auction where buyers do not know with certainty whether their opponents are informed. It provides all the proofs of the results of the paper. §5 summarizes the results of the paper.

2. High-Low Bidding: Does It Imply Collusive Behavior?

Studies of the first policy question date back to a 1971 article by Capen, Clapp and Campbell, who organize an interesting set of bid data that challenges researchers for an explanation. Specifically, they observe that two companies, Atlantic Richfield and Humble, as equal partners in exploratory efforts, bid differently on 55 tracts in the Alaska North Slope Sale. On one tract Humble's bid is about 0.03 of Atlantic Richfield's bid and on another Humble's bid is about 17 times higher than Atlantic Richfield's. This phenomenon has since been coined "high-low bidding."

Two explanations of the phenomenon are offered in the literature. One explanation relies on the presence of information. Capen, Clapp and Campbell (1971) consider the case of imperfect information on the true value of the auctioned object in a single object auction. They suggest that all buyers have acquired information but might have inferred different information signals from the same data base and thus have different

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value estimates. They do not consider the information acquisition decisions of the buyers. In contrast, Lee (1982) explicitly considers the information acquisition decisions of the buyers. Specifically, he assumes that the value of the auctioned object can take on a high or low value. A piece of perfect information, available at a cost, will reveal the true value of the object. A two-stage game model, consisting of an information acquisition game and a bidding game played in that sequence, is analyzed. In the information acquisition game, each buyer chooses the probability of being informed. An information acquisition equilibrium is established. Once the information acquisition game is played, the information acquisition decisions of the buyers are made public. If buyers end up both being informed or both being uninformed, they submit identical bids in the bidding game. If only one buyer is informed and the information indicates a high value for the tract, a mixed strategy bidding equilibrium is established. The informed buyer generates his bid from a bid distribution which stochastically dominates the bid distribution of the uninformed buyer. The two bid distributions share a common support. On the average, the informed buyer bids higher than the bid by the uninformed one. If only one buyer is informed and the information indicates a low value for the tract, the informed buyer bids the low value and thus bids lower than that bid by the uninformed buyer. This explanation of the high-low bidding phenomenon obviously implies variation in bids but whether it adequately explains the wide variation in bids remains questionable.

Another explanation makes use of multiple object auctions to explain the high-low bidding phenomenon. Buyers are assumed to know the true value of the auctioned object. Engelbrecht-Wiggans and Weber (1979) consider the case where buyers have positive value for one and only one object. A mixed strategy equilibrium is analyzed. Palfrey (1980) considers the case where buyers face budgetary constraints. A pure strategy equilibrium is analyzed. In equilibrium both models result in buyers submitting high bids on some object(s) and low bids on others. This explanation of the high-low bidding phenomenon faces two difficulties. First, it is hard to believe that an additional object (e.g. a resource lease) has zero value (even when there is an imperfect resale market). Second, the high-low bidding phenomenon is common across high price and low price resources. It is hard to believe that oil companies face budgetary constraints in bidding for relatively low price resources such as hydrothermal types of geothermal resources.

This paper contributes to the first set of explanations of the high-low bidding phenomenon. Information acquisition activities are explicitly modeled. I shall assume that the value of an auctioned object can take on a high or a low value. A piece of information, available at a cost, will reveal the true value of the object. In contrast to the two stage game model of Lee (1982), I shall assume that buyers simultaneously choose the probability of being informed and bid distributions (depending on the information revealed) from one of which a bid is generated, without knowing whether their opponents are informed. An equilibrium of the model is then analyzed.

An interesting result of this paper is that the model provides an explanation of the high-low bidding phenomenon. When a buyer is informed that the auctioned object is of high value, he never bids less than that of an uninformed buyer. Technically speaking, the bid distributions do not share a common support. On the other hand, when a buyer is informed that the auctioned object is of low value, he bids the low value and, thus, never bids higher than that of an uninformed buyer. Thus, wide variation in bids is expected. The model also explains small variation in bids. For example, when both buyers are informed that the auctioned object is of low value, both buyers enter identical low bids. Also, when both buyers are informed that the auctioned object is of high value, their bids are generated from the same bid distribution.
3. Public Information: Is It Profitable to a Seller?

The study of the second policy question of public information provision by a seller in a first price auction is due to a paper by Hugart (1975). In a study of asymmetric information in resource leasing, he concludes that there is evidence of excessive investment in socially wasteful information and hence a seller should purchase and disseminate information free of charge to all potential buyers. Reece (1978) provides a justification of Hugart's proposal by showing, in a numerical example, that a seller can capture a greater portion of the economic rent from the buyers through public information provision. Milgrom and Weber (1982), in an interesting paper, substantiate that there is incentive for a seller to reveal truthfully the information that he possesses. This ensures that buyers would believe the information provided by a seller.

Lee (1982) extends the literature by asking under what condition it pays a seller to acquire and disseminate information free of charge to the buyers. Using a two-stage game model, he provides three precise statements of the condition under which it pays a seller to provide information. The first statement is that if the cost of information to a seller is less than the expected bonus bid loss of the seller due to the presence of a single informed buyer, then it pays the seller to provide public information. The second statement is that if the cost of information to a seller is less than the profit that the seller can capture from a single informed buyer, then it pays the seller to provide public information. The third statement is that if the cost of information to a seller is less than the joint expected information cost of the buyers, then it pays the seller to provide public information because the seller can reap the gains in efficiency.

An interesting result of this paper is that the third statement of the public information condition is not an artifact of the special feature of the two-stage game model, namely, that buyers know whether their opponents are informed when they bid. The paper proves that the statement holds true even if buyers do not know with certainty whether their opponents are informed. Thus the third statement of the public information condition seems to be more general than expected.

In the process of analyzing an equilibrium of the model in this paper and comparing it with that of Lee (1982), I show that if the provision of public information is not feasible, a seller will be better off under the two-stage game model than under the alternate formulation of the model. Hence, an institutional alternative exists to improve the payoff to a seller, namely, that a seller should make known to buyers and pursue the policy that before the start of an auction the seller would announce which buyer has acquired information. This option is feasible whenever information can be generated only through an appraisal of the auctioned object which requires the consent of the seller.

4. First Price Auction Model

A first price auction is one where the highest bidder wins the auctioned object and pays the amount he bids. The auctioned object is assumed to take on one of two values, designated by \( V_H \) for a high value object, and \( V_L \) for a low value object. The likelihood that the object will take on the high value is given by a probability \( p \) and the low value by a probability \( 1 - p \). Let there be two buyers of the object. The probability values are public knowledge to the buyers. A piece of perfect information that can reveal the true value of the object is available at a cost \( K \). Assume that

\[
p(V_H - V_L) > K/(1 - p),
\]

i.e. conditional on the true value of the object being high, the ex ante benefit of having the piece of information is greater than the information cost adjusted for the likelihood that the value of the object is low.
Consider buyer 1. Suppose that he purchases the piece of information. If it reveals that the value of the object is high, he generates a bid from a bid distribution \( F(\cdot) \). If it reveals that the value of the object is low, he submits a bid \( V_L \). If he does not purchase the information, he generates his bid from a bid distribution \( M(\cdot) \). Let the corresponding bid distributions for buyer 2 be \( G(\cdot) \) and \( N(\cdot) \). Assume that buyer 1 has a subjective probability assessment that buyer 2 is informed. Let this probability be \( r \).

First, it will be shown that the supports of the two bid distributions do not overlap. To show that, a contradiction is derived under an opposite assumption. If buyer 1 is informed, his profit is given by

\[
\pi_I = p \int_{V_F}^{V_H} (V_H - b) \left[ rG(b) + (1 - r)N(b) \right] dF(b) - K
\]

where \([V_F, V_H]\) is the support of the bid distribution \( F(\cdot) \). If buyer 1 is not informed, his profit is given by

\[
\pi_U = \int_{V_M}^{V_H} \left( p(V_H - b) \left[ rG(b) + (1 - r)N(b) \right] + (1 - p)(V_L - b) \left[ r + (1 - r)N(b) \right] \right) dM(b)
\]

where \([V_M, V_H]\) is the support of the bid distribution \( M(\cdot) \). Buyer 1 chooses a bid distribution \( F(\cdot) \) to maximize \( \pi_I \), and a bid distribution \( M(\cdot) \) to maximize \( \pi_U \). In a symmetric equilibrium, the two profit maximization conditions are given by

\[
(V_H - b) \left[ rF(b) + (1 - r)M(b) \right] = C_1, \quad \text{for } b \in [V_F, V_H],
\]

\[
p(V_H - b) \left[ rF(b) + (1 - r)M(b) \right] + (1 - p)(V_L - b) \left[ r + (1 - r)M(b) \right] = C_2,
\]

for \( b \in [V_M, V_H] \), where \( C_1 \) and \( C_2 \) are constants.

Since we assume that the supports of \( F(\cdot) \) and \( M(\cdot) \) overlap, let \( b \in [V_F, V_H] \cap [V_M, V_H] \). Upon substituting (4) into (5) and rearranging terms, we have:

\[
M(b) = \frac{(C_2 - pC_1)/(1 - p)(V_L - b) - r}{1 - r}.
\]

Now, in equilibrium, \( r \) must be the value so that \( pC_1 - C_2 = K \), i.e. as long as both buyers choose to acquire information with positive probability, each has the same expected profit when informed as when uninformed. Hence, we have

\[
M(b) = \frac{K/(1 - p)(b - V_L) - r}{1 - r}.
\]

We note immediately that \( M'(b) < 0 \), violating the usual property of a probability distribution. Thus, the supports of \( F(\cdot) \) and \( M(\cdot) \) do not overlap.

Given the above result and if buyer 1 is informed, his expected profit must be given by

\[
\pi_I = p \int_{V_F}^{V_H} (V_H - b) \left[ rG(b) + (1 - r) \right] dF(b) - K.
\]

If buyer 1 is not informed, his profit must be given by

\[
\pi_U = \int_{V_M}^{V_H} \{ r(1 - p)(V_L - b) \left[ r(1 - r)(V_H - b)N(b) \right] \} dM(b),
\]

where \( V_e \equiv pV_H + (1 - p)V_L \). In a symmetric equilibrium, the profit maximization
conditions are given by
\[(V_H - b)[rF(b) + (1 - r)] = C_3, \quad \text{for } b \in [V_F, V_M], \quad \text{and} \quad (4')\]
\[r(1 - p)(V_L - b) + (1 - r)(V_e - b)M(b) = C_4 \quad (5')\]
for \(b \in [V_M, V_F]\), where \(C_3\) and \(C_4\) are constants. In equilibrium, it must be the case that the expected profit of both buyers is zero. The reason is that in equilibrium they will compete away all potential profits. Specifically, \(C_4 = 0\). Thus, (5') implies
\[M(b) = r(1 - p)(b - V_L) \quad (6)\]
\[(1 - r)(V_e - b) \quad (6)\]
Now the fact that \(M(V_M) = 1\) implies, via (6), that \(V_M = \frac{1}{1 - rp} [V_e - rpV_H]\). Note \(V_M < V_e\); in words: an uninformed buyer always bids less than the expected value of the object. The intuition is as follows. Condition (3') contains a "winner's curse" term, \(\int_{V_M}^{V_F} r(1 - p)(V_L - b) dM(b) < 0\). An uninformed buyer always faces a positive probability that he wins an object which has a value less than what he bids. To avoid the winner's curse problem, an uninformed but sophisticated buyer bids strictly less than the expected value of the object.

Moreover, the fact that \(M(V_F) = 0\) implies, via (6), that \(V_F = V_L\). We note that condition (4') and the zero profit equilibrium condition, \(pC_3 = K\), imply that
\[F(b) = \frac{K/p(V_H - b) - (1 - r)}{r} \quad (7)\]
Also, \(F(V_F) = 1\) implies \(V_F = V_H - K/p\), and \(F(V_F) = 0\) implies \(V_F = V_H - K/p(1 - r)\).

Finally, an equilibrium result, \(V_F = V_M\), gives us the equilibrium value of \(r\) as
\[r = \frac{p(1 - p)(V_H - V_L) - K}{p[(1 - p)(V_H - V_L) - K]} \quad (8)\]

In summary, when a buyer is informed that the auctioned object is of high value, he never bids less than an uninformed buyer's bid. When a buyer is informed that the auctioned object is of low value, he never bids higher than an uninformed buyer. These results provide the basis for explaining the high-low bidding phenomenon. Since these results are generated from a noncooperative game model, we can conclude that high-low bidding does not imply collusion among buyers. This completes a study of the first policy question.

To address the second policy question, we note that the seller's expected profit is given by
\[\pi_S = 2(1 - r)^2 \int_{V_M}^{V_F} b M(b) dM(b) + 2r(1 - r) \left\{ p \int_{V_F}^{V_H} b dF(b) + (1 - p) \int_{V_M}^{V_F} b dM(b) \right\}
+ 2r^2 \int_{V_F}^{V_H} b F(b) dF(b) + r^2(1 - p)V_L \quad (9)\]
Substituting (6) and (7) into (9) and integrating by parts several times, we have
\[\pi_S = V_e - 2rK \quad (10)\]
If the seller chooses to provide the information free of charge to the buyers, it is
obvious that both buyers will use the information and the seller’s expected profit is \( \pi_S = V_e - K \). Thus, the expected profit of the seller is higher under the public information case if \( K < 2rK \). In words, if the information cost to the seller is less than the joint expected information cost of the two buyers, then it pays the seller to provide public information because that would allow him to reap the gains in efficiency.

To study the third policy question, we recall that in the two-stage game model analyzed by Lee (1982), if the seller does not provide free information, the seller’s expected profit is given by

\[
\pi_S = V_e - 2\bar{r}K
\]

Comparing (8) and (12), it is clear that \( r > \bar{r} \). Suppose that the provision of public information is not feasible (e.g., \( K > 2rK \)). If a seller knows which buyer has acquired information, the seller will be better off to announce that fact to the buyers before they submit bids. Thus an institutional alternative exists that would improve the payoff to a seller, namely, the seller should inform the buyer and pursue the policy that just before the start of an auction the seller would announce which buyer has acquired information. This option is feasible whenever information can be generated only through an appraisal of the auctioned object which requires the consent of the seller.

\[
\bar{r} = \frac{p(1-p)(V_H - V_L) - K}{p(1-p)(V_H - V_L)}
\]

5. Conclusion

Three interesting results emerge from an analysis of a first price auction model. First, it demonstrates that high-low bidding does not imply collusive behavior among buyers. Moreover, the model explains both small and wide variations of bids even in a context where information reveals the true value of an auctioned object. The significant aspect of imperfect information in this model is that buyers are uncertain whether their opponents are informed.

Second, it specifies a precise and economically intuitive condition under which it pays a seller to provide public information, namely, if the cost of information to a seller is less than the joint expected information cost of the buyers, then it pays the seller to provide public information. The seller reaps the gains in efficiency.

Third, if the provision of public information is not a feasible alternative but a seller will know which buyer will have acquired information, it pays the seller to make known to the buyers and to pursue the policy that just before the start of an auction the seller will announce the information acquisition decisions of the buyers. Thus, an institutional alternative exists that can improve the seller’s payoff.\(^1\)

\(^1\)The author would like to thank Professor Paul Milgrom for pointing out that the assumption that the value of the auctioned object can take on only two values implies the bidders’ posterior value estimates are affiliated. Thus, results from the paper by Milgrom and Weber (1982) are also applicable here.

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References


