5 How Chemists Measure Atoms and Molecules

Learning Objectives

As you work through this chapter you will learn how to:

- calculate the number of atoms or molecules in a sample.
- perform calculations involving mole units.
- use the molar mass of a substance in calculations.
- determine the mass percent composition of a compound.
- determine the theoretical yield of a product in a chemical reaction.
- determine the limiting reactant in a chemical reaction.

5.1 A Sure Bet

Long before television, the Internet and computer games a popular activity among children was the game of marbles. In the basic game, players try to knock out small stone or glass marbles (mibs) from a circular playing area using a slightly larger shooter marble (taw). Typically a player carried a set of glass marbles in a cloth pouch and the game was played on just about any smooth hard surface. Marble tournaments are still held today all around the world. Expressions such as “losing your marbles”, “playing for keeps” and “knuckle down” can be traced back to the game of marbles.

With a little bit of information, you can use a collection of marbles to amaze your friends and win a friendly wager. You’ll need a set of identical glass or stone marbles and an opaque container like a small cardboard box or even a paper bag.[1] Let's imagine using a group of marbles and a small cardboard box with lid similar to that shown in Figure 5.1. Ask a friend to secretly place as many marbles in the box as he or she wishes and then close the box. The bet is that you will be able to determine how many marbles are in the box without opening it (and in fact without even touching the box of marbles!).

Figure 5.1 Set of marbles and small cardboard box

In order to make good on your claim, you will need to know in advance the mass of the container and the typical mass of one of the marbles. Then all that you need to do is to weigh the box of marbles prepared by your friend. At home you can conveniently weigh objects such as the box or one marble using an inexpensive digital scale such as that shown in Figure 5.2. The object to be weighed is simply placed on the scale and the mass is displayed in a digital readout in grams. In order to minimize errors, it is necessary to obtain the mass of the objects to three significant figures. If you use marbles this means obtaining masses to the nearest 0.01 g.

[1]You can use any set of small objects that are identical in size, such as paper clips, however, it is better if the mass of the object is significant compared to the container you use. Coins could be used, however, the composition of various coins has changed over the years and so has their mass. For example, pennies minted before 1983 have a different composition than that for those minted after 1983 so a mix of pennies would not work.

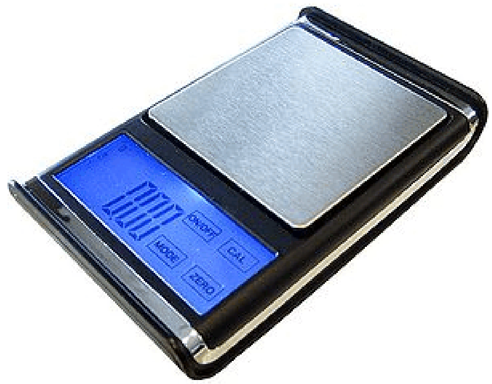

Figure 5.2 Inexpensive digital kitchen scale

Let's assume you weigh several individual marbles and find that the "average" marble in your collection weighs 5.21 g. You also determine that the box weighs 12.84 g. Since 1 marble corresponds to 5.21 g, this provides you with a conversion factor between mass and marbles. Now your friend secretly places several marbles in the box and then places the box of marbles on the scale and it reads 49.14 g. In order to figure out how many marbles are in the box you first calculate the total mass of the marbles in the box. This is determined by subtracting out the mass of the box.

$$\begin{aligned} \text{total marble mass} &= (\text{mass of box + marbles}) - (\text{mass of box only}) \\ &= 49.14\text{ g} - 12.84\text{ g} = 36.30\text{ g} \end{aligned}$$

Now, to get the unknown number of marbles in the box, you use your conversion factor in the following way.

$$\text{number of marbles} = \text{total marble mass (g)} \times \frac{1\,\text{marble}}{5.21\,\text{g}} = 36.30\,\cancel{\text{g}} \times \frac{1\,\text{marble}}{5.21\,\cancel{\text{g}}} = 6.97\text{ marbles}$$

Since there must be a whole number of marbles in the box, the closest whole number is 7 so you tell your friend there must be 7 marbles in the box (and you win the bet!). This approach to counting objects by weighing can work for any type of object, even tiny invisible ones. This is exactly how chemists determine the number of atoms or molecules in a sample.

5.2 Counting Invisible Objects

If, for example, you want to determine the number of atoms in a sample of gold using the same approach as was used for the marbles, you need to know the mass of a single gold atom. However, remember from Chapter 3 that an individual atom is very small with a mass on the order of 10^{-22} g. It is not so easy to weigh individual atoms as was done for the marbles.[2] In the 1800s chemists, such as John Dalton, were able to assign *relative masses*, called atomic weights, to atoms. They did this by comparing the masses of two elements that react with each other. Dalton chose hydrogen (assumed to have an atomic weight of 1) as the reference point for his scale of atomic weights. Scientists continue to use a scale of relative atomic masses, however, now the mass of atoms and molecules is determined very precisely using a technique called **mass spectrometry**. In this approach, the mass of an ionized atom or molecule is determined by monitoring its motion in a magnetic field. The present system of atomic weights compares the mass of an atom to 1/12 the atomic mass of one carbon-12 (^{12}C) atom. Thus, an **atomic weight** (A_r) is a ratio of masses $\dfrac{\text{mass of one atom}}{\frac{1}{12}\text{mass of one }^{12}\text{C atom}}$ and has no units associated with it.[3]

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[2]Scientists have been able to measure the mass of individual atoms using carbon nanotubes. DOE/Lawrence Berkeley National Laboratory (2008, July 29). Golden Scales: Nanoscale Mass Sensor Can Be Used To Weigh Individual Atoms And Molecules. ScienceDaily. Retrieved January 28, 2011, from http://www.sciencedaily.com-/releases/2008/07/080728192940.htm.

[3]The symbol AW is often used by chemists to designate atomic weight.

Since the mass of individual atoms is a very, very small number when expressed in SI units such as g or kg, it is convenient to express atomic masses using the **atomic mass unit** (u).[4] One atomic mass unit is defined as exactly 1/12 the mass of a single ^{12}C atom. Thus, one ^{12}C atom weighs exactly 12 u. The atomic mass unit has been determined to correspond to a value of 1.6605 x 10^{-24} g.[5] With this unit, the mass of a proton (1.0073 u) and the mass of a neutron (1.0087 u) are just sightly larger than one atomic mass unit and the mass of an electron is 0.00054858 u.

The atomic weight of gold is 197.0. Since this atomic weight represents a ratio of $\dfrac{\text{mass of one gold atom (in u)}}{\frac{1}{12}\text{ mass of one } ^{12}\text{C atom } (= 1\text{u})}$, this numerical value also corresponds to the mass of one gold atom in atomic mass units. This information can now be used to determine the number of gold atoms in a sample of pure gold, as shown in Example 5.1.

[4]The official name is *unified atomic mass unit*. An equivalent unit is the **dalton** (Da) which is commonly used in biochemistry; 1 Da = 1 u.

[5]The atomic mass unit equivalent in grams has been determined to 10 significant figures. Atomic weights of many elements are known to 6 or more significant figures.

Example 5.1

Problem

Determine the number of gold atoms in one ounce (28.35 g) of gold (Au).

Solution

What we know: g Au; atomic weight of Au

Desired answer: number of Au atoms

The mass of gold can be converted into number of gold atoms using the atomic weight of gold. Recall that the numerical value of the atomic weight of gold refers to the mass of a single gold atom in atomic mass units. First, convert this value to an equivalent mass in grams.

$$\frac{197.0\ \cancel{\text{u Au}}}{1\ \text{Au atom}} \times \frac{1.6605 \times 10^{-24}\ \text{g Au}}{1\ \cancel{\text{u Au}}} = \frac{3.271 \times 10^{-22}\ \text{g Au}}{\text{Au atom}}$$

Now use this result as the conversion factor between mass of gold and atoms of gold.

$$28.35\ \cancel{\text{g Au}} \times \frac{1\ \text{Au atom}}{3.271 \times 10^{-22}\ \cancel{\text{g Au}}} = 8.667 \times 10^{22}\ \text{atoms Au}$$

Note: As you might have expected, the number of these very small atoms in our sample is enormous.

Check for Understanding 5.1 **Solution**

1. The atomic weight of sodium is 22.99. What is the mass, in grams, of a billion sodium atoms?

This approach to relating the number of atoms of an element to the sample mass using the mass of one atom works fine for elements like gold and sodium. For most elements, however, there is an added complication. For instance, if you were to carefully analyze a sample of copper wire using mass spectrometry you would discover that not all of the copper atoms in the sample have the same mass. One type of copper atom weighs 62.9296 u and another weighs 64.9278 u. Copper, unlike gold, has two stable atomic forms called **isotopes**. Isotopes are atoms of the same element that have different masses. In order for these atoms to be of the same element they must have the same atomic number (number of protons) and the same number of electrons. Thus, the difference in mass must result from the atoms having different numbers of neutrons (and hence different mass numbers). For copper, the two stable isotopes are ^{63}Cu and ^{65}Cu. So what does the atomic weight of copper refer to, and how can we relate the mass of a copper sample to number of copper atoms as we did for gold?

To answer these questions, let's return to the marble example briefly. Recall that we were using "identical" marbles, but suppose your friend had placed some of the larger shooter marbles in the box with the smaller ones. Now your average marble mass does not represent the mix of marbles in the box and your calculated number of marbles will be in error. What you need is the average mass of a marble in this mix of marbles. Let's assume that your friend used a 50-50 mix; half are identical small marbles and half are identical large marbles. The average mass of a marble in this mix would be the average of the two marble masses. This can be represented mathematically as:

average marble mass (50:50 mix) = ½ (smaller marble mass + larger marble mass)
= ½ (smaller marble mass) + ½ (larger marble mass)

However, your friend may have used only one larger shooter and the rest small marbles. To determine the average marble mass for any combination of large and small marbles, notice that the ½ factor in the equation above represents the fraction of that particular type of marble. Thus, to get the average marble mass for any mix you can use the following relationship.

average marble mass = (fraction of small marbles)(smaller marble mass)
+ (fraction of large marbles)(larger marble mass)

The average marble mass calculated in this way is known as a **weighted average**. A weighted average can be calculated for as many different types of marbles as you wish. All that you need to know is the mass of each marble type and the fraction of the total for this marble type.

For elements that have more than one stable isotope[6], a weighted average is used for the mass of one atom of that element. In the case of copper, 69.15% of the atoms in a typical copper sample are ^{63}Cu and 30.85% are ^{65}Cu. Thus, the weighted average mass of a copper atom is calculated by:

$$\begin{aligned}\text{weighted average mass of Cu atoms} &= (\text{fractional isotope abundance})(^{63}\text{Cu mass}) \\ &\quad + (\text{fractional isotope abundance})(^{65}\text{Cu mass}) \\ &= (0.6915)(62.9296\ \text{u}) + (0.3085)(64.9278\ \text{u}) = 63.55\ \text{u}\end{aligned}$$

Therefore, the atomic weight of copper is 63.55. This value can be used to determine the number of copper atoms in a sample of copper as was done for gold.

Take note that no atom of copper weighs 63.55 u. This weighted average mass is useful only if the fractional abundance of the copper isotopes is the same for all samples of copper. This same caution applies to all elements that have more than one stable isotope. Scientists have determined that for some elements there is a significant variation in the isotopic composition from one type of sample to another. Starting in 2011, the atomic weights of 10 elements (H, Li, B, C, N, O, Si, S, Cl and Tl) are expressed as intervals that reflect this variation in isotopic abundance. For most instructional purposes, including this course, the conventional atomic weights for these elements are sufficient. The periodic table shown on the next page includes the atomic weights to be used in this course. Generally you will be provided a periodic table with such numerical information for use on quizzes and exams. It is very important that you understand and can apply all of the information available from this periodic table.

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[6]Only 26 elements have a single stable isotope.

metals

nonmetals

metalloids

26 **Fe** 55.84 — atomic number (Z), chemical symbol, atomic weight

1A	2A	3B	4B	5B	6B	7B		8B		1B	2B	3A	4A	5A	6A	7A	8A
1 **H** 1.008																	2 **He** 4.003
3 **Li** 6.941	4 **Be** 9.012											5 **B** 10.81	6 **C** 12.01	7 **N** 14.01	8 **O** 16.00	9 **F** 19.00	10 **Ne** 20.18
11 **Na** 22.99	12 **Mg** 24.31											13 **Al** 26.98	14 **Si** 28.09	15 **P** 30.97	16 **S** 32.07	17 **Cl** 35.45	18 **Ar** 39.95
19 **K** 39.10	20 **Ca** 40.08	21 **Sc** 44.96	22 **Ti** 47.87	23 **V** 50.94	24 **Cr** 52.00	25 **Mn** 54.94	26 **Fe** 55.84	27 **Co** 58.93	28 **Ni** 58.69	29 **Cu** 63.55	30 **Zn** 65.39	31 **Ga** 69.72	32 **Ge** 72.63	33 **As** 74.92	34 **Se** 78.96	35 **Br** 79.90	36 **Kr** 83.80
37 **Rb** 85.47	38 **Sr** 87.62	39 **Y** 88.91	40 **Zr** 91.22	41 **Nb** 92.91	42 **Mo** 95.94	43 **Tc** [98]	44 **Ru** 101.1	45 **Rh** 102.9	46 **Pd** 106.4	47 **Ag** 107.9	48 **Cd** 112.4	49 **In** 114.8	50 **Sn** 118.7	51 **Sb** 121.8	52 **Te** 127.6	53 **I** 126.9	54 **Xe** 131.3
55 **Cs** 132.9	56 **Ba** 137.3	57 **La*** 138.9	72 **Hf** 178.5	73 **Ta** 180.9	74 **W** 183.8	75 **Re** 186.2	76 **Os** 190.2	77 **Ir** 192.2	78 **Pt** 195.1	79 **Au** 197.0	80 **Hg** 200.6	81 **Tl** 204.4	82 **Pb** 207.2	83 **Bi** 209.0	84 **Po** [209]	85 **At** [210]	86 **Rn** [222]
87 **Fr** [223]	88 **Ra** [226]	89 **Ac**‡ [227]	104 **Rf** [267]	105 **Db** [262]	106 **Sg** [266]	107 **Bh** [264]	108 **Hs** [269]	109 **Mt** [268]	110 **Ds** [281]	111 **Rg** [272]	112 **Cn** [285]	113 **Uut** [286]	114 **Fl** [289]	115 **Uup** [289]	116 **Lv** [293]	117 **Uus** [294]	118 **Uuo** [294]

*	58 **Ce** 140.1	59 **Pr** 140.9	60 **Nd** 144.2	61 **Pm** [145]	62 **Sm** 150.4	63 **Eu** 152.0	64 **Gd** 157.2	65 **Tb** 158.9	66 **Dy** 162.5	67 **Ho** 164.9	68 **Er** 167.3	69 **Tm** 168.9	70 **Yb** 173.0	71 **Lu** 175.0
‡	90 **Th** 232.0	91 **Pa** 231.0	92 **U** 238.0	93 **Np** [237]	94 **Pu** [244]	95 **Am** [243]	96 **Cm** [247]	97 **Bk** [247]	98 **Cf** [251]	99 **Es** [252]	100 **Fm** [257]	101 **Md** [258]	102 **No** [259]	103 **Lr** [262]

Figure 5.3 Periodic Table of the elements showing atomic weights[7]

[7]Elements which have no known stable isotopes (that is, all atoms of the element are radioactive) and are not found in nature have the atomic weight of the most stable isotope in brackets.

5.3 The Mole

Since a very large number of atoms (and molecules) are associated with typical samples of material, scientists have created a unit that allows such quantities to be more conveniently expressed. This unit is the **mole** (mol). The mole is a counting unit like the dozen and is linked to the reference point (^{12}C) of the atomic weight system. One mole is defined as the number of atoms of ^{12}C in exactly 12 grams of ^{12}C (Note that 12 grams of carbon corresponds to a 60-carat diamond!). The **Avogadro constant** (N_A), often referred to as *Avogadro's number*, expresses the number of elementary entities (atoms, molecules, etc.) there are per mole of a substance. It has been experimentally determined to have a value (to 4 significant figures) of 6.022×10^{23} mol^{-1} (Note that mol^{-1} means the same as 1/mol.). Thus, one mole of anything is the amount of that substance that contains 6.022×10^{23} items. The value of the Avogadro constant is so large it is difficult to comprehend just how many items are associated with a mole of something. For example, if we take a mole of pennies and distribute these equally to the approximately 7 billion people on earth, then each individual would have nearly a trillion dollars! Consequently, the mole is a good unit for counting very small objects such as atoms and molecules. Since the mole is so commonplace in chemistry you should memorize the value of the Avogadro constant.

$$\boxed{\mathbf{N_A = 6.022 \times 10^{23}\ mol^{-1}}}$$

Since a mole is defined in terms of a number of grams numerically equal to the atomic mass (in u) of ^{12}C, a similar connection can be made for the other elements. The number of grams numerically equal to the weighted average atomic mass (in u) of an element will contain one mole of atoms of that element. For example, 1 mole of silver weighs 107.9 g and 1 mole of sulfur weighs 32.07 g. We can determine this by noting the atomic weights of these elements listed on the periodic table in Figure 5.3.

The mass of a substance that corresponds to one mole of that material is called the **molar mass** (M) of that substance. In chemistry molar masses typically are expressed in units of g/mol. This quantity serves as the conversion factor between the mass of a substance and the moles of that substance. You can use it to convert from either mass to moles or from moles to mass.[8] The Avogadro constant (N_A) is used to convert from moles to a specific number of fundamental units (atoms, molecules, ions) present, or to convert from a number of particles to the corresponding number of moles.

g element $\xleftrightarrow[\textbf{mass}]{\textbf{molar}}$ **mol element** $\xleftrightarrow{\mathbf{N_A}}$ **atoms element**

Molar mass will be one of the most frequently used conversion factors in this course.

The steps of the problem-solving strategy described in Section 2.6 for problems involving calculations are listed below . This approach is used to set up and solve the many problems involving calculations in this text. You should review these steps before studying subsequent *Examples* and before starting work on the *Check for Understanding* problems and the *Exercises* in this chapter.

<u>Strategy for Solving Calculation Problems</u>

1. Read the entire problem very carefully, making note of key terms and units.

2. Identify the question to be answered and make special note of the units for the answer.

3. Look at all the information that is given, including units for numerical values.

4. Think about relationships that connect the given information to the question and its plausible answer and create your solution map, identifying each step.

5. Identify all the conversion factors needed for your solution map.

6. Confirm that the relevant units cancel to give you the desired units for the answer.

7. Do the calculations carefully. Think about whether or not your answer makes sense.

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[8]The symbol $\leftrightarrow$ means a process that can go in either direction.

A key to this strategy is the creation of a solution map that outlines how the starting units are converted to the units of the desired answer. For example, if you are given information about the mass (in g) of substance A and asked for the number of moles of substance A, then the solution map would look like: g A ➔ mol A. Since molar mass has units of g/mol, this is the logical choice for the conversion factor needed in this calculation. However, remember that conversion factors can always be expressed in two forms, one the inverse of the other. Thus, molar mass can be used as $\frac{\text{g}}{\text{mol}}$ or as $\frac{\text{mol}}{\text{g}}$. Which form you use depends on the unit conversion needed. In this case, we need a conversion factor with units of $\frac{\text{mol}}{\text{g}}$ which will cancel *g* and introduce *mol* in the numerator, so the unit conversion looks like:

$$\cancel{\text{g}} \times \frac{\text{mol}}{\cancel{\text{g}}} = \text{mol}$$

Check for Understanding 5.2 **Solution**

1. Write the solution map for the conversion of grams of sodium to atoms of sodium. Indicate the numerical ratio that is the conversion factor in each step.

The use of the mole and molar mass in calculations involving elements is illustrated in Examples 5.2 and 5.3.

Example 5.2

Problem

How many iron (Fe) atoms are present in 0.79 mmol of iron?

Solution

What we know: mmol Fe

Desired answer: number of Fe atoms

The solution map for this calculation is:

mmol Fe ➔ mol Fe ➔ atoms Fe

The conversion factor needed in the first step is the relationship between *mmol* and *mol*. Since 1 mmol = 10^{-3} mol, use $\frac{10^{-3}\ \text{mol Fe}}{1\ \text{mmol Fe}}$ in order to convert units properly.

The conversion factor needed in the second step is the Avogadro constant. It is used in the form $\frac{6.022 \times 10^{23}\ \text{atoms Fe}}{1\ \text{mol Fe}}$ in order to convert units properly.

Putting these together yields:

$$0.79\ \cancel{\text{mmol Fe}} \times \frac{10^{-3}\ \cancel{\text{mol Fe}}}{1\ \cancel{\text{mmol Fe}}} \times \frac{6.022 \times 10^{23}\ \text{atoms Fe}}{1\ \cancel{\text{mol Fe}}} = 4.8 \times 10^{20}\ \text{atoms Fe}$$

Note that 2 significant figures are appropriate for the answer.

Example 5.3

Problem

What is the mass (in kg) of 18.4 mol of mercury (Hg)?

Solution

What we know: mol Hg

Desired answer: kg Hg

The solution map for this calculation is

mol Hg ➔ g Hg ➔ kg Hg

The conversion factor needed in the first step is the molar mass of mercury. The numerical value for the molar mass is obtained from the periodic table and is expressed in the form $\frac{200.6\,\text{g Hg}}{1\,\text{mol Hg}}$ in order to convert units properly.

The conversion factor needed in the second step is the relationship between *kg* and *g*. Since 1kg = 10^3 g, use $\frac{1\,\text{kg Hg}}{10^3\,\text{g Hg}}$ in order to convert units properly.

Putting these together yields:

$$18.4\,\cancel{\text{mol Hg}} \times \frac{200.6\,\cancel{\text{g Hg}}}{1\,\cancel{\text{mol Hg}}} \times \frac{1\,\text{kg Hg}}{10^3\,\cancel{\text{g Hg}}} = 3.69\,\text{kg Hg}$$

Note that 3 significant figures are appropriate for the answer.

Check for Understanding 5.3

Solutions

1. What is the mass, in grams, of one cobalt atom?

2. How many atoms of aluminum are present in a piece of aluminum foil that weighs 6.27 g?

5.4 Molar Mass of a Compound

The molar mass of a compound is the mass in grams of one mole of formula units of the compound. The molar mass of a compound is obtained by summing the molar masses of all of the atoms represented in the chemical formula of the compound. If there is more than one atom of an element in the chemical formula, its molar mass is added as many times as the element appears in the formula. The calculation of the molar mass of a compound is illustrated in Examples 5.4 and 5.5.

Example 5.4

Problem

What is the molar mass of water?

Solution

What we know: water; atomic weights of elements

Desired answer: g water/mol

The chemical formula for water is H_2O. The formula indicates that in 1 mole of water molecules there are 2 moles of hydrogen atoms and 1 mole of oxygen atoms. From the periodic table we see that 1 mole of hydrogen atoms weighs 1.008 g and that 1 mole of oxygen atoms weighs 16.00 g. Thus, one mole of water will weigh:

$$\left(2\,\cancel{\text{mol H}} \times \frac{1.008\,\text{g}}{1\,\cancel{\text{mol H}}} = 2.016\,\text{g}\right) + \left(1\,\cancel{\text{mol O}} \times \frac{16.00\,\text{g}}{1\,\cancel{\text{mol O}}} = 16.00\,\text{g}\right) = 18.02\,\text{g}$$

The molar mass of water is 18.02 g/mol.

Example 5.5

Problem

What is the molar mass of calcium nitrate?

Solution

What we know: calcium nitrate; atomic weights of elements

Desired answer: g calcium nitrate/mol

The chemical formula for calcium nitrate is $Ca(NO_3)_2$. The formula indicates that in 1 mole of calcium nitrate there are 1 mole of calcium, 2 moles of nitrogen and 6 moles of oxygen. From the periodic table we see that 1 mole of calcium atoms weighs 40.08 g, 1 mole of nitrogen atoms weighs 14.01 g and 1 mole of oxygen atoms weighs 16.00 g. Thus, one mole of $Ca(NO_3)_2$ will weigh:

$$1\ \mathrm{Ca}\quad 1\ \cancel{\mathrm{mol\,Ca}} \times \frac{40.08\,\mathrm{g}}{1\ \cancel{\mathrm{mol\,Ca}}} = 40.08\,\mathrm{g}$$

$$2\ \mathrm{N}\quad 2\ \cancel{\mathrm{mol\,N}} \times \frac{14.01\,\mathrm{g}}{1\ \cancel{\mathrm{mol\,N}}} = 28.02\,\mathrm{g}$$

$$6\ \mathrm{O}\quad 6\ \cancel{\mathrm{mol\,O}} \times \frac{16.00\,\mathrm{g}}{1\ \cancel{\mathrm{mol\,O}}} = 96.00\,\mathrm{g}$$

$$\overline{164.10\ \mathrm{g}}$$

The molar mass of calcium nitrate is 164.10 g/mol.

Check for Understanding 5.4

Solution

1. Calculate the molar mass of sodium phosphate (Na_3PO_4).

Analogous to the situation for chemical elements, the molar mass of a compound connects the macroscopic world to the invisible microscopic one by serving as the conversion factor between a readily measurable mass of a compound and the number of moles of that compound. You can use it to convert from mass of a compound to moles of a compound, or from moles of a compound to the corresponding mass of the compound. The Avogadro constant (N_A) is used to convert from moles to a specific number of fundamental units (molecules, formula units) present, or to convert from a number of molecules or formula units to the corresponding number of moles of a compound.

$$\boxed{\textbf{g compound}} \underset{\textbf{mass}}{\overset{\textbf{molar}}{\longleftrightarrow}} \boxed{\textbf{mol compound}} \overset{\mathbf{N_A}}{\longleftrightarrow} \boxed{\textbf{basic units of compound}}$$

Examples 5.6, 5.7 and 5.8 illustrate how to use the molar mass of a compound in calculations.

Example 5.6

Problem

What is the mass in grams of 0.295 mol $MgCl_2$?

Solution

What we know: mol $MgCl_2$

Desired answer: g $MgCl_2$

The solution map for this calculation is:

mol $MgCl_2$ ➔ g $MgCl_2$

The conversion factor needed is the molar mass of $MgCl_2$. From the periodic table we can get the molar masses of magnesium and chlorine and add them as follows. 24.31 g Mg + 2(35.45 g) Cl = 95.21 g. Thus, the molar mass of $MgCl_2$ is 95.21 g/mol. It is used in the form $\frac{95.21\,g\,MgCl_2}{1\,mol\,MgCl_2}$ in order to convert units properly.

Applying this yields:

$$0.295\,\cancel{mol\,MgCl_2} \times \frac{95.21\,g\,MgCl_2}{1\,\cancel{mol\,MgCl_2}} = 28.1\,g\,MgCl_2$$

Example 5.7

Problem

How many moles of CH_3OH molecules are there in 0.19 g of CH_3OH?

Solution

What we know: g CH_3OH

Desired answer: mol CH_3OH

The solution map for this calculation is:

g CH_3OH ➔ mol CH_3OH

The conversion factor needed is the molar mass of CH_3OH. From the periodic table we can get the molar masses of carbon, hydrogen and oxygen and add them as follows. 12.01 g C + 4(1.008 g) H + 16.00 g O = 32.04 g. Thus, the molar mass of CH_3OH is 32.04 g/mol. It is used in the form $\frac{1\,\text{mol}\,CH_3OH}{32.04\,\text{g}\,CH_3OH}$ in order to convert units properly.

Applying this yields:

$$0.19\,\cancel{\text{g}\,CH_3OH} \times \frac{1\,\text{mol}\,CH_3OH}{32.04\,\cancel{\text{g}\,CH_3OH}} = 0.0059\,\text{mol}\,CH_3OH$$

Example 5.8

Problem

How many molecules of CO_2 are there in 112 g of CO_2?

Solution

What we know: g CO_2

Desired answer: molecules CO_2

The solution map for this calculation is:

g CO_2 ➔ mol CO_2 ➔ molecules CO_2

The conversion factor needed in the first step is the molar mass of CO_2. From the periodic table we can get the molar masses of carbon and oxygen and add them as follows. 12.01 g C + 2(16.00 g) O = 44.01 g. Thus, the molar mass of CO_2 is 44.01 g/mol. It is used in the form $\frac{1\,\text{mol}\,CO_2}{44.01\,\text{g}\,CO_2}$ in order to convert units properly.

The conversion factor needed in the second step is the Avogadro constant. It is used in the form $\frac{6.022\times10^{23}\ \text{molecules}\,CO_2}{1\,\text{mol}\,CO_2}$ in order to convert units properly.

Putting these together yields:

$$112\,\cancel{\text{g}\,CO_2}\ \text{x}\ \frac{1\,\cancel{\text{mol}\,CO_2}}{44.01\,\cancel{\text{g}\,CO_2}}\ \text{x}\ \frac{6.022\times10^{23}\ \text{molecules}\,CO_2}{1\,\cancel{\text{mol}\,CO_2}} = 1.53\times10^{24}\ \text{molecules}\,CO_2$$

Check for Understanding 5.5 Solutions

1. Calculate the number of moles in a sample of sodium bicarbonate ($NaHCO_3$) that weighs 6.22 g.

2. How many molecules are present in 1.05 g of iodine (I_2)?

3. What is the mass in grams of 25 molecules of CCl_2F_2?

5.5 Mass Percent Composition of Compounds

Since compounds always have the same specific composition, it is possible to establish quantitative relationships that are useful conversion factors. For example, since water has the composition H_2O, we note there are 2 atoms of H for each atom of O. Such a relationship can be expressed as an atom ratio, and can be "scaled up" by multiplying both numerator and denominator by the Avogadro constant.

For water there are: $\dfrac{2\,\text{atoms H}}{1\,\text{atom O}}$ or $\dfrac{2\,\text{mol H}}{1\,\text{mol O}}$

For water it is also true that: $\dfrac{2\,\text{atoms H}}{1\,\text{molecule H}_2\text{O}}$ or $\dfrac{2\,\text{mol H}}{1\,\text{mol H}_2\text{O}}$

Check for Understanding 5.6 Solutions

1. What is the numerical ratio between the number of hydrogen atoms and oxygen atoms in $(NH_4)_3PO_4$?

2. What is the numerical ratio between the number of moles of sulfur and moles of $Al_2(SO_4)_3$?

Example 5.9 illustrates how such relationships can be applied to useful conversions.

Example 5.9

Problem

How many moles of hydrogen atoms are in 3.6 mol of ammonium carbonate?

Solution

What we know: mol ammonium carbonate

Desired answer: mol H

The solution map for this calculation is:

mol ammonium carbonate ➔ mol H

As in this case, you will often need to provide a correct chemical formula so continue to review chemical names and formulas. The chemical formula for ammonium carbonate is $(NH_4)_2CO_3$. The formula indicates that in 1 mole of ammonium carbonate there are 8 moles of hydrogen atoms. This relationship can be applied as follows.

$$3.6\ \cancel{\text{mol}\ (NH_4)_2CO_3} \times \frac{8\ \text{mol H}}{1\ \cancel{\text{mol}\ (NH_4)_2CO_3}} = 29\ \text{mol H}$$

Check for Understanding 5.7

Solutions

1. Calculate the number of moles of barium nitrate that contain 0.81 mol of oxygen atoms.

2. How many carbon atoms are present in 16.0 g C_3H_8?

3. How many grams of silver are present in 25.0 g AgBr?

Another very useful piece of information that is associated with the definite composition of compounds is **percent** (%), or parts per hundred. A number percent refers to the number of items of a specified type in a group of 100 items total. For example, if 12% of the students in a class are left-handed, this means 12/100, or 0.12, of the class has this characteristic. It is important to note that the percent sign (%) relates to implicit units, so if this quantity is to be used as a conversion factor it is important to indicate the specific items and the total items referred to. If 12% of the class is left-handed, then there are 12 left-handed students/100 students in the class. This means that a number percent is calculated by:

$$\text{number percent} = \frac{\text{no. items of interest}}{\text{total no. of items}} \text{ x } 100$$

Example 5.10 illustrates the use of number percent.

In chemistry, **mass percent** is often a more useful quantity. A mass percent is calculated by:

$$\text{mass percent} = \frac{\text{mass of component of interest}}{\text{total mass}} \text{ x } 100$$

Note that the mass of the component and the total mass must be expressed in the same units.

Example 5.10

Problem

If 68% of a class of 93 students are biology majors and 57% of the biology majors are female, how many students in the class are female biology majors?

Solution

What we know: students in class; % biology majors; % female biology majors

Desired answer: number of female biology majors

The solution map for this problem is

students in class ➔ biology majors ➔ female biology majors

The conversion factor needed in the first step is $\frac{68\text{ biol. majors}}{100\text{ students in class}}$.

The conversion factor needed in the second step is $\frac{57\text{ female biol. majors}}{100\text{ biol. majors}}$.

Putting these together yields:

$$93\ \cancel{\text{students in class}} \times \frac{68\ \cancel{\text{biol. majors}}}{100\ \cancel{\text{students in class}}} \times \frac{57\text{ female biol. majors}}{100\ \cancel{\text{biol. majors}}} = 36\text{ female biol. majors}$$

The specific composition of a compound is frequently expressed in terms of the mass percent of each constituent element. For example, NaCl is 39.34% sodium (Na) and 60.66% chlorine (Cl) by mass. Notice that the mass percent values total 100% since these two elements are the only ones present. Also, according to the law of constant composition, these mass percent values apply to any sample of sodium chloride. The calculation of the mass percent composition of a compound is illustrated in Example 5.11.

Example 5.11

Problem

Determine the mass percent of the constituent elements in calcium nitrate.

Solution

Known information: calcium nitrate; atomic weights of elements

Desired answer: mass % for each constituent element

The chemical formula for calcium nitrate is $Ca(NO_3)_2$. Since the mass percent values apply to any sample of calcium nitrate, it is convenient to consider one mole of this compound. In Example 5.5 we determined the molar mass of $Ca(NO_3)_2$ to be 164.10 g/mol. Let 164.10 g represent the total mass of material. The mass of each element (= component mass) associated with one mole of calcium nitrate was also calculated. This information is used to obtain the mass percent for the three elements present.

$$\text{mass \% Ca} \quad \frac{40.08\,\text{g Ca}}{164.10\,\text{g Ca(NO}_3)_2} \times 100 = 24.42\,\%$$

$$\text{mass \% N} \quad \frac{28.02\,\text{g N}}{164.10\,\text{g Ca(NO}_3)_2} \times 100 = 17.07\,\%$$

$$\text{mass \% O} \quad \frac{96.00\,\text{g O}}{164.10\,\text{g Ca(NO}_3)_2} \times 100 = 58.50\,\%$$

99.99%

The slight variation of the sum from 100% is due to rounding off the mass percent values for the individual elements.

In summary, you can calculate the mass percent composition of a compound by comparing the total mass of each element present in one mole of compound to the molar mass of the compound. If the formula of the compound is not known, then the mass percent composition can be obtained by analyzing a specific mass of material and measuring the mass of each element present.

Check for Understanding 5.8 Solution

1. What is the mass percent of carbon in $C_6H_{12}O_6$?

5.6 Interpreting Chemical Reactions Quantitatively

We learned that in a chemical reaction atoms are neither created not destroyed, so a balanced chemical equation contains information that you can use to establish quantitative relationships between reactants and products. **Stoichiometry** (pronounced *stoy-kee-ahm-eh-tree*) is the area of chemistry that deals with the quantitative relationships between reactants and products in a chemical reaction. Consider the balanced equation for one of the single displacement reactions described in Chapter 3 in which zinc metal reacts with hydrochloric acid to produce zinc chloride and hydrogen gas.

$$Zn(s) \quad + \quad 2HCl(aq) \quad \rightarrow \quad ZnCl_2(aq) \quad + \quad H_2(g)$$

At the most fundamental level this equation suggests that 1 atom of zinc reacts with 2 molecules of HCl to produce 1 formula unit of $ZnCl_2$ and 1 molecule of H_2.

$$\begin{array}{ccccccc} Zn(s) & + & 2HCl(aq) & \rightarrow & ZnCl_2(aq) & + & H_2(g) \\ \text{1 atom} & & \text{2 molecules} & & \text{1 formula unit} & & \text{1 molecule} \end{array}$$

When such a reaction is carried out in the laboratory, many, many atoms of zinc react with many, many molecules of HCl, however, they always react in a 1:2 ratio.

The balanced equation can be "scaled up" by multiplying each reactant amount and each product amount by the Avogadro constant to yield the following quantitative relationship.

Zn(s)	+	2HCl(aq)	→	$ZnCl_2$(aq)	+	H_2(g)
1 mol		2 mol		1 mol		1 mol

These mole quantities of reactants and products can be used to create mole ratios. A **mole ratio** is a ratio of moles of any two substances involved in a chemical reaction. The coefficients in the balanced equation for the reaction are used to establish the numerical part of this ratio. For the above equation, there are 12 possible mole ratios. Look carefully at the equation above to verify each of the 12 ratios given below.

$$\frac{1\,\text{mol Zn}}{2\,\text{mol HCl}} \quad \frac{1\,\text{mol Zn}}{1\,\text{mol ZnCl}_2} \quad \frac{1\,\text{mol Zn}}{1\,\text{mol H}_2} \quad \frac{2\,\text{mol HCl}}{1\,\text{mol ZnCl}_2} \quad \frac{2\,\text{mol HCl}}{1\,\text{mol H}_2} \quad \frac{1\,\text{mol ZnCl}_2}{1\,\text{mol H}_2}$$

$$\frac{2\,\text{mol HCl}}{1\,\text{mol Zn}} \quad \frac{1\,\text{mol ZnCl}_2}{1\,\text{mol Zn}} \quad \frac{1\,\text{mol H}_2}{1\,\text{mol Zn}} \quad \frac{1\,\text{mol ZnCl}_2}{2\,\text{mol HCl}} \quad \frac{1\,\text{mol H}_2}{2\,\text{mol HCl}} \quad \frac{1\,\text{mol H}_2}{1\,\text{mol ZnCl}_2}$$

These ratios can be used to convert between moles of one substance and moles of another substance involved in this reaction. Such *mole-to-mole* calculations are illustrated in Examples 5.12 and 5.13.

Example 5.12

Problem

How many moles of Fe_2O_3 will be produced from the reaction of 1.00 mol FeS according to the equation below?

$$4FeS(s) + 7O_2(g) \rightarrow 2Fe_2O_3(s) + 4SO_2(g)$$

Solution

What we know: mol FeS; balanced equation relating FeS and Fe_2O_3

Desired answer: mol Fe_2O_3

The solution map for this problem is:

mol FeS ➔ mol Fe_2O_3

The conversion factor needed is the mole ratio for these two substances from the balanced equation in the form $\frac{2\,\text{mol}\,Fe_2O_3}{4\,\text{mol}\,FeS}$ in order to convert units properly.

Applying this yields:

$$1.00\,\cancel{\text{mol FeS}} \times \frac{2\,\text{mol}\,Fe_2O_3}{4\,\cancel{\text{mol FeS}}} = 0.500\,\text{mol}\,Fe_2O_3$$

Note that the numerical part of a mole ratio involves exact numbers with an unlimited number of significant figures.

Example 5.13

Problem

How many moles of O_2 are needed to react with 6.50 mol FeS according to the equation below?

$$4FeS(s) + 7O_2(g) \rightarrow 2Fe_2O_3(s) + 4SO_2(g)$$

Solution

What we know: mol FeS; balanced equation relating FeS and O_2

Desired answer: mol O_2

The solution map for this problem is:

mol FeS ➔ mol O_2

The conversion factor needed is the mole ratio for these two substances from the balanced equation in the form $\frac{7\,\text{mol}\,O_2}{4\,\text{mol}\,\text{FeS}}$ in order to convert units properly.

Applying this yields:

$$6.50\ \cancel{\text{mol FeS}} \times \frac{7\,\text{mol}\,O_2}{4\ \cancel{\text{mol FeS}}} = 11.4\,\text{mol}\,O_2$$

In the last few sections of the text you have seen how to form mathematical relationships that allow you to make the following calculations.

Calculation	Conversion factor
mol substance ↔ g substance	molar mass
mol substance ↔ number of atoms or molecules	Avogadro constant
mol compound ↔ mol constituent element	atom ratio based on formula of compound
g constituent element ↔ g compound	mass percent
mol reactant or product ↔ mol of another reactant or product	mole ratio from balanced equation

These relationships can be used to form the overall solution map for stoichiometric calculations shown in Figure 5.4. This map applies to the general reaction:

reactant A → product B

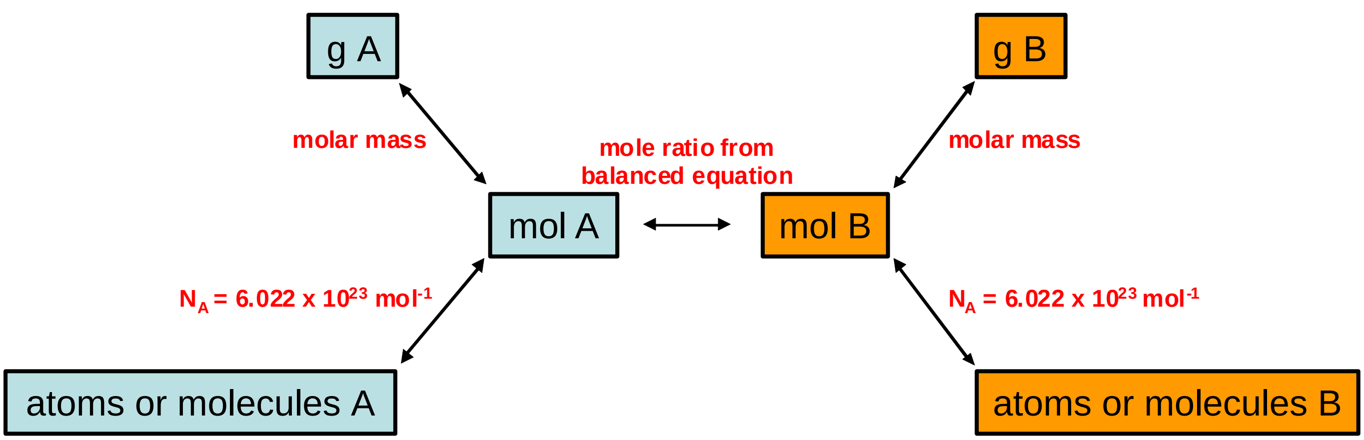


Figure 5.4 Solution map for stoichiometric calculations

This map indicates the number of steps and the conversion factors needed for performing various calculations that you will need to make in this course. For example, to convert from molecules of reactant A to mass of product B three steps are required.

The conversion factors are N_A, mol B/mol A and the molar mass of B. It is very important that you understand the role of each conversion factor and how to obtain it, and that you practice these calculations by doing many different types of problems. Examples 5.14, 5.15 and 5.16 illustrate applications of this solution map.

Example 5.14

Problem

How many grams of ammonia (NH_3) can be produced from 0.512 mol of hydrogen gas reacting with nitrogen gas according to the equation below?

$$N_2(g) \ + \ 3H_2(g) \ \rightarrow \ 2NH_3(g)$$

Solution

What we know: mol H_2; balanced equation relating NH_3 and H_2

Desired answer: g NH_3

The solution map for this problem is:

mol H_2 ➔ mol NH_3 ➔ g NH_3

The conversion factor needed in the first step is the mole ratio for these two substances from the balanced equation in the form $\frac{2\,\text{mol NH}_3}{3\,\text{mol H}_2}$.

The conversion factor needed in the second step is the molar mass of NH_3. From the periodic table we can get the molar masses of nitrogen and hydrogen and add them as follows. 14.01 g N + 3(1.008 g) H = 17.03 g. Thus, the molar mass of NH_3 is 17.03 g/mol.

Putting these together yields:

$$0.512\,\cancel{\text{mol H}_2} \ \text{x} \ \frac{2\,\cancel{\text{mol NH}_3}}{3\,\cancel{\text{mol H}_2}} \ \text{x} \ \frac{17.03\,\text{g NH}_3}{1\,\cancel{\text{mol NH}_3}} \ = \ 5.81\,\text{g NH}_3$$

Example 5.15

Problem

How many moles of oxygen gas are produced when 40.6 g of potassium chlorate decomposes according to the equation below?

$$2KClO_3(s) \rightarrow 2KCl(s) + 3O_2(g)$$

Solution

What we know: g $KClO_3$; balanced equation relating $KClO_3$ and O_2

Desired answer: mol O_2

The solution map for this problem is:

$$\text{g } KClO_3 \rightarrow \text{mol } KClO_3 \rightarrow \text{mol } O_2$$

The conversion factor needed in the first step is the molar mass of $KClO_3$. From the periodic table we can get the molar masses of potassium, chlorine and oxygen and add them as follows. 39.10 g K + (35.45 g) Cl + 3(16.00 g) O = 122.55 g. Thus, the molar mass of $KClO_3$ is 122.55 g/mol. It is used in the form $\frac{1\,\text{mol}\,KClO_3}{122.55\,\text{g}\,KClO_3}$ in order to cancel units properly.

The conversion factor needed in the second step is the mole ratio for these two substances from the balanced equation in the form $\frac{3\,\text{mol}\,O_2}{2\,\text{mol}\,KClO_3}$.

Putting these together yields:

$$40.6\,\cancel{\text{g}\,KClO_3} \times \frac{1\,\cancel{\text{mol}\,KClO_3}}{122.55\,\cancel{\text{g}\,KClO_3}} \times \frac{3\,\text{mol}\,O_2}{2\,\cancel{\text{mol}\,KClO_3}} = 0.497\,\text{mol}\,O_2$$

Example 5.16

Problem

What mass (in grams) of carbon dioxide is produced by the complete combustion of 95.0 grams of propane (C_3H_8)? The balanced equation for this combustion reaction is shown below.

$$C_3H_8(l) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$$

Solution

What we know: g C_3H_8; balanced equation relating C_3H_8 and CO_2

Desired answer: g CO_2

The solution map for this problem is:

$$\text{g } C_3H_8 \rightarrow \text{mol } C_3H_8 \rightarrow \text{mol } CO_2 \rightarrow \text{g } CO_2$$

The conversion factor needed in the first step is the molar mass of C_3H_8. From the periodic table we can get the molar masses of carbon and hydrogen and add them as follows. 3(12.01 g) C + 8(1.008 g) H = 44.09 g. Thus, the molar mass of C_3H_8 is 44.09 g/mol. It is used in the form $\frac{1\,\text{mol}\,C_3H_8}{44.09\,\text{g}\,C_3H_8}$ in order to cancel units properly.

The conversion factor needed in the second step is the mole ratio for these two substances from the balanced equation in the form $\frac{3\,\text{mol}\,CO_2}{1\,\text{mol}\,C_3H_8}$.

The conversion factor needed in the last step is the molar mass of CO_2. From the periodic table we can get the molar masses of carbon and oxygen and add them as follows. 12.01 g C + 2(16.00 g) O = 44.01 g. Thus, the molar mass of CO_2 is 44.01 g/mol.

Putting these together yields:

$$95.0\,\cancel{\text{g}\,C_3H_8} \times \frac{1\,\cancel{\text{mol}\,C_3H_8}}{44.09\,\cancel{\text{g}\,C_3H_8}} \times \frac{3\,\cancel{\text{mol}\,CO_2}}{1\,\cancel{\text{mol}\,C_3H_8}} \times \frac{44.01\,\text{g}\,CO_2}{1\,\cancel{\text{mol}\,CO_2}} = 284\,\text{g}\,CO_2$$

Check for Understanding 5.9 Solutions

1. Based on the reaction between iron(III) oxide and carbon, how many moles of carbon are needed to produce 1.0 kmol of iron?

$$Fe_2O_3(s) + 3C(s) \rightarrow 2Fe(s) + 3CO(g)$$

2. What mass of $NaHCO_3$ will react with 0.00257 mol HCl?

$$NaHCO_3(s) + HCl(aq) \rightarrow NaCl(aq) + H_2CO_3(aq)$$

3. How many grams of sodium metal are needed to produce 125 g hydrogen gas by the reaction below?

$$2Na(s) + 2H_2O(l) \rightarrow 2NaOH(aq) + H_2(g)$$

5.7 Theoretical Yield and Limiting Reactant

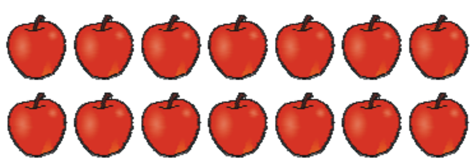
If you have ever made a batch of cookies from a detailed recipe you have probably found that the number of cookies you ultimately end up with is very different than what the recipe indicates. This is a result of a number of factors. Cookie size varies, cookie dough is lost on bowls and spoons, some cookies may be discarded because they cooked for too long, and inevitably you have been sampling your delicious product hot from the oven. A similar situation is found for chemical reactions. The balanced equation indicates exactly how much product will form from a specific amount of reactants. The maximum amount of product that can be obtained from a given amount of reactants is called the **theoretical yield** of the reaction. Often, however, side reactions occur that consume reactants and products or you may not be able to fully recover the product that forms so the amount of product that you obtain, called the **actual yield**, is less than expected. An important area of chemistry research involves working to find reactions and the necessary conditions for which the actual yield is very close to the theoretical yield. This is especially important for industrial processes that involve very large scale reactions, or processes such as the production of pharmaceuticals that involve many steps.

Often, chemical processes are carried out under conditions in which one reactant is present in excess. This means that you will completely consume the other reactant(s) and will have some of the reactant in excess left over. When one or more of the reactants in a chemical reaction are completely consumed the reaction ceases and no additional product can be formed. A reactant that is completely consumed is called a **limiting reactant** because it limits the amount of product that can form. The maximum amount of product that can form from the limiting reactant is the theoretical yield of a reaction.

To illustrate the idea of a limiting reactant and theoretical yield consider the following situation. Suppose you and your friends wish to put together fruit baskets for gifts. You want each basket to contain 1 grapefruit, 2 apples and 3 bananas. If you collect 8 grapefruit, 14 apples and 27 bananas, how many baskets can you prepare?

Starting Materials

8 grapefruit 

14 apples 

27 bananas 

Product

1 basket = 

At first glance you might imagine that the fruit present in the smallest amount (the grapefruit) would be used up first. This would be the case if you used the same number of each item to make your basket, however, this is not the situation here. In order to determine the maximum number of baskets you can prepare (the theoretical yield), determine how many baskets you could make from each type of fruit.

The yield from the grapefruit is: $8\,\cancel{\text{grapefruit}} \times \dfrac{1\,\text{basket}}{1\,\cancel{\text{grapefruit}}} = 8\,\text{baskets}$

The yield from the apples is: $14\,\cancel{\text{apples}} \times \dfrac{1\,\text{basket}}{2\,\cancel{\text{apples}}} = 7\,\text{baskets}$

The yield from the bananas is: $27\,\cancel{\text{bananas}} \times \dfrac{1\,\text{basket}}{3\,\cancel{\text{bananas}}} = 9\,\text{baskets}$

Once you have made 7 fruit baskets all of the apples will be gone and you can make no more. Thus, the apples are the limiting starting material and the maximum amount of product is 7 baskets. After you have used all of the apples there will be some grapefruit and bananas remaining. To determine the excess of each of these, calculate how many were used to make the 7 baskets and subtract this amount from the starting quantity.

The number of grapefruit used is: $7\,\cancel{\text{baskets}} \times \dfrac{1\,\text{grapefruit}}{1\,\cancel{\text{basket}}} = 7\,\text{grapefruit}$

Therefore the number of grapefruit remaining is 8 - 7 = 1.

The number of bananas used is: $7\,\cancel{\text{baskets}} \times \dfrac{3\,\text{bananas}}{1\,\cancel{\text{basket}}} = 21\,\text{bananas}$

Therefore, the number of bananas remaining is 27 - 21 = 6.

This same logic can be applied to chemical reactions. Consider, for example, the combustion of acetylene (C_2H_2). The balanced equation for this reaction is shown below.

$$2C_2H_2(l) \ + \ 5O_2(g) \ \rightarrow \ 4CO_2(g) \ + \ 2H_2O(l)$$

Suppose 75 mol C_2H_2 and 160 mol O_2 are mixed together and allowed to react. What is the maximum number of moles of CO_2 that could be produced? The first step is to decide which substance is the limiting reactant. As was done for the fruit baskets, calculate the amount of product that can form from each specified amount of reactant. The limiting reactant produces the smaller quantity, which will be the theoretical yield. All of the calculations needed are familiar ones using parts of the general solution map shown in Figure 5.4. To get the moles of CO_2 from the starting mole amounts of C_2H_2 and O_2, use moles ratios from the balanced equation.

$$75\ \cancel{\text{mol}\ C_2H_2}\ \text{x}\ \frac{4\ \text{mol}\ CO_2}{2\ \cancel{\text{mol}\ C_2H_2}} = 150\ \text{mol}\ CO_2$$

$$160\ \cancel{\text{mol}\ O_2}\ \text{x}\ \frac{4\ \text{mol}\ CO_2}{5\ \cancel{\text{mol}\ O_2}} = 130\ \text{mol}\ CO_2$$

Since O_2 produces the smaller amount of CO_2 it will be consumed first even though a larger number of moles of O_2 are present initially. The 130 moles of CO_2 represents the maximum yield from these reaction conditions and is the theoretical yield. You can also determine how much of the reactant in excess (C_2H_2) is left over. Calculate how many moles of C_2H_2 are needed to produce the theoretical yield and subtract this from the starting quantity.

$$130\ \cancel{\text{mol}\ CO_2}\ \text{x}\ \frac{2\ \text{mol}\ C_2H_2}{4\ \cancel{\text{mol}\ CO_2}} = 65\ \text{mol}\ C_2H_2$$

Therefore, the number of moles of C_2H_2 remaining is 75 mol - 65 mol = 10. mol. Other examples of such calculations are shown below.

Example 5.17

Problem

What is the theoretical yield (in grams) of silver if 8.75 g copper is mixed with 0.350 mol $AgNO_3(aq)$? The balanced equation for this reaction is shown below.

$$Cu(s) + 2AgNO_3(aq) \rightarrow Cu(NO_3)_2(aq) + 2Ag(l)$$

Solution

What we know: g Cu; mol $AgNO_3$; balanced equation relating Cu, $AgNO_3$ and Ag

Desired answer: maximum g Ag produced

First determine the limiting reactant by calculating how many grams of silver can form from each starting amount of reactant. The solution maps for these calculations are:

g Cu ➔ mol Cu ➔ mol Ag ➔ g Ag

mol $AgNO_3$ ➔ mol Ag ➔ g Ag

For the first calculation the conversion factors needed are the molar mass of Cu, the Ag/Cu mole ratio and finally the molar mass of Ag.

Putting these together yields:

$$8.75\,\cancel{\text{g Cu}} \times \frac{1\,\cancel{\text{mol Cu}}}{63.55\,\cancel{\text{g Cu}}} \times \frac{2\,\cancel{\text{mol Ag}}}{1\,\cancel{\text{mol Cu}}} \times \frac{107.9\,\text{g Ag}}{1\,\cancel{\text{mol Ag}}} = 29.7\,\text{g Ag}$$

For the second calculation the conversion factors needed are the $Ag/AgNO_3$ mole ratio and the molar mass of Ag.

Putting these together yields:

$$0.350\,\cancel{\text{mol AgNO}_3} \times \frac{2\,\cancel{\text{mol Ag}}}{2\,\cancel{\text{mol AgNO}_3}} \times \frac{107.9\,\text{g Ag}}{1\,\cancel{\text{mol Ag}}} = 37.8\,\text{g Ag}$$

Since the starting amount of copper produces the smaller amount of silver, copper is the limiting reactant and the theoretical yield is 29.7 g Ag.

Example 5.18

Problem

How many grams of $AgNO_3$ remain in Example 5.17?

Solution

What we know: mol $AgNO_3$; g Ag produced; balanced equation relating Ag and $AgNO_3$

Desired answer: g $AgNO_3$ remaining

Since copper was found to be the limiting reactant, the $AgNO_3$ is in excess. Use the theoretical yield of Ag to calculate how much $AgNO_3$ was used and subtract this from the starting quantity. Note that the starting moles of $AgNO_3$ will have to be converted to grams. The solution maps for these calculations are:

g Ag ➔ mol Ag ➔ mol $AgNO_3$ ➔ g $AgNO_3$ used

mol $AgNO_3$ initially ➔ g $AgNO_3$ initially

g $AgNO_3$ remaining = g $AgNO_3$ initially - g $AgNO_3$ used

For the first calculation the conversion factors needed are the molar mass of Ag, the $AgNO_3$/Ag mole ratio and finally the molar mass of $AgNO_3$.

Putting these together yields:

$$29.7\,\cancel{\text{g Ag}} \times \frac{1\,\cancel{\text{mol Ag}}}{107.9\,\cancel{\text{g Ag}}} \times \frac{2\,\cancel{\text{mol AgNO}_3}}{2\,\cancel{\text{mol Ag}}} \times \frac{169.9\,\text{g AgNO}_3}{1\,\cancel{\text{mol AgNO}_3}} = 46.8\,\text{g AgNO}_3\text{ used}$$

For the second calculation the conversion factor needed is the molar mass of $AgNO_3$.

$$0.350\,\cancel{\text{mol AgNO}_3} \times \frac{169.9\,\text{g AgNO}_3}{1\,\cancel{\text{mol AgNO}_3}} = 59.5\,\text{g AgNO}_3\text{ initially}$$

Finally, the mass of $AgNO_3$ that remains equals 59.5 g - 46.8 g = 12.7 g.

In summary, in order to determine the theoretical yield from a given set of reaction conditions, first determine which reactant is the limiting reactant by calculating the yield from each starting amount of reactant. The reactant producing the least amount of a product is limiting and the amount of product formed from the limiting reactants is the theoretical yield.

Check for Understanding 5.10 **Solutions**

1. What is the maximum number of moles of $Al_2(SO_4)_3$ that can be produced from the reaction of 95 moles $Al(OH)_3$ and 120 moles H_2SO_4? First write a balanced chemical equation for the reaction.

2. What is the theoretical yield in grams of silicon for the reaction between 2.5 kg $SiCl_4$ and 1.6 kg Mg?

$$SiCl_4(s) \ + \ 2Mg(s) \ \rightarrow \ Si(s) \ + \ 2MgCl_2(s)$$

3. When a mixture of 58 g CO and 9.2 g H_2 reacts, which reactant is in excess? How many grams of the reactant in excess remain?

$$CO(g) \ + \ 2H_2(g) \ \rightarrow \ CH_3OH(l)$$

Chapter 5 Keywords

Glossary

mass spectrometry	mole	mole ratio
atomic weight	Avogadro constant	theoretical yield
atomic mass unit	molar mass	actual yield
Dalton	percent	limiting reactant
isotopes	mass percent	
weighted average	stoichiometry	

Supplementary Chapter 5 *Check for Understanding* questions

Chapter 5 Exercises

Answers

(You may use a periodic table as needed.)

1. If a single molecule of a substance weighs 7.64×10^{-23} g, what is its molar mass?

2. Which of the following contains the smallest number of molecules?

   A. 1.0 g H_2O
   B. 1.0 g NH_3
   C. 1.0 g H_2
   D. 1.0 g O_2
   E. all of the above are the same

3. Ethanol (C_2H_5OH) has a density of 0.789 g/mL. How many ethanol molecules are in a teaspoon (~ 5 mL) of ethanol?

4. 10^{23} atoms of zinc weigh nearly the same as:

   A. a mole of zinc (Zn).
   B. a half of a mole of carbon (C).
   C. two moles of xenon (Xe).
   D. a mole of boron (B).

5. In your own words describe what the atomic weight of an element means.

6. What is the molar mass of $C_3H_5(NO_3)_3$?

7. How many moles of ammonia are in 61 g of NH_3?

8. Create a solution map for calculating how many hydrogen atoms are present in a 75-g sample of CH_3CH_2OH and then make this calculation.

9. What mass of N_2O contains 0.50 mol of nitrogen atoms?

10. A 1979 penny is 95.05% copper by mass. If the penny weighs 3.098 g, how many copper atoms does the penny contain?

11. What is the mass percent of oxygen in K_2SO_4?

12. A substance was analyzed and found to contain only sulfur and fluorine. If there was 0.1570 g fluorine present in a 0.2011-g sample of this substance, what is the mass percent composition of this material?

13. How many moles of HNO_3 does a 60.0-g sample of a nitric acid solution that is 70.4% HNO_3 (by mass) contain?

14. How many grams of Fe_2O_3 contain 15.0 g of iron?

15. The balanced equation $2Cu(s) + O_2(g) \rightarrow 2CuO(s)$ tells us that 1 mol of Cu

A. reacts with 1 mol O_2.
B. produces 1 mol CuO.
C. reacts with 32 g of O_2.
D. produces 2 mol CuO.
E. does both A and C.

16. Create a solution map for calculating the mass (in grams) of ammonia that can be produced from 5 kg of hydrogen gas and an excess of nitrogen according to the reaction $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$.

17. According to the reaction $4Li(s) + O_2(g) \rightarrow 2Li_2O(s)$, how many grams of Li_2O can be produced from each gram of lithium that reacts?

18. The unbalanced equation for the reaction of Na_2O_2 with water is:

$$Na_2O_2 + H_2O \rightarrow NaOH + O_2$$

What mass of oxygen gas will be produced if 3.25 g of sodium peroxide reacts with a large excess of water?

19. What mass of O_2 is needed to react completely with 4.20 g CH_4 in a combustion reaction?

20. How many grams of oxygen gas can be formed from the decomposition of 0.437 mol HgO?

21. For the reaction $2CH_4(g) + 3O_2(g) + 2NH_3(g) \rightarrow 2HCN(g) + 6H_2O(g)$, if you start with 5.0 g of each reactant, which substance is the limiting reactant?

22. How many moles of NH_3 can be produced from the reaction of 4.00 mol N_2 and 10.0 mol H_2?

23. For the reaction $CaCO_3(s) + 2HCl(aq) \rightarrow CaCl_2(aq) + CO_2(g) + H_2O(l)$, how many grams of CO_2 can be produced if 68.1 g $CaCO_3$ (molar mass =100.09 g/mol) is mixed with 51.6 g HCl (molar mass =36.46 g/mol)?

24. How does the actual yield of a chemical reaction compare to the theoretical yield of the reaction?

25. The unbalanced equation below describes the formation of $C_2H_{2.}$

$$CaC_2(s) + H_2O(l) \rightarrow Ca(OH)_2(s) + C_2H_2(g)$$

What is the maximum number of grams of C_2H_2 that can be produced if 8.00 g of CaC_2 and 8.00 g of H_2O are allowed to react?

26. When 2.34 grams of aluminum reacts with 1.50 grams of hydrochloric acid, which reactant is in excess? How much of this reactant is left over?

Chapter 5

Check for Understanding 5.1

1. The atomic weight of sodium is 22.99. What is the mass, in grams, of a billion sodium atoms?

Answer: 3.817×10^{-14} g

Solution

What we know: number of Na atoms; atomic weight of Na

Desired answer: g Na

The number of sodium atoms can be converted into mass using the atomic weight of sodium. Recall that the numerical value of the atomic weight of sodium refers to the mass of a single sodium atom in atomic mass units. First, convert this value to an equivalent mass in grams.

$$\frac{22.99\ \cancel{\text{u Na}}}{1\ \text{Na atom}} \times \frac{1.6605 \times 10^{-24}\ \text{g Na}}{1\ \cancel{\text{u Na}}} = \frac{3.817 \times 10^{-23}\ \text{g Na}}{\text{Na atom}}$$

Now use this result as the conversion factor between atoms of sodium and mass of sodium.

$$1 \times 10^{9}\ \cancel{\text{atoms Na}} \times \frac{3.817 \times 10^{-23}\ \text{g Na}}{\cancel{\text{atom Na}}} = 3.817 \times 10^{-14}\ \text{g Na}$$

Check for Understanding 5.2

1. Write the solution map for the conversion of grams of sodium to atoms of sodium. Indicate the numerical ratio that is the conversion factor in each step.

Answer: The solution map is: g Na ➔ mol Na ➔ atoms Na

The conversion factor for the first step is $\dfrac{1\ \text{mol Na}}{22.99\ \text{g Na}}$.

The conversion factor for the second step is $\dfrac{6.022 \times 10^{23}\ \text{atoms Na}}{1\ \text{mol Na}}$.

Solution

Each step of the solution map requires a conversion factor.

For g Na ➔ mol Na, we need the molar mass of sodium expressed in the form $\frac{1\,\text{mol Na}}{22.99\,\text{g Na}}$ which will cancel *g Na* and introduce *mol Na* into the numerator. The numerical value for the molar mass is obtained from the periodic table.

For mol Na ➔ atoms Na the conversion factor needed is the Avogadro constant expressed in the form $\frac{6.022 \times 10^{23}\,\text{atoms Na}}{1\,\text{mol Na}}$ so that *mol Na* will cancel.

Check for Understanding 5.3

1. What is the mass, in grams, of one cobalt atom?

Answer: 9.786×10^{-23} g

Solution

What we know: number of Co atoms

Desired answer: g Co/atom

The solution map for this calculation is:

atom Co ➔ mol Co ➔ g Co

The conversion factor needed in the first step is the Avogadro constant expressed in the form $\frac{1\,\text{mol Co}}{6.022 \times 10^{23}\,\text{atoms Co}}$.

The conversion factor needed in the second step is the molar mass of cobalt. The numerical value for the molar mass is obtained from the atomic weight of cobalt (58.93) and is expressed in the form $\frac{58.93\,\text{g Co}}{1\,\text{mol Co}}$.

Putting these together yields:

$$1\,\cancel{\text{atom Co}} \times \frac{1\,\cancel{\text{mol Co}}}{6.022 \times 10^{23}\,\cancel{\text{atoms Co}}} \times \frac{58.93\,\text{g Co}}{1\,\cancel{\text{mol Co}}} = 9.786 \times 10^{-23}\,\text{g Co}$$

2. How many atoms of aluminum are present in a piece of aluminum foil that weighs 6.27 g?

Answer: 1.40×10^{23} atoms Al

Solution

What we know: g Al

Desired answer: number of Al atoms

The solution map for this calculation is:

g Al → mol Al → atoms Al

The conversion factor needed in the first step is the molar mass of aluminum. The numerical value for the molar mass is obtained from the periodic table and is expressed in the form $\frac{1\,\text{mol Al}}{26.98\,\text{g Al}}$.

The conversion factor needed in the second step is the Avogadro constant in the form $\frac{6.022 \times 10^{23}\,\text{atoms Al}}{1\,\text{mol Al}}$.

Putting these together yields:

$$6.27\,\cancel{\text{g Al}} \times \frac{1\,\cancel{\text{mol Al}}}{26.98\,\cancel{\text{g Al}}} \times \frac{6.022 \times 10^{23}\,\text{atoms Al}}{1\,\cancel{\text{mol Al}}} = 1.40 \times 10^{23}\,\text{atoms Al}$$

Check for Understanding 5.4

1. Calculate the molar mass of sodium phosphate.

Answer: 163.94 g/mol

Solution

What we know: sodium phosphate; atomic weights of elements

Desired answer: g sodium phosphate/mol

The chemical formula for sodium phosphate is Na_3PO_4. The formula indicates that in 1 mole of sodium phosphate there are 3 moles of sodium, 1 mole of phosphorus and 4 moles of oxygen. From the periodic table we see that 1 mole of sodium atoms weighs 22.99 g, 1 mole of phosphorus atoms weighs 30.97 g and 1 mole of oxygen atoms weighs 16.00 g. Thus, one mole of Na_3PO_4 will weigh:

$$3\ \text{Na} \quad 3\ \cancel{\text{mol Na}} \times \frac{22.99\ \text{g}}{1\ \cancel{\text{mol Na}}} = 68.97\ \text{g}$$

$$1\ \text{P} \quad 1\ \cancel{\text{mol P}} \times \frac{30.97\ \text{g}}{1\ \cancel{\text{mol P}}} = 30.97\ \text{g}$$

$$4\ \text{O} \quad 4\ \cancel{\text{mol O}} \times \frac{16.00\ \text{g}}{1\ \cancel{\text{mol O}}} = 64.00\ \text{g}$$

$$\overline{163.94\ \text{g}}$$

The molar mass of sodium phosphate is 163.94 g/mol.

Check for Understanding 5.5

1. Calculate the number of moles in a sample of sodium bicarbonate that weighs 6.22 g.

Answer: 0.0740 mol

Solution

What we know: g sodium bicarbonate; atomic weights of elements

Desired answer: mol sodium bicarbonate

The solution map for this calculation is:

g sodium bicarbonate ➔ mol sodium bicarbonate

The conversion factor needed is the molar mass of sodium bicarbonate. The chemical formula for sodium bicarbonate is $NaHCO_3$. From the periodic table we see that 1 mole of sodium atoms weighs 22.99 g, 1 mole of hydrogen atoms weighs 1.008 g, 1 mole of carbon atoms weighs 12.01 g and 1 mole of oxygen atoms weighs 16.00 g. Thus, one mole of $NaHCO_3$ will weigh:

1 Na $1\,\cancel{\text{mol Na}} \times \dfrac{22.99\,\text{g}}{1\,\cancel{\text{mol Na}}} = 22.99\,\text{g}$

1 H $1\,\cancel{\text{mol H}} \times \dfrac{1.008\,\text{g}}{1\,\cancel{\text{mol H}}} = 1.008\,\text{g}$

1 C $1\,\cancel{\text{mol C}} \times \dfrac{12.01\,\text{g}}{1\,\cancel{\text{mol C}}} = 12.01\,\text{g}$

3 O $3\,\cancel{\text{mol O}} \times \dfrac{16.00\,\text{g}}{1\,\cancel{\text{mol O}}} = 48.00\,\text{g}$

84.01 g

The molar mass of sodium bicarbonate is 84.01 g/mol.

Using this result yields:

$$6.22\,\cancel{\text{g NaHCO}_3} \times \frac{1\,\text{mol NaHCO}_3}{84.01\,\cancel{\text{g NaHCO}_3}} = 0.0740\,\text{mol NaHCO}_3$$

2. How many molecules are present in 1.05 g of iodine (I_2)?

Answer: 2.49×10^{21} molecules

<u>Solution</u>

What we know: g I_2

Desired answer: number of I_2 molecules

The solution map for this calculation is

g I_2 → mol I_2 → molecules I_2

The conversion factor needed in the first step is the molar mass of I_2. From the periodic table we see that 1 mole of iodine atoms weighs 126.9 g. Thus, one mole of I_2 will weigh:

2 I $\quad 2\,\cancel{\text{mol I}} \times \dfrac{126.9\,\text{g}}{1\,\cancel{\text{mol I}}} = 253.8\,\text{g}$

This is needed in the form $\dfrac{1\,\text{mol}\,I_2}{253.8\,\text{g}\,I_2}$.

The conversion factor needed in the second step is the Avogadro constant in the form $\dfrac{6.022 \times 10^{23}\ \text{molecules}\ I_2}{1\,\text{mol}\,I_2}$.

Putting these together yields:

$$1.05\,\cancel{\text{g}\,I_2} \times \frac{1\,\cancel{\text{mol}\,I_2}}{253.8\,\cancel{\text{g}\,I_2}} \times \frac{6.022 \times 10^{23}\ \text{molecules}\ I_2}{1\,\cancel{\text{mol}\,I_2}} = 2.49 \times 10^{21}\,\text{molecules}\ I_2$$

3. What is the mass in grams of 25 molecules of CCl_2F_2?

Answer: 5.0×10^{-21} g

Solution

What we know: molecules CCl_2F_2

Desired answer: g CCl_2F_2

The solution map for this calculation is:

molecules CCl_2F_2 ➔ mol CCl_2F_2 ➔ g CCl_2F_2

The conversion factor needed in the first step is the Avogadro constant in the form $\dfrac{1\,\text{mol}\,CCl_2F_2}{6.022 \times 10^{23}\ \text{molecules}\ CCl_2F_2}$.

The conversion factor needed in the second step is the molar mass of CCl_2F_2. From the periodic table we can get the molar masses of carbon, fluorine and chlorine and add them as follows. 12.01 g C + 2(35.45 g) Cl + 2(19.00 g) F = 120.91 g. Thus the molar mass of CCl_2F_2 is 120.91 g/mol. This is used in the form $\dfrac{120.91\,\text{g}\,CCl_2F_2}{1\,\text{mol}\,CCl_2F_2}$ to convert units properly.

Putting these together yields:

$$25\,\cancel{\text{molecules}\,CCl_2F_2} \times \frac{1\,\cancel{\text{mol}\,CCl_2F_2}}{6.022\times10^{23}\,\cancel{\text{molecules}\,CCl_2F_2}} \times \frac{120.91\,\text{g}\,CCl_2F_2}{1\,\cancel{\text{mol}\,CCl_2F_2}} = 5.0\times10^{-21}\,\text{g}\,CCl_2F_2$$

Check for Understanding 5.6

1. What is the numerical ratio between the number of hydrogen atoms and oxygen atoms in $(NH_4)_3PO_4$?

Answer: $\frac{12\,\text{H atoms}}{4\,\text{O atoms}}$

Solution

In each formula unit of $(NH_4)_3PO_4$ there are 12 H atoms and 4 O atoms.

2. What is the numerical ratio between the number of moles of sulfur and moles of $Al_2(SO_4)_3$?

Answer: $\frac{3\,\text{mol S}}{1\,\text{mol}\,Al_2(SO_4)_3}$

Solution

There are 3 S atoms in each formula unit of $Al_2(SO_4)_3$. This ratio can be converted to a ratio of moles by multiplying each quantity by the Avogadro constant.

Check for Understanding 5.7

1. Calculate the number of moles of barium nitrate that contain 0.81 mol of oxygen atoms.

Answer: 0.14 mol

Solution

What we know: mol oxygen atoms

Desired answer: mol barium nitrate

The solution map for this calculation is:

mol oxygen atoms ➔ mol barium nitrate

The chemical formula for barium nitrate is $Ba(NO_3)_2$. The formula indicates that in 1 mole of barium nitrate there are 6 moles of oxygen atoms. This relationship can be applied as follows.

$$0.81\ \cancel{\text{mol O}} \times \frac{1\ \text{mol Ba(NO}_3)_2}{6\ \cancel{\text{mol O}}} = 0.14\ \text{mol Ba(NO}_3)_2$$

2. How many carbon atoms are present in 16.0 g C_3H_8?

Answer: 6.56×10^{23} atoms

Solution

What we know: g C_3H_8

Desired answer: number of carbon atoms

The solution map for this calculation is:

g C_3H_8 ➔ mol C_3H_8 ➔ mol C ➔ atoms C

The conversion factor needed in the first step is the molar mass of C_3H_8. From the periodic table we can get the molar masses of carbon and hydrogen and add them as follows. 3(12.01 g) C + 8(1.008 g) H = 44.09 g. Thus the molar mass of C_3H_8 is 40.09 g/mol. This is needed in the form $\frac{1\ \text{mol C}_3\text{H}_8}{44.09\ \text{g C}_3\text{H}_8}$.

The formula of C_3H_8 indicates that in 1 mole of C_3H_8 there are 3 moles of carbon atoms. This relationship can be applied as the conversion factor in the second step in the form $\frac{3\ \text{mol C}}{1\ \text{mol C}_3\text{H}_8}$.

The conversion factor needed in the last step is the Avogadro constant in the form $\frac{6.022 \times 10^{23}\ \text{atoms C}}{1\ \text{mol C}}$.

Putting these together yields:

$$16.0\ \cancel{\text{g C}_3\text{H}_8} \times \frac{1\ \cancel{\text{mol C}_3\text{H}_8}}{44.09\ \cancel{\text{g C}_3\text{H}_8}} \times \frac{3\ \cancel{\text{mol C}}}{1\ \cancel{\text{mol C}_3\text{H}_8}} \times \frac{6.022 \times 10^{23}\ \text{atoms C}}{1\ \cancel{\text{mol C}}} = 6.56 \times 10^{23}\ \text{atoms C}$$

3. How many grams of silver are present in 25.0 g AgBr?

Answer: 14.4 g

Solution

What we know: g AgBr

Desired answer: g Ag

The solution map for this calculation is:

g AgBr ➔ mol AgBr ➔ mol Ag ➔ g Ag

The conversion factor needed in the first step is the molar mass of AgBr. From the periodic table we can get the molar masses of silver and bromine and add them as follows. 107.9 g Ag + 79.90 g Br = 187.8 g. Thus the molar mass of AgBr is 187.8 g/mol. This is needed in the form $\frac{1\,\text{mol AgBr}}{187.8\,\text{g AgBr}}$.

The formula of AgBr indicates that in 1 mole of AgBr there is 1 mole of silver. This relationship can be applied as the conversion factor in the second step in the form $\frac{1\,\text{mol Ag}}{1\,\text{mol AgBr}}$.

The conversion factor needed in the last step is the molar mass of Ag expressed in the form $\frac{107.9\,\text{g Ag}}{1\,\text{mol Ag}}$.

Putting these together yields:

$$25.0\,\cancel{\text{g AgBr}} \times \frac{1\,\cancel{\text{mol AgBr}}}{187.8\,\cancel{\text{g AgBr}}} \times \frac{1\,\cancel{\text{mol Ag}}}{1\,\cancel{\text{mol AgBr}}} \times \frac{107.9\,\text{g Ag}}{1\,\cancel{\text{mol Ag}}} = 14.4\,\text{g Ag}$$

Check for Understanding 5.8

1. What is the mass percent of carbon in $C_6H_{12}O_6$?

Answer: 40.00 %

Solution

What we know: $C_6H_{12}O_6$; atomic weights of elements

Desired answer: mass % for C in $C_6H_{12}O_6$

Since the mass percent values apply to any sample of $C_6H_{12}O_6$, it is convenient to consider one mole of this compound. From the periodic table we can get the molar masses of carbon, hydrogen and oxygen and add them as follows. 6(12.01 g) C + 12(1.008 g) H + 6(16.00 g) O = 180.16 g. Thus the molar mass of $C_6H_{12}O_6$ is 180.16 g/mol, so 180.16 g represents the total mass of material. The formula indicates that 1 mole of $C_6H_{12}O_6$ contains 6 moles of carbon. Therefore the mass of carbon (=component mass) present is:

$$6\ \cancel{\text{mol C}} \times \frac{12.01\,\text{g C}}{1\ \cancel{\text{mol C}}} = 72.06\,\text{g C}$$

These values are used to obtain the mass percent for carbon.

$$\text{mass}\,\%\,\text{C} = \frac{72.06\,\text{g C}}{180.16\,\text{g}\ C_6H_{12}O_6} \times 100 = 40.00\,\%$$

Check for Understanding 5.9

1. Based on the reaction between iron(III) oxide and carbon, how many moles of carbon are needed to produce 1.0 kmol of iron?

$$Fe_2O_3(s) + 3C(s) \rightarrow 2Fe(s) + 3CO(g)$$

Answer: 1500 mol

Solution

What we know: kmol Fe; balanced equation relating Fe and C

Desired answer: mol C

The solution map for this problem is:

kmol Fe ➔ mol Fe ➔ mol C

The conversion factor needed in the first step is that between *kmol* and *mol* in the form $\frac{10^3\ \text{mol Fe}}{1\,\text{kmol Fe}}$.

The conversion factor needed in the second step is the mole ratio for these two substances from the balanced equation in the form $\frac{3\,\text{mol C}}{2\,\text{mol Fe}}$.

Putting these together yields:

$$1.0\ \cancel{\text{kmol Fe}} \times \frac{10^3\ \cancel{\text{mol Fe}}}{1\ \cancel{\text{kmol Fe}}} \times \frac{3\ \text{mol C}}{2\ \cancel{\text{mol Fe}}} = 1500\ \text{mol C}$$

2. What mass of $NaHCO_3$ will react with 0.00257 mol HCl?

$$NaHCO_3(s) + HCl(aq) \rightarrow NaCl(aq) + H_2CO_3(aq)$$

Answer: 0.216 g

Solution

What we know: mol HCl; balanced equation relating HCl and $NaHCO_3$

Desired answer: g $NaHCO_3$

The solution map for this problem is:

mol HCl → mol $NaHCO_3$ → g $NaHCO_3$

The conversion factor needed in the first step is the mole ratio for these two substances from the balanced equation in the form $\frac{1\ \text{mol NaHCO}_3}{1\ \text{mol HCl}}$.

The conversion factor needed in the second step is the molar mass of $NaHCO_3$ expressed in the form $\frac{84.01\ \text{g NaHCO}_3}{1\ \text{mol NaHCO}_3}$.

Putting these together yields:

$$0.00257\ \cancel{\text{mol HCl}} \times \frac{1\ \cancel{\text{mol NaHCO}_3}}{1\ \cancel{\text{mol HCl}}} \times \frac{84.01\ \text{g NaHCO}_3}{1\ \cancel{\text{mol NaHCO}_3}} = 0.216\ \text{g NaHCO}_3$$

3. How many grams of sodium metal are need to produce 125 g hydrogen gas by the reaction below?

$$2Na(s) + 2H_2O(l) \rightarrow 2NaOH(aq) + H_2(g)$$

Answer: 2.85×10^3 g

Solution

What we know: g H_2; balanced equation relating H_2 and Na

Desired answer: g Na

The solution map for this problem is:

g H_2 → mol H_2 → mol Na → g Na

The conversion factor needed in the first step is the molar mass of H_2 expressed in the form $\frac{1\,\text{mol}\,H_2}{2.016\,\text{g}\,H_2}$.

The conversion factor needed in the second step is the mole ratio for these two substances from the balanced equation in the form $\frac{2\,\text{mol Na}}{1\,\text{mol}\,H_2}$.

The conversion factor needed in the last step is the molar mass of Na expressed in the form $\frac{22.99\,\text{g Na}}{1\,\text{mol Na}}$.

Putting these together yields:

$$125\,\cancel{\text{g}\,H_2} \times \frac{1\,\cancel{\text{mol}\,H_2}}{2.016\,\cancel{\text{g}\,H_2}} \times \frac{2\,\cancel{\text{mol Na}}}{1\,\cancel{\text{mol}\,H_2}} \times \frac{22.99\,\text{g Na}}{1\,\cancel{\text{mol Na}}} = 2.85 \times 10^3\,\text{g Na}$$

Check for Understanding 5.10

1. What is the maximum number of moles of $Al_2(SO_4)_3$ that can be produced from the reaction of 95 moles $Al(OH)_3$ and 120 moles H_2SO_4? First write a balanced chemical equation for the reaction?

Answer: 40. mol

Solution

The balanced equation for the reaction is:

$$2Al(OH)_3(s) + 3H_2SO_4(aq) \rightarrow Al_2(SO_4)_3(aq) + 6H_2O(l)$$

What we know: mol $Al(OH)_3$; mol H_2SO_4; balanced equation relating $Al(OH)_3$, H_2SO_4 and $Al_2(SO_4)_3$

Desired answer: maximum moles $Al_2(SO_4)_3$ produced

First determine the limiting reactant by calculating how many moles of $Al_2(SO_4)_3$ can form from each starting amount of reactant. The solution maps for these calculations are:

mol $Al(OH)_3$ → mol $Al_2(SO_4)_3$

mol H_2SO_4 → mol $Al_2(SO_4)_3$

For the first calculation the conversion factor needed is the $Al_2(SO_4)_3/Al(OH)_3$ mole ratio from the balanced equation.

Applying this yields:

$$95\ \cancel{\text{mol Al(OH)}_3} \times \frac{1\,\text{mol Al}_2(\text{SO}_4)_3}{2\ \cancel{\text{mol Al(OH)}_3}} = 48\,\text{mol Al}_2(\text{SO}_4)_3$$

For the second calculation the conversion factor needed is the $Al_2(SO_4)_3/H_2SO_4$ mole ratio from the balanced equation.

Applying this yields:

$$120\ \cancel{\text{mol H}_2\text{SO}_4} \times \frac{1\,\text{mol Al}_2(\text{SO}_4)_3}{3\ \cancel{\text{mol H}_2\text{SO}_4}} = 40.\,\text{mol Al}_2(\text{SO}_4)_3$$

Since the starting amount of H_2SO_4 produces the smaller amount of $Al_2(SO_4)_3$, sulfuric acid is the limiting reactant and the maximum moles of $Al_2(SO_4)_3$ is 40. moles.

2. What is the theoretical yield in grams of silicon for the reaction between 2.5 kg $SiCl_4$ and 1.6 kg Mg?

$$SiCl_4(s) + 2Mg(s) \rightarrow Si(s) + 2MgCl_2(s)$$

Answer: 4.1×10^2 g

Solution

What we know: kg $SiCl_4$; kg Mg; balanced equation relating $SiCl_4$, Mg and Si

Desired answer: maximum g Si produced

First determine the limiting reactant by calculating how many grams of silicon can form from each starting amount of reactant. The solution maps for these calculations are:

kg $SiCl_4$ → g $SiCl_4$ → mol $SiCl_4$ → mol Si → g Si

kg Mg → g Mg → mol Mg → mol Si → g Si

For the first calculation the conversion factors needed are that between *kg* and *g*, the molar mass of $SiCl_4$, the Si/$SiCl_4$ mole ratio and finally the molar mass of Si.

Putting these together yields:

$$2.5\,\cancel{\mathrm{kg\,SiCl_4}} \times \frac{10^3\,\cancel{\mathrm{g\,SiCl_4}}}{1\,\cancel{\mathrm{kg\,SiCl_4}}} \times \frac{1\,\cancel{\mathrm{mol\,SiCl_4}}}{169.89\,\cancel{\mathrm{g\,SiCl_4}}} \times \frac{1\,\cancel{\mathrm{mol\,Si}}}{1\,\cancel{\mathrm{mol\,SiCl_4}}} \times \frac{28.09\,\mathrm{g\,Si}}{1\,\cancel{\mathrm{mol\,Si}}} = 4.1 \times 10^2\,\mathrm{g\,Si}$$

For the second calculation the conversion factors needed are that between *kg* and *g*, the molar mass of Mg, the Si/Mg mole ratio and finally the molar mass of Si.

Putting these together yields:

$$2.5\,\cancel{\mathrm{kg}\,Mg} \times \frac{10^3\,\cancel{\mathrm{g}\,Mg}}{1\,\cancel{\mathrm{kg}\,Mg}} \times \frac{1\,\cancel{\mathrm{mol}\,Mg}}{24.31\,\cancel{\mathrm{g}\,Mg}} \times \frac{1\,\cancel{\mathrm{mol\,Si}}}{2\,\cancel{\mathrm{mol}\,Mg}} \times \frac{28.09\,\mathrm{g\,Si}}{1\,\cancel{\mathrm{mol\,Si}}} = 9.2 \times 10^2\,\mathrm{g\,Si}$$

Since the starting amount of $SiCl_4$ produces the smaller amount of silicon, $SiCl_4$ is the limiting reactant and the theoretical yield is 4.1×10^2 g Si.

3. When a mixture of 58 g CO and 9.2 g H_2 reacts, which reactant is in excess? How many grams of the reactant in excess remain?

$$CO(g) + 2H_2(g) \rightarrow CH_3OH(l)$$

Answers: H_2

0.7 g

<u>Solution</u>

What we know: g CO; g H_2; balanced equation relating CO, H_2 and CH_3OH

Desired answer: g excess reactant remaining

To find which reactant is in excess, determine the limiting reactant by calculating how many moles of CH_3OH can form from each starting amount of reactant. The solution maps for these calculations are:

g CO ➔ mol CO ➔ mol CH_3OH

g H_2 ➔ mol H_2 ➔ mol CH_3OH

For the first calculation the conversion factors needed are the molar mass of CO and the CH_3OH/CO mole ratio.

Putting these together yields:

$$58\,\cancel{\text{g CO}} \times \frac{1\,\cancel{\text{mol CO}}}{28.01\,\cancel{\text{g CO}}} \times \frac{1\,\text{mol CH}_3\text{OH}}{1\,\cancel{\text{mol CO}}} = 2.1\,\text{mol CH}_3\text{OH}$$

For the second calculation the conversion factors needed are the molar mass of H_2 and the CH_3OH/H_2 mole ratio.

Putting these together yields:

$$9.2\,\cancel{\text{g H}_2} \times \frac{1\,\cancel{\text{mol H}_2}}{2.016\,\cancel{\text{g H}_2}} \times \frac{1\,\text{mol CH}_3\text{OH}}{2\,\cancel{\text{mol H}_2}} = 2.3\,\text{mol CH}_3\text{OH}$$

Since the starting amount of CO produces the smaller amount of CH_3OH, CO is the limiting reactant and H_2 is the reactant in excess.

Now use the theoretical yield of CH_3OH to calculate how much H_2 was used and subtract this from the starting quantity. The solution maps for these calculations are:

mol CH_3OH ➔ mol H_2 ➔ g H_2 used

g H_2 remaining = g H_2 initially - g H_2 used

For the first calculation the conversion factors needed are the H_2/CH_3OH mole ratio and the molar mass of H_2.

Putting these together yields:

$$2.1\,\cancel{\text{mol CH}_3\text{OH}} \times \frac{2\,\cancel{\text{mol H}_2}}{1\,\cancel{\text{mol CH}_3\text{OH}}} \times \frac{2.016\,\text{g H}_2}{1\,\cancel{\text{mol H}_2}} = 8.5\,\text{g H}_2\text{ used}$$

Therefore, the mass of H_2 that remains equals 9.2 g - 8.5 g = 0.7 g.

Chapter 5

1. 46.0 g/mol

2. B

3. 5×10^{22} molecules

4. D

6. 227.10 g/mol

7. 3.6 mol

8. g CH_3CH_2OH → mol CH_3CH_2OH → mol H → atoms H

   5.9×10^{24} atoms

9. 11 g

10. 2.790×10^{22} atoms

11. 36.72%

12. 21.93% S
    78.07% F

13. 0.670 mol

14. 21.4 g

15. B

16. kg H_2 → g H_2 → mol H_2 → mol NH_3 → g NH_3

17. 2.152 g

18. 0.667 g

19. 16.8 g

20. 6.83 g

21. O_2

22. 6.67 mol

23. 29.9 g

24. The actual yield is always less than the theoretical yield.

25. 3.25 g

26. Al is in excess
    1.97 g remain