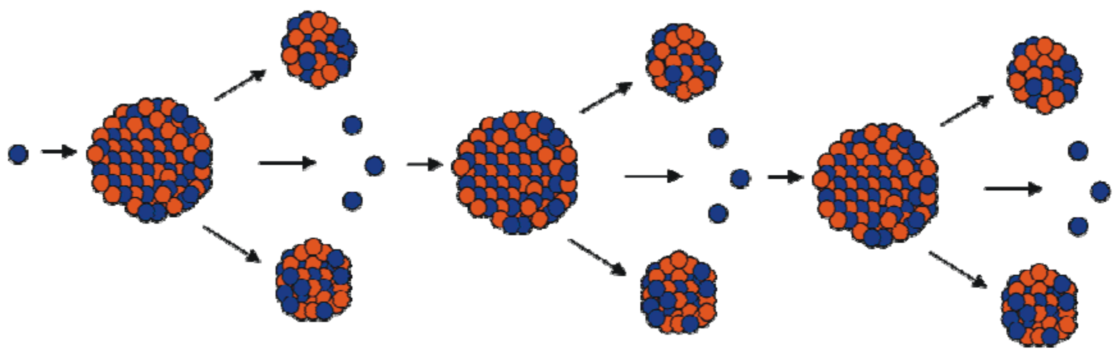
Chem 481 Lecture Material
4/17/09

Nuclear Reactors

A controlled, self-sustaining fission "chain reaction" is the source of enormous energy in a nuclear reactor. Recall that the energy release when a large atom splits apart (fissions) results from the formation of smaller nuclides that have a higher binding energy per nucleon. The fission product nuclides also favor a lower neutron-to-proton ratio so neutrons are released during fission.

Nuclear Fission Chain Reaction



A *fissile* nuclide is one that can fission by absorption of a neutron of any energy, most notably thermal neutrons. ^{235}U is a naturally-occurring fissile nuclide while ^{233}U and ^{239}Pu are fissile nuclides that can be produced by nuclear reactions. A *fissionable* nuclide is one that can fission by absorption of a neutron. This includes all fissile nuclides and those nuclides that fission only by absorbing high-energy neutrons. Some examples of nuclides requiring high-energy neutrons to fission are ^{232}Th, ^{238}U and ^{239}Pu. Nuclides that can be transformed into fissile materials are called *fertile*. For example, ^{238}U can be converted into ^{239}Pu by the following steps.

$$^{238}U(n,\gamma)^{239}U \xrightarrow{\beta^-} {}^{239}Np \xrightarrow{\beta^-} {}^{239}Pu$$

Fission neutrons are formed with an average energy of about 2 MeV. Since the cross section for capture resulting in fission increases with decreasing neutron energy (see figure and table below), fission neutrons must be slowed down (moderated) to be more effective in sustaining the chain reaction. The ideal moderator has the following nuclear properties.

- large scattering cross section
- small absorption cross section
- large energy loss per collision with neutron

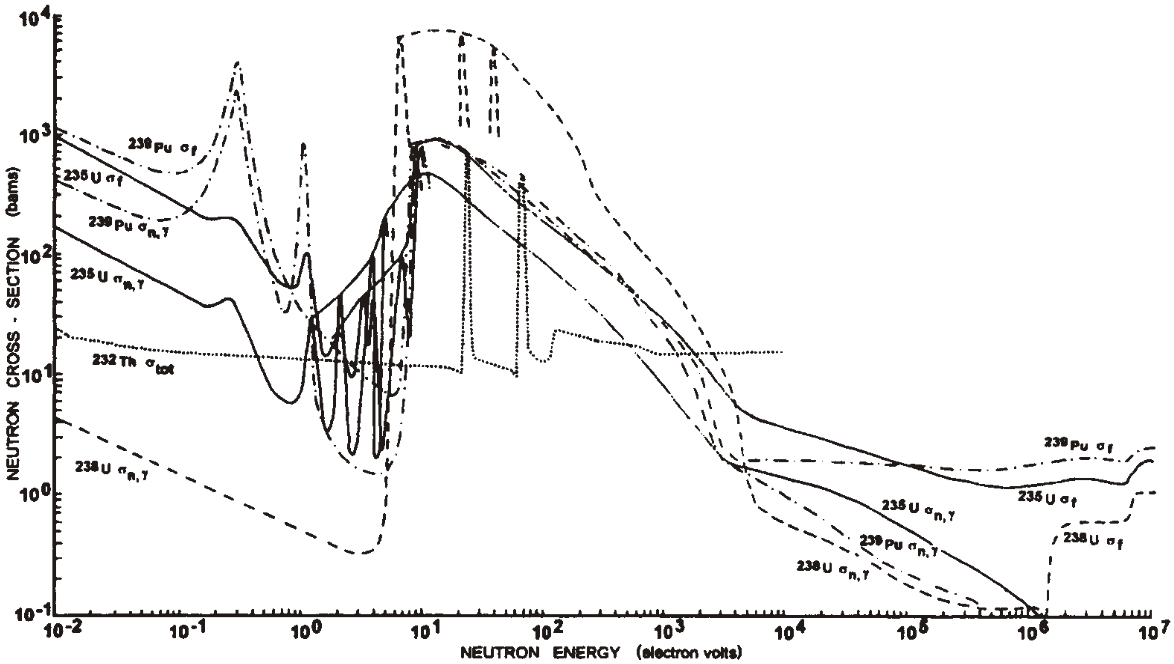


FIG. 19.3. Cross sections for n-capture ($\sigma_{n,\gamma}$), fission (σ_f), and total (σ_{tot}) as a function of neutron energy.

Table 5-2 Neutron cross-sections in ^{235}U and ^{238}U.

Cross-section	^{235}U (b)	^{238}U (b)
σ_f (thermal)	584	0
σ_f (fast)	~1.1	~0.55
σ_γ (thermal)	107	0.05
σ_s (elastic)	9.0	4.55
σ_s (inelastic)		2.10

The most complete measure of the effectiveness of a moderator is the moderating ratio (MR). This is given by

$$MR = \frac{\xi \Sigma_s}{\Sigma_a}$$

where ξ is the average logarithmic energy loss per collision;

Σ_s is the macroscopic cross section for scattering (cm^{-1});

Σ_a is the macroscopic cross section for absorption (cm^{-1}).

The larger the moderating ratio, the more effective the moderator in slowing down neutrons without absorbing them. The moderating properties of various materials are given in the table below.

TABLE 2
Moderating Properties of Materials

Material	ξ	Number of Collisions to Thermalize	Macroscopic Slowing Down Power	Moderating Ratio
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Helium	0.427	42	9×10^{-6}	51
Beryllium	0.207	86	0.154	126
Boron	0.171	105	0.092	0.00086
Carbon	0.158	114	0.083	216

Most (>99%) of the neutrons produced in fission are released within about 10^{-13} s of the fission event. These are known as *prompt neutrons*. A small fraction of the fission neutrons are produced some time after the fission event and are called *delayed neutrons*. They are emitted immediately following the beta decay of a fission product know as a delayed neutron precursor. One example of a delayed neutron precursor is ^{87}Br which decays as follows.

$$^{87}\text{Br} \xrightarrow[\text{55.9 s}]{\beta^-} {^{87}\text{Kr}} \xrightarrow[\text{instantaneous}]{n} {^{86}\text{Kr}}\ \text{(stable)}$$

It is convenient to combine known precursors into groups with appropriately averaged properties. The groups vary slightly depending upon the fissile material. The properties of delayed neutron groups for ^{235}U are shown in the table below.

TABLE 3
Delayed Neutron Precursor Groups
for Thermal Fission in Uranium-235

Group	Half-Life (sec)	Delayed Neutron Fraction	Average Energy (MeV)
1	55.7	0.00021	0.25
2	22.7	0.00142	0.46
3	6.2	0.00127	0.41
4	2.3	0.00257	0.45
5	0.61	0.00075	0.41
6	0.23	0.00027	-
Total	-	0.0065	-

While delayed neutrons are only a small fraction (β) of the neutron population, they play a vital role in the control of a nuclear reactor because they significantly increase the neutron lifetime. The weighted mean time for the production of ^{235}U delayed neutrons is 12.3 s. Since delayed neutrons are produced at lower energies than prompt neutrons from fission, they are less likely to be captured by non-fission events and thus are slightly more effective than the fraction suggests. This enhanced fraction (β_{eff}) varies from reactor to reactor. For the reactor at UCI, β_{eff} = 0.0070.

Reactor Physics

On the average, fission of ^{235}U produces 2.42 neutrons per event. In order to sustain a chain reaction at least one of these neutrons must cause another fission event. However, some neutrons will be absorbed and not cause fission and others will leak out of the reactor. In order to maintain a steady fission rate the number of neutrons must remain constant. The multiplication factor (k) describes what is happening to the neutron population as a function of time.
When

$k = 1$ (critical)	neutron population (and reactor power) is steady
$k > 1$ (supercritical)	neutron population (and reactor power) is increasing
$k < 1$ (subcritical)	neutron population (and reactor power) is decreasing

For an infinitely large reactor there is no leakage and the infinite multiplication factor (k_∞) is used.

$$k_\infty = \frac{\textit{neutron production in one generation}}{\textit{neutron absorption in preceding generation}}$$

There are four factors that describe the ability of the fuel and moderator materials to multiply the neutron population. Each factor represents a process that increases or decreases the neutron population from the previous generation. This relationship is known as the *four factor formula*.

$$k_\infty = \varepsilon \, p \, f \, \eta$$

where ε is the fast fission factor;

p is the resonance escape probability;

f is the thermal utilization factor;

η is the reproduction factor.

The fast fission factor represents the increase in the fast neutron population due to fast fission events. The value of ε is slightly larger than one.

$$\varepsilon = \frac{\textit{number of fast neutrons produced by all fissions}}{\textit{number of fast neutrons produced by thermal fissions}}$$

As fast neutrons slow down there is a chance they will be captured in the resonance energy region (about 6 eV - 200 eV) of ^{238}U. These neutrons will not result in fission and lead to a decrease in the neutron population. The value of p is slightly less than one.

$$p = \frac{\textit{number of neutrons that reach thermal energy}}{\textit{number of fast neutrons produced by all fissions}}$$

Note that the product (ε p) is the ratio of the number of fast neutrons that survive the thermalization process compared to the number of fast neutrons originally starting the generation.

The thermal utilization factor describes how effectively thermal neutrons are absobed by the fuel. The value of f will always be less than one.

$$f = \frac{\textit{number of thermal neutrons absorbed by the fuel}}{\textit{number of neutrons that reach thermal energy}}$$

The reproduction factor is a measure of the extent to which neutrons absorbed in the fuel cause fission. The value of η is less than the fission yield (2.42 for ^{235}U) because not all absorptions in the fuel result in fission.

$$\eta = \frac{\textit{number of fast neutrons produced by thermal fission}}{\textit{number of thermal neutrons absorbed by the fuel}}$$

For any real reactor it is necessary to account for neutrons that leak out of the reactor. An effective multiplication factor (k_{eff}) and a six factor formula are used.

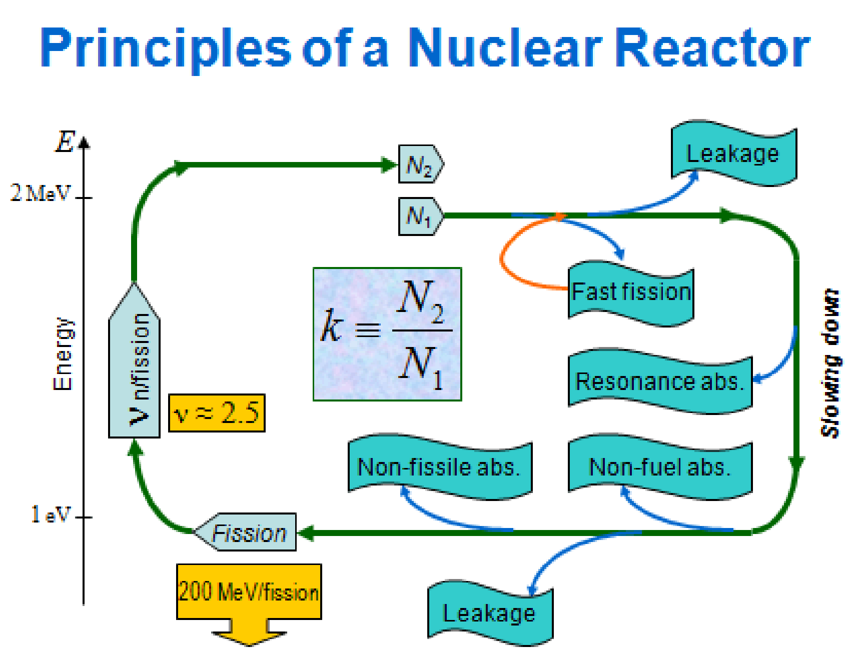
$$k_{eff} = \frac{\textit{neutron production in one generation}}{\textit{neutron absorption} + \textit{leakage in preceding generation}}$$

$$k_{eff} = \varepsilon\, L_f\, p\, L_t\, f\, \eta$$

where $$L_f = \frac{\textit{number of fast neutrons that do not leak from reactor}}{\textit{number of fast neutrons produced by all fissions}}$$

$$L_t = \frac{\textit{number of thermal neutrons that do not leak from reactor}}{\textit{number of neutrons that reach thermal energies}}$$

The non-leakage factors depend not only on the composition of the reactor but also on the size and shape of the reactor. The figure below summarizes the various factors that affect the neutron population.



Reactivity

If there were N_o neutrons in the preceding generation, then there are $N_o\ k_{eff}$ neutrons in the present generation. The change in the neutron population is $N_o\ k_{eff} - N_o$. Expressing this gain or loss as a fraction of the present generation population gives

$$\frac{N_0 k_{eff} - N_0}{N_0 k_{eff}}$$

This fractional change in neutron population per generation is known as *reactivity* (ρ).

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k}$$

where δk is the excess multiplication factor and $k = k_{eff}$.

The reactivity can be positive or negative depending upon the value of k_{eff}. A reactor with a positive reactivity is supercritical and one with a negative reactivity is subcritical. Reactivity is also expressed in units of %δk/k which is the reactivity multiplied by 100. Since k_{eff} is usually very close to one, $\delta k = \rho\ k_{eff} \sim \rho$.

The relative size of reactivity additions compared to β_{eff} is very important in the control of a reactor. Consequently, a unit of reactivity called the *dollar* has been defined. The reactivity of a system is $1.00 if $\rho = \beta_{eff}$ (= 0.0070 for the UCI reactor). Another way to think of this is

$$\$ = \frac{\rho}{\beta_{eff}}$$

One hundredth of a dollar is 1 cent. Thus, a $2.00 reactivity insertion would be a δk/k of 0.0140 (2 x 0.0070). The value of the dollar varies with the kind of fuel because β_{eff} is different for various types of reactors.

Reactor Kinetics

Now consider what happens to the neutron population as a function of time following a change in k. First, look at the case where k > 1. For N neutrons the gain each generation is N δk. If the generation time or neutron lifetime is ℓ (in seconds), the gain in neutrons each second would be N δk / ℓ. Thus, the rate of change in the number of neutrons is given by

$$\frac{dN}{dt} = \frac{N\,\delta k}{l}$$

If at time zero there were N_0 neutrons, then at time *t* the neutron population would be

$$N = N_0 e^{(\delta k/l)t}$$

The neutron lifetime can be estimated from the thermal velocity (2200 m/s) and the average distance a neutron travels before being absorbed. For the UCI reactor ℓ has been estimated to be 43 μs.

The above equation is simplified by defining a reactor period (T) such that T = ℓ / δk. Thus,

$$N = N_0 e^{\frac{t}{T}}$$

The reactor period represents the length of time required to change the neutron population (or reactor power level) by a factor of e (2.718).

As an example, assume the excess reactivity (δk or ρ) is $1.75 and $\ell = 4.3 \times 10^{-5}$ s. What is the increase in power after 0.1 second?

First, note that $1.75 converts to a value of δk ≃ ρ = $($\beta_{eff}$) = ($1.75)(0.0070) = 0.012.

$$\frac{N}{N_0} = \frac{P}{P_0} = e^{\frac{0.012}{4.3 \times 10^{-5}\,s} 0.1\,s} = 1.3 \times 10^{12}$$

When $\delta k > \beta_{eff}$ the reactor is said to be prompt critical. This rapid rise in power associated with prompt criticality, while desirable in a nuclear weapon, is very difficult to control in a reactor. Fortunately, delayed neutrons make control much easier because they increase the effective neutron lifetime (ℓ_{eff}). When delayed neutrons are taken into account, the effective neutron lifetime can be approximated by

$$l_{eff} = l + \frac{\beta_{eff} - \rho}{\lambda_{eff}}$$

where λ_{eff} is the effective decay constant for the delayed neutron precursors (= 1/12.3 s = 0.0813 s^{-1} for ^{235}U). For the UCI reactor and a reactivity change of 0.0025 ($0.36), the reactor period can be calculated as

$$T = \frac{l_{eff}}{\rho} = \frac{l + \frac{(\beta_{eff} - \rho)}{\lambda}}{\rho} = \frac{4.3 \, x \, 10^{-5}\, s + (0.0070 - 0.0025)/0.0813\, s^{-1}}{0.0025} = 22\, s$$

and the increase in power after one second would be

$$\frac{P}{P_0} = e^{\frac{t}{T}} = E^{\frac{1\,s}{22\,s}} = 1.05$$

This indicates that the reactor power is increasing much more slowly and hence is more easily controlled. Thus, one can insert a small amount of reactivity ($\rho < \beta_{eff}$) and after a small *prompt jump* the reactor power will increase more slowly as the delayed neutrons get produced (see figure below).

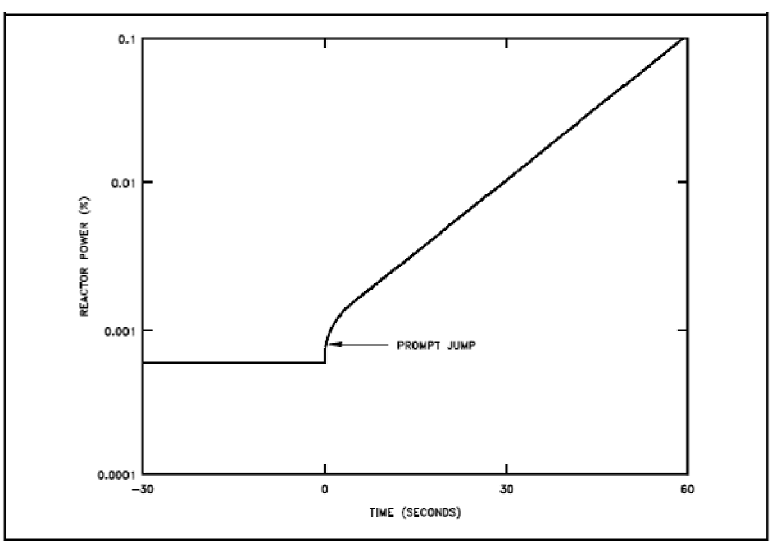


Figure 2 Reactor Power Response to Positive Reactivity Addition