

Chem 481 Lecture Material

1/30/09

Nature of Radioactive Decay

The Standard Model in physics postulates that all particles in nature are composed of quarks and leptons and that they interact by exchange force carrier (virtual) particles (see tables below). Protons and neutrons are composites consisting of three quarks, uud for protons and udd for neutrons. The quarks in a proton or neutron are held together by the fundamental strong force involving the exchange of gluons as described by quantum chromodynamics (QCD). Protons and neutrons interact via the residual strong force (nuclear force) that involves quarks in one nucleon interacting with quarks in another nucleon. The nuclear force can also be described in terms of the exchange of π mesons between nucleons. This interaction is stronger than the electromagnetic force and is responsible for keeping a collection of protons in the nucleus intact. An excellent summary of the fundamentals of forces and particles can be found at: http://www.particleadventure.org/frameless/decay_intro.html

FERMIONS					
Leptons <small>spin = 1/2</small>			Quarks <small>spin = 1/2</small>		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	-1	d down	0.006	-1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	-1	s strange	0.1	-1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	-1	b bottom	4.3	-1/3

PROPERTIES OF THE INTERACTIONS					
Property \ Interaction	Gravitational	Weak	Electromagnetic	Strong	
		(Electroweak)		Fundamental	Residual
Acts on:	Mass – Energy	Flavor		Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons		Electrically charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0		γ	Gluons
Strength relative to electromag for two u quarks at:	10^{-41}	0.8		1	25
10^{-18} m	10^{-41}	10^{-4}		1	60
3×10^{-17} m	10^{-36}	10^{-7}		1	Not applicable to hadrons
for two protons in nucleus					20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$					
Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
\bar{p}	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	-1	1.672	3/2

Mesons $q\bar{q}$					
Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	-1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

BOSONS					
force carriers spin = 0, 1, 2, ...					
Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge	Name	Mass GeV/c^2	Electric charge
γ photon	0	0	g gluon	0	0
W^-	80.4	-1			
W^+	80.4	+1			
Z^0	91.187	0			

Conservation Laws

In addition to the conservation of energy, charge, momentum, etc., other conservation laws govern particle interactions and decay processes. Two important conservation laws are the conservation of baryon number and the conservation of lepton number. For nuclear decay processes and nuclear reactions, the conservation of charge and baryon number are manifested in the balancing of subscripts and superscripts in the equation for the reaction.

Alpha Decay

Alpha (α) decay involves the emission of a packet of 2 protons and 2 neutrons from a nucleus. The α particle has a +2 charge and usually represented in α decay reactions as ${}^4_2\text{He}$ (the ultimate fate of the charged α particle is to pick up 2 electrons and become an atom of ${}^4\text{He}$). The emitted α particles are either monoenergetic or consist of a few monoenergetic groups.

general α decay equation: ${}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}\text{X} + {}^4_2\text{He} + Q$

example: ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^4_2\text{He} + Q$

For alpha decay, $Q = \frac{931.5 \text{ MeV}}{\text{amu}} (M_Z - [M_{Z-2} + M_{{}^4\text{He}}])$

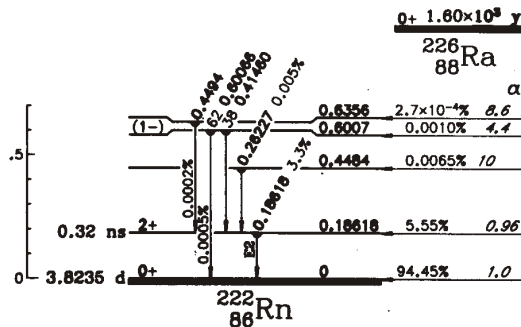
For the α decay of ${}^{226}_{88}\text{Ra}$,

$$Q = \frac{931.5 \text{ MeV}}{\text{amu}} (226.025403 \text{ amu} - 222.017570 \text{ amu} - 4.002603250) = 4.871_5 \text{ MeV}$$

The Chart of the Nuclides lists for ${}^{226}\text{Ra}$: α 4.78442, 4.602 ...
 γ 186.2

This means that alpha particles of several energies are emitted, with 4.78 MeV being the dominant one, along with a gamma ray of 186.2 keV. The gamma emission results from the ${}^{222}\text{Rn}$ being formed in an excited state which then de-excites by photon emission. Detailed information about decay schemes is found in the Table of Isotopes. The information found for ${}^{226}\text{Ra}$ is shown below. Note that the decay scheme is a plot of relative energy vs Z.

${}^{226}_{88}\text{Ra}$
 A: 23.66573y {ANDT 19 175(77)}
 σ_a : 81 {PC77 Holden}; 202 (reactor spectrum) {PC77 Holden}
 σ_f : $<7 \times 10^{-4}$ (subcadmium) {PC77 Holden}
 σ : a {JINC 7 305(58)}
 $t_{1/2}$: 1599.7y *calorim* {Nukl 8 244(66)}; 1602y *calorim* {Iso 5 141(59)}; 1617.1y *sp act* {AnPs 13v1 680(56)}; 1622.3y *sp act* {NNEs 14Bd4 1675(49)}; 1590y *sp act* {RMP 3 427(31)}; *others*: {Atyo 7 445(59), ZETF 34 756(58)}
 Class: A; Ident: chem, genet {RMP 3 427(31)}
 Prod: natural source {Bk64 Hyde2}
 α : α_0 4.7845025 mag {Metr 7 65(71), Cf71 Tedton 1}
 α_0 4.784510 (94.45%), α_{186} 4.601510 (5.55%), α_{448} 4.343510 (0.00653%), α_{601} 4.194520 (0.00101%), α_{636} 4.163520 (2.75x10⁻⁴%) mag {JPPa 24 854(63)}
 α_0 4.7845 (94.61%), α_{186} 4.60195 (5.41%), α_{448} 4.342510 (0.00513%), α_{601} 4.196540 (7.1x10⁻⁴%) mag {C158 NPPa 910, ANDT 12 479(73)}
others: {CR 236 1016(53), PR 68 129(52), CR 233 945(51), PR 83 390(51), CR 229 191(49)}
 γ : (norm: $\gamma_{0.186}$ (7.33%), from level scheme, JWD), (intensity relative to γ_{100} for $\gamma_{0.609}$ (with ${}^{214}\text{Bi}$ in equilibrium)) 0.1861804 ($t_{1/2}$ 8.20 12) *Ge(Li)* {CuP 48 2606(70), AECL-PRCMA-4 39(68)}
 (intensity relative to γ_{100} for $\gamma_{0.609}$ (with ${}^{214}\text{Bi}$ in equilibrium)) 0.18621112 ($t_{1/2}$ 9.00 10) *Ge(Li)* {NIM 142 329(77)}
 (intensity relative to γ_{100} for $\gamma_{0.609}$ (with ${}^{214}\text{Bi}$ in equilibrium)) $\gamma_{0.186}$ ($t_{1/2}$ 9.9131) *Ge(Li)* {NIM 74 353(69)}



In order to conserve momentum when the alpha particle is emitted it, the much heavier product nucleus recoils with a small amount of energy (E_r) analogous to the recoil felt when firing a gun. This means that the kinetic energy of the alpha particle (E_α) is only a portion of the decay energy. Thus, one can write:

$$Q = E_r + E_\alpha = \frac{1}{2}m_r v_r^2 + \frac{1}{2}m_\alpha v_\alpha^2$$

and $m_r v_r = m_\alpha v_\alpha$ (assuming a stationary parent)

$$m_r v_r^2 = \frac{m_\alpha^2 v_\alpha^2}{m_r}$$

$$Q = \frac{1}{2} \frac{m_\alpha^2 v_\alpha^2}{m_r} + \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{2} m_\alpha v_\alpha^2 \left(1 + \frac{m_\alpha}{m_r}\right) = E_\alpha \left(1 + \frac{m_\alpha}{m_r}\right)$$

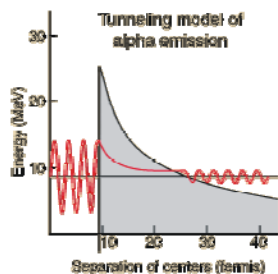
$$E_\alpha = Q \frac{m_r}{m_\alpha + m_r} \approx Q \frac{m_r}{m_Z}$$

$$\text{For } ^{226}\text{Ra}, E_\alpha = 4.871_5 \text{ MeV} \left(\frac{222.017570 \text{ amu}}{4.002603250 \text{ amu} + 222.017570 \text{ amu}} \right) = 4.785_2 \text{ MeV}$$

$$E_r = Q - E_\alpha \approx Q \frac{m_\alpha}{m_Z} = 0.0863 \text{ MeV}$$

The recoil energy of the daughter is sufficient to break chemical bonds if the parent is bound to other atoms.

Although nuclei with $A \geq 140$ -150 are unstable with respect to alpha decay, this decay mode is usually observed only if $A \geq 210$. The range of α particles energies is from about 2-10 MeV (usually 5-7 MeV), yet α decay half lives range from $\sim 10^{-7}$ s to $> 10^{19}$ y. There is a strong inverse relationship between E_α and half life. Since E_α is much less than the Coulomb barrier of the nucleus, the particle should be trapped inside the nucleus. George Gamow (1928), and independently, R.W. Gurney and E.U. Condon, proposed that alpha emission occurs via quantum mechanical tunneling (see figure below).



This led to the relationship known as the Geiger-Nuttall rule which in its modern form is:

$$\ln \lambda = -a_1 \frac{Z}{\sqrt{E}} + a_2$$

where λ is the decay constant ($= \ln 2 / t_{1/2}$);
 Z is the parent atomic number;
 E is the decay energy;
 a_1 and a_2 are constants.

The energy released in alpha emission is equal to the E_B of the α particle (28.3 MeV) minus the energy needed to remove 2 p and 2 n from the nucleus. E_B/A decreases with increasing A and around $A = 150$ α emission is exoergic, but measurable decay rates are not observed until $A > 210$ because the Coulomb barrier increases more slowly with A than does the decay energy. The emission of other charged particles is not common because their E_B is not so large compared to that of the most loosely bound nucleons in the nucleus (for example, E_B for ^2H is only 2 MeV).

Negatron (β^-) Decay

In β^- decay a neutron is converted to a proton with the subsequent emission of a high-energy electron and an electron antineutrino ($\bar{\nu}_e$).

general β^- decay equation: ${}_Z^AX \rightarrow {}_{Z+1}^AX + {}_{-1}^0e + {}_0^0\bar{\nu}_e + Q$

example: ${}_{25}^{56}\text{Mn} \rightarrow {}_{26}^{56}\text{Fe} + {}_{-1}^0e + {}_0^0\bar{\nu}_e + Q$

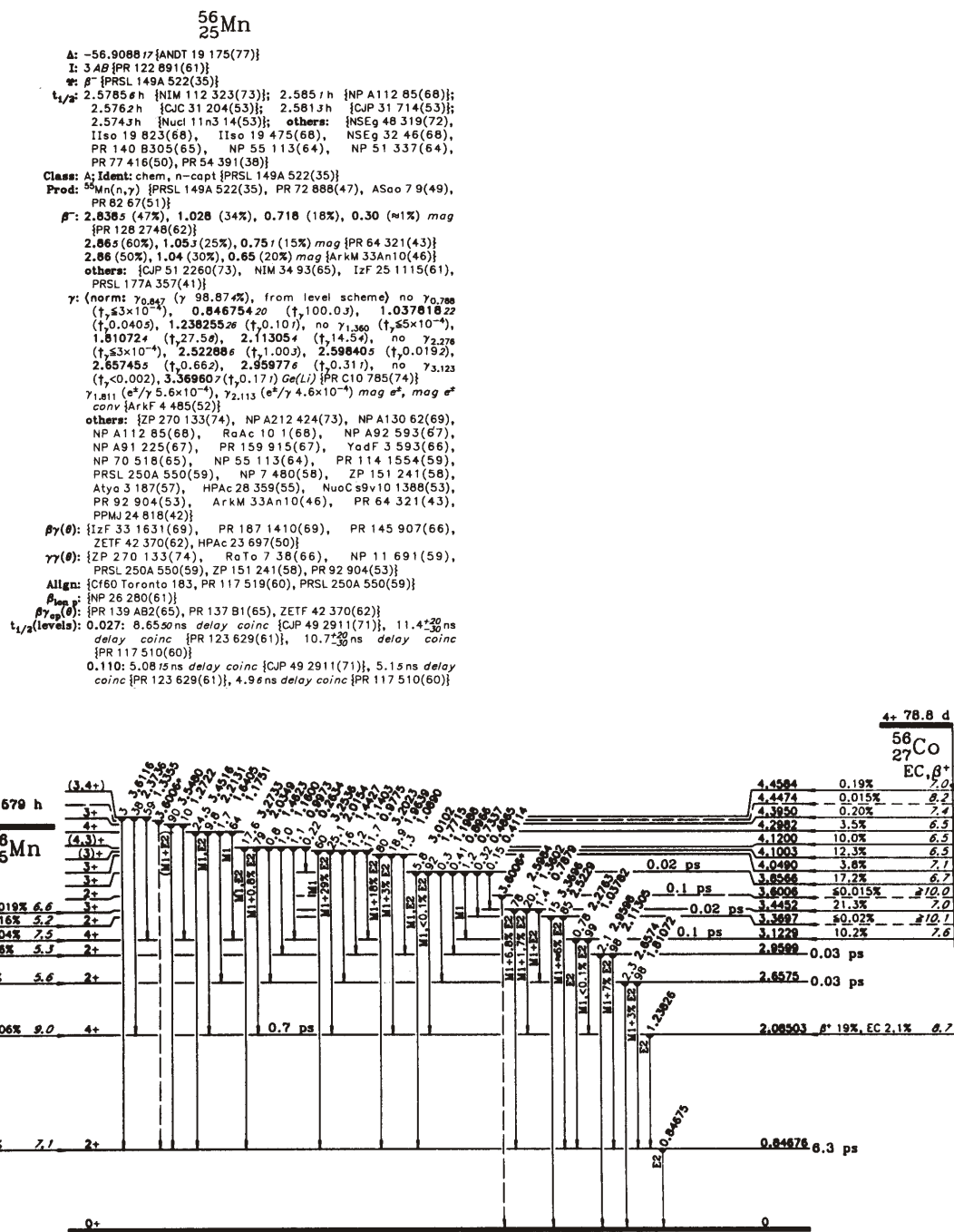
For β^- decay, $Q = \frac{931.5 \text{ MeV}}{\text{amu}} (M_Z - M_{Z+1})$ (Note that the mass of e^- is taken into account by the daughter electrons.)

Alternatively, $Q = \Delta_Z - \Delta_{Z+1}$

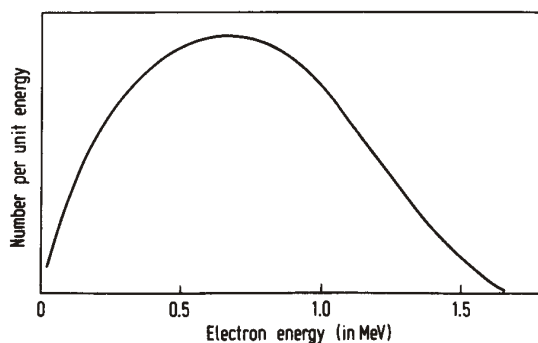
For the β^- decay of $^{56}_{25}\text{Mn}$,

$$Q = -56.907 \text{ MeV} - (-60.603 \text{ MeV}) = 3.696 \text{ MeV}$$

Decay information for ^{56}Mn from the Table of Isotopes is given below.



Notice that ^{56}Mn does not decay to the ground state of ^{56}Fe . The closest the decay gets is to the 847-keV excited state which rapidly de-excites with emission of a gamma ray of this energy. Thus, one would expect $E_{\beta^-} = 3.696 \text{ MeV} - 0.847 \text{ MeV} = 2.849 \text{ MeV}$. Surprisingly, a continuous range of β^- energies are observed as illustrated in the beta spectrum for ^{32}P below.



To account for this and to allow for conservation of spin, Wolfgang Pauli proposed in 1930 that another particle with spin $\frac{1}{2}$ and no charge was emitted during the decay process. It was not until 1956 that evidence for the existence of neutrinos and antineutrinos was obtained by Reines and Cowan.

In the Standard Model the decay of the neutron is associated with a quark transformation in which one of the down quarks in a neutron is converted to an up quark by the weak interaction. Since the down quark has a charge of $-\frac{1}{3}$ and the up quark has a charge of $\frac{2}{3}$, this process is mediated by a virtual W^- particle, which carries away a (-1) charge (thus conserving charge). The new up quark rebounds away from the emitted W^- . The neutron has now become a proton. An electron and an electron antineutrino emerge from the virtual W^- boson and are emitted from the nucleus. This process is diagrammed below.

