Chapter 2

Solutions to Supplementary *Check for Understanding* Problems

Scientific Notation

1. Write each of the following values in proper scientific notation.

a) 0.2660 mg   b) 19,370 m   c) 0.00035900 g   d) 87.420 s

Answers: a) 2.660×10^{-1} mg

b) 1.937×10^{4} m

c) 3.5900×10^{-4} g

d) 8.7420×10^{1} s

Solutions

First express the original quantity as a number between 1 and 10. Then multiply by the appropriate power of ten. This is determined by noting how many places, and in which direction, the decimal point was moved in the first step. For (a) and (c) the decimal point is moved to the right thereby making the number larger. This means that the power of ten to use is negative so that you are multiplying by a value smaller than 1. For (b) and (d) the decimal point is moved to the left thereby reducing the number. This means that the power of ten to use is positive so that you are multiplying by a value greater than 1. Note that the trailing zeros are significant for (a), (c) and (d) because of the presence of the decimal point

2. Write each of the following values in decimal form.

a) 1.28×10^{-7} m   b) 4.33×10^{4} L   c) 0.0506×10^{4} kg   d) 669×10^{-5} cm^3

Answers: a) 0.000000128 m

b) 43300 L

c) 5060 kg

d) 0.00669 cm^3

Solutions

a) Since the power of ten is a negative value, the number is less than one. Move the decimal point 7 places to the left using zeros to fill in six of the needed places.

b) Since the power of ten is a positive value, the number is greater than one. Move the decimal point 4 places to the right. Because the number only has 3 significant figures do not indicate the decimal point.

c) Since the power of ten is a positive value, move the decimal point 4 places to the right. Because the number only has 3 significant figures do not indicate the decimal point.

d) Since the power of ten is a negative value, move the decimal point 5 places to the left using zeros to fill in two of the needed places.

Significant Figures

1. How many significant figures are in each of the following measurements?

a) 33.04 L b) 0.0099100 m c) 2500 s d) 6.1800×10^{-8} g

Answers: a) 4 sig. fig.

b) 5 sig. fig.

c) 2 sig. fig.

d) 5 sig. fig.

Solutions

Recall that all digits in a measurement are significant except trailing zeros in numbers without a decimal point and all leading zeros.

a) All of the digits are significant.

b) The leading zeros are not significant, but because of the decimal point the trailing zeros are significant, so there are 5 significant figures.

c) Because there is no decimal point, the trailing zeros are not significant so there are just 2 significant figures.

d) All of the digits shown in a quantity expressed in proper scientific notation are significant.

2. Round off each of the following to 2 significant figures.

a) 2.65849 cm b) 472.8 s c) 0.028411 km d) 17069 ft

Answers: a) 2.7 cm

b) 470 s or 4.7×10^2 s

c) 0.028 km

d) 17000 ft or 1.7×10^4 ft

Solutions

a) The tenths place is rounded up because the next digits (5849) are not exactly 500000...

b) The trailing zero is not significant because no decimal point is shown.

c) The leading zeros are not significant and the thousandths place is left unchanged because the next digit (4) is less than 5.

d) The trailing zeros are not significant because no decimal point is shown.

3. For each of the following, calculate the result and round it off to the proper number of significant figures. The units of these measurements have been omitted for clarity. Do not use a calculator.

a) $6.127 \times 10^5 + 4.53 \times 10^2 =$

b) $9.45 \times 10^{-2} - 2.66 \times 10^{-3} =$

c) $6.018 \times 10^{-3} \div 2.00 \times 10^{-7} =$

Answers: a) 6.132×10^5

b) 9.18×10^{-2}

c) 3.01×10^4

Solutions

a) First covert 4.53×10^2 into a number having a power of ten exponent equal to 5. This requires increasing the exponent by three units (making a thousandfold increase in the power of ten) so the decimal number must be decreased a thousandfold.

$4.53 \times 10^2 \rightarrow 0.00453 \times 10^5$

Then add the decimal numbers and use the common power of ten.

$$\begin{array}{rl} & 6.127 \times 10^5 \\ + & 0.00453 \times 10^5 \\ \hline & 6.13153 \times 10^5 \end{array} \rightarrow 6.132 \times 10^5$$

Note that the uncertain digit (in blue) in the first measurement is in the thousandths place while the uncertain digit in the second measurement is in the hundred thousandths place. Thus, the sum is rounded off to the (larger) thousandths place. Since the digits to be dropped (53) are greater than 50, the preceding digit is increased by 1.

b) First covert 2.66×10^{-3} into a number having a power of ten exponent equal to -2. This requires increasing the exponent by one unit (making a tenfold increase in the power of ten) so the decimal number must be decreased tenfold.

$2.66 \times 10^{-3} \rightarrow 0.266 \times 10^{-2}$

Then add the decimal numbers and use the common power of ten.

$$\begin{array}{rl} & 9.45 \times 10^{-2} \\ - & 0.266 \times 10^{-2} \\ \hline & 9.184 \times 10^{-2} \end{array} \rightarrow 9.18 \times 10^{-2}$$

Note that the uncertain digit (in blue) in the first measurement is in the hundredths place while the uncertain digit in the second measurement is in the thousandths place. Thus, the sum is rounded off to the (larger) hundredths place. Since the digit to be dropped (4) is less than 5, the preceding digit is unchanged.

c) The calculated result is: $\dfrac{6.018 \times 10^{-3}}{2.00 \times 10^{-7}} = 3.009 \times 10^{4}$

Note the number of significant figures in each measurement in this calculation.

6.018×10^{-3} (4 sig. fig.) 2.00×10^{-7} (3 sig. fig.)

Since 3 significant figures is the fewest number in any measurement, the result is rounded off to 3 significant figures. Since the first digit to be dropped (9) is greater than 5, the preceding digit is increased by 1.

$3.009 \times 10^{4} \rightarrow 3.01 \times 10^{4}$

Unit Conversions

1. Write the solution map for the conversion of TL into nL. What is the conversion factor for each step?

Answers: The solution map is TL → L → nL

The conversion factor for the first step is: $\dfrac{10^{12}\ \text{L}}{1\,\text{TL}}$.

The conversion factor for the second step is: $\dfrac{1\,\text{nL}}{10^{-9}\ \text{L}}$.

Solutions

Each step of the solution map requires a conversion factor.

For TL → L, we need a conversion factor with units of $\dfrac{\text{L}}{\text{TL}}$ which will cancel *TL* and introduce *L* into the numerator.

Since 1 TL = 10^{12} L we use $\dfrac{10^{12}\ \text{L}}{1\,\text{TL}}$.

For L → nL, we need a conversion factor with units of $\frac{\text{nL}}{\text{L}}$ to cancel *L*.

Since 1 nL = 10^{-9} L we use $\frac{1\,\text{nL}}{10^{-9}\,\text{L}}$.

2. How many meters does an automobile traveling 65 mph move in one second?

Answer: 29 m

Solution

Using conversion factors that you readily know results in the following solution map.

$$\frac{\text{miles}}{\text{hr}} \rightarrow \frac{\text{miles}}{\text{min}} \rightarrow \frac{\text{miles}}{\text{s}} \rightarrow \frac{\text{ft}}{\text{s}} \rightarrow \frac{\text{in}}{\text{s}} \rightarrow \frac{\text{cm}}{\text{s}} \rightarrow \frac{\text{m}}{\text{s}}$$

Putting all of the conversion factors together yields:

$$\frac{65\,\cancel{\text{miles}}}{\cancel{\text{hr}}} \times \frac{1\,\cancel{\text{hr}}}{60\,\cancel{\text{min}}} \times \frac{1\,\cancel{\text{min}}}{60\,\text{s}} \times \frac{5280\,\cancel{\text{ft}}}{1\,\cancel{\text{mile}}} \times \frac{12\,\cancel{\text{in}}}{1\,\cancel{\text{ft}}} \times \frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}} \times \frac{10^{-2}\,\text{m}}{1\,\cancel{\text{cm}}} = \frac{29\,\text{m}}{\text{s}}$$

3. Make each of the following conversion and express the answer in proper scientific notation.

   a) 411 nm to cm

   b) 1.19 g/mL to μg/kL

Answers: a) 4.11×10^{-5} cm

b) 1.19×10^{12} μg/kL

Solutions

a) The solution map for this calculation is:

$$\text{nm} \rightarrow \text{m} \rightarrow \text{cm}$$

The conversion factor needed in the first step is $\frac{10^{-9}\,\text{m}}{1\,\text{nm}}$.

The conversion factor needed in the second step is $\frac{1\,\text{cm}}{10^{-2}\,\text{m}}$.

Putting these together yields:

$$411\,\cancel{\text{nm}} \times \frac{10^{-9}\,\cancel{\text{m}}}{1\,\cancel{\text{nm}}} \times \frac{1\,\text{cm}}{10^{-2}\,\cancel{\text{m}}} = 4.11 \times 10^{-5}\,\text{cm}$$

b) The solution map for this calculation is:

$$\frac{\text{g}}{\text{mL}} \rightarrow \frac{\text{g}}{\text{L}} \rightarrow \frac{\text{g}}{\text{kL}} \rightarrow \frac{\mu\text{g}}{\text{kL}}$$

The conversion factor needed in the first step is $\frac{1\,\text{mL}}{10^{-3}\,\text{L}}$.

The conversion factor needed in the second step is $\frac{10^{3}\,\text{L}}{1\,\text{kL}}$.

The conversion factor needed in the last step is $\frac{1\,\mu\text{g}}{10^{-6}\,\text{g}}$.

Putting these together yields:

$$\frac{1.19\,\cancel{\text{g}}}{\cancel{\text{mL}}} \times \frac{1\,\cancel{\text{mL}}}{10^{-3}\,\cancel{\text{L}}} \times \frac{10^{3}\,\cancel{\text{L}}}{1\,\text{kL}} \times \frac{1\,\mu\text{g}}{10^{-6}\,\cancel{\text{g}}} = \frac{1.19 \times 10^{12}\,\mu\text{g}}{\text{kL}}$$

4. The price of gold recently reached \$1780 per ounce. What is the value of 150 g of gold at this price? Note that gold is priced by the troy ounce (1 lb = 14.58 troy ounces).

Answer: \$8600

Solution

What we know: g gold; \$/troy oz gold; troy oz/lb; g/lb

Desired answer: \$

The solution map for this problem is:

g gold → lb gold → troy oz gold → \$

The conversion factor needed in the first step is $\frac{1\,\text{lb gold}}{454\,\text{g gold}}$.

The conversion factor needed in the second step is $\frac{14.58\,\text{troy oz gold}}{1\,\text{lb gold}}$.

The conversion factor needed in the last step is $\frac{\$1780}{1\,\text{troy oz gold}}$.

Putting these together yields:

$$150\,\cancel{\text{g gold}} \times \frac{1\,\cancel{\text{lb gold}}}{454\,\cancel{\text{g gold}}} \times \frac{14.58\,\cancel{\text{troy oz gold}}}{1\,\cancel{\text{lb gold}}} \times \frac{\$1780}{1\,\cancel{\text{troy oz gold}}} = \$8600$$

5. Express each of the following quantities as a value between 1 and 10 by converting them to the appropriate scaled SI unit.

a) 0.0025 L b) 0.00000843 s c) 1600 m

Answers: a) 2.5 mL

b) 8.43 μs

c) 1.6 km

<u>Solutions</u>

First write each value in scientific notation. Then replace the power of ten with the appropriate SI scaling prefix.

a) 0.0025 L ➔ 2.5×10^{-3} L ➔ 2.5 mL

b) 0.00000843 s ➔ 8.43×10^{-6} s ➔ 8.43 μs

c) 1600 m ➔ 1.6×10^{3} m ➔ 1.6 km

6. The average density of canola oil is 0.92 g/mL. What is the mass in grams of 1 gallon of canola oil.

Answer: 3.5×10^3 g

Solution

What we know: gal canola oil; g canola oil/mL; L/gal; mL/L

Desired answer: g canola oil

The solution map for this problem is:

gal canola oil ➔ L canola oil ➔ mL canola oil ➔ g canola oil

The conversion factor needed in the first step is $\frac{3.78\ \text{L canola oil}}{1\ \text{gal canola oil}}$.

The conversion factor needed in the second step is $\frac{1\ \text{mL canola oil}}{10^{-3}\ \text{L canola oil}}$.

The conversion factor needed in the last step is $\frac{0.92\ \text{g canola oil}}{1\ \text{mLcanola oil}}$.

Putting these together yields:

$$1\ \cancel{\text{gal canola oil}} \times \frac{3.78\ \cancel{\text{L canola oil}}}{1\ \cancel{\text{gal canola oil}}} \times \frac{1\ \cancel{\text{mL canola oil}}}{10^{-3}\ \cancel{\text{L canola oil}}} \times \frac{0.92\ \text{g canola oil}}{1\ \cancel{\text{mLcanola oil}}} = 3.5 \times 10^3\ \text{g canola oil}$$

7. Suppose you have a 295-g sample of aluminum (density = 2.7 g/cm^3) and a 438-g sample of iron (density = 9.0 g/cm^3). Which sample will occupy the larger volume?

Answer: 295 g aluminum

Solution

Use the density of each material to convert its mass into volume and compare the results.

$$295\ \cancel{\text{g aluminum}} \times \frac{1\ \text{cm}^3\ \text{aluminum}}{2.7\ \cancel{\text{g aluminum}}} = 110\ \text{cm}^3\ \text{aluminum}$$

$$438 \, \cancel{\text{g iron}} \times \frac{1 \, \text{cm}^3 \text{ iron}}{9.0 \, \cancel{\text{g iron}}} = 49 \, \text{cm}^3 \text{ iron}$$

Thus the aluminum sample has the larger volume.

8. Pure silver has a density of 10.5 g/cm^3 at room temperature. If a pure silver spoon is fully submersed in a graduated cylinder containing 26.50 mL of water and the water level rises to 28.11 mL, what is the mass of the spoon?

Answer: 16.9 g

Solution

The volume of the spoon equals the difference between the final and initial water levels. Use the density of silver to convert the spoon volume into mass. Recall that 1 mL = 1 cm^3.

spoon volume = 28.11 mL - 26.50 mL = 1.61 mL = 1.61 cm^3

$$1.61 \, \cancel{\text{cm}^3 \text{ silver}} \times \frac{10.5 \, \text{g silver}}{1 \, \cancel{\text{cm}^3 \text{ silver}}} = 16.9 \, \text{g silver}$$

9. Imagine that you need special postage stamps that are sold in booklets at a cost of $43.20 per 5 booklets. If each booklet contains 3 sheets of 6 stamps each and a total of 210 stamps are needed, what will be the cost of the stamps?

Answer: $100.80

Solution

What we know: number of stamps; stamps/sheet; sheets/booklet; $/booklet

Desired answer: $

The solution map for this problem is:

numbers of stamps ➔ sheets of stamps ➔ booklets ➔ $

The conversion factor needed in the first step is $\frac{1 \, \text{sheet}}{6 \, \text{stamps}}$.

The conversion factor needed in the second step is $\frac{1\,\text{booklet}}{3\,\text{sheets}}$.

The conversion factor needed in the last step is $\frac{\$43.20}{5\,\text{booklets}}$.

Putting these together yields:

$$210\,\cancel{\text{stamps}} \times \frac{1\,\cancel{\text{sheet}}}{6\,\cancel{\text{stamps}}} \times \frac{1\,\cancel{\text{booklet}}}{3\,\cancel{\text{sheets}}} \times \frac{\$43.20}{5\,\cancel{\text{booklets}}} = \$100.80$$

Problem-Solving Strategies

1. Suppose you wished to build a wooden box that will hold a ton of gravel. What additional information would you need in order to decide how much lumber to purchase for the construction of this box? Explain your reasoning.

Solution

The key to knowing how much lumber to purchase is an estimate of how big the box must be (the box volume). In order to make this estimate you will need an estimate of the volume of the ton of gravel. Since you know the sample mass and want the sample volume, the gravel density will serve as the necessary conversion factor. Since there are many different types of gravel, there is no single answer for the gravel density. A typical value is about 100 lb/ft^3. This means a ton of gravel will occupy about 20 ft^3 so a box 1 ft x 4 ft x 5 ft should work. This is probably smaller than you might have imagined.

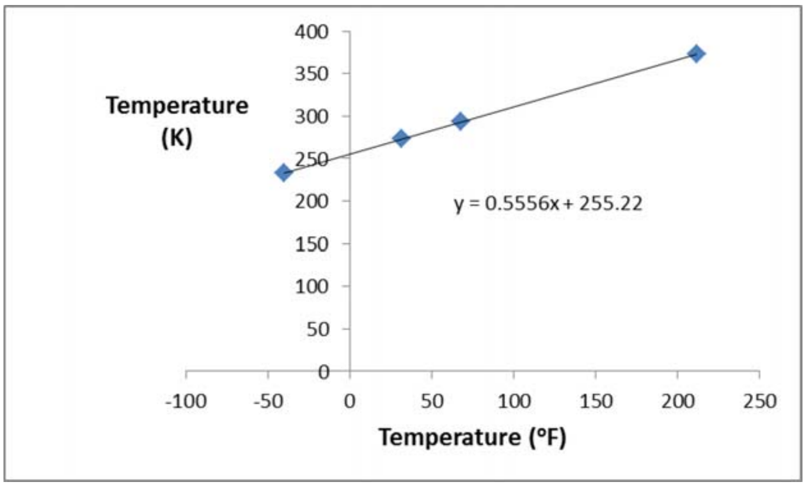
3. The data in the table below show various Fahrenheit temperatures and their corresponding Kelvin temperatures. Use this information to determine the equivalent Fahrenheit temperature when the Kelvin temperature is zero. Assume that Fahrenheit temperature is the independent variable.

°F	K
212	373
68	293
32	273
-40	233

Answer: -459.4 °F

Solution

First, use software such as Excel to plot the Kelvin temperature versus the Fahrenheit temperature and fit the data with a straight line. The equation for this line allows you to solve for the Fahrenheit temperature (x) when the Kelvin temperature (y) is zero.



$$x = \frac{y - 255.22\,\text{K}}{0.5556\,\text{K}/^\circ\text{F}} = \frac{0\,\cancel{\text{K}} - 255.22\,\cancel{\text{K}}}{0.5556\,\cancel{\text{K}}/^\circ\text{F}} = -459.4\,^\circ\text{F}$$