Chapter 4: Variability

1. a. The range is 7 points and the interquartile range is 2 points (from 2.5 to 4.5).
   b. The range is 27 points and the interquartile range is 2 points (from 2.5 to 4.5).
   c. The range is completely determined by the extreme scores. The interquartile range is not influenced by the most extreme scores.

2. a. SS is the sum of squared deviation scores.
   b. Variance is the mean squared deviation.
   c. Standard deviation is the square root of the variance. It provides a measure of the standard distance from the mean.

3. The standard deviation, $\sigma = 20$ points, measures the standard distance between a score and the mean.

4. SS cannot be less than zero because it is computed by adding squared deviations. Squared deviations are always greater than or equal to zero.

5. A standard deviation of zero indicates there is no variability. In this case, all of the scores in the sample have exactly the same value.

6. A standard deviation of $s = 5$ indicates that the scores are scattered around the mean with the average distance between X and M equal to 5 points. More specifically, around 70% of the scores should be within 5 points of the mean and about 95% of the scores should be within 10 points of the mean.

7. a. The range is 5 points, the interquartile range is 1 point (from 2.5 to 3.5), and the standard deviation is $\sqrt{1.25} = 1.12$.
   b. After adding two points to every score, the range is still 4, the interquartile range is still 1,
   and the standard deviation is still 1.12. Adding a constant to every score does not affect variability.

8. An *unbiased* estimate means that on average the sample statistic will provide an accurate representation of the corresponding population parameter. More specifically, if you take all the possible samples and compute the variance for each sample, then the average of all the sample variances will be exactly equal to the population variance.

9. a. Median = 3.5. Semi-interquartile range = 2 (Q1 = 2.5 and Q3 = 6.5)
   b. The median is still 3.5 and the semi-interquartile range is still 2 points.
   c. One extreme score does not influence the median or the semi-interquartile range.

10. a. The mean is $M = 4$ and the standard deviation is $s = \sqrt{9} = 3$.
    b. The new mean is $M = 6$ and the new standard deviation is $\sqrt{49} = 7$.
    c. Changing one score changes both the mean and the standard deviation.

11. a. With a standard deviation of 10 points, a score of $X = 38$ would not be considered extreme. It is within one standard deviation of the mean.
    b. With a standard deviation of only 2 points, a score of $X = 38$ is extreme. In this case, the score is located above the mean by a distance equal to four times the standard deviation.

12. a. Your score is 5 points above the mean. If the standard deviation is 2, then your score is an extremely high value (out in the tail of the distribution). However, if the standard deviation is 10, then your score is only slightly above average. Thus, you would prefer $\sigma = 2$.
    b. If you are located 5 points below the mean then the situation is reversed. A standard deviation of 2 gives you an extremely low score, but with a standard deviation of 10 you are only slightly below average. Here, you would prefer $\sigma = 10$.

13. a. The new mean is $\mu = 35$ and the standard deviation is still $\sigma = 5$.
    b. The new mean is $\mu = 90$ and the new standard deviation is $\sigma = 15$.

14. The mean is $M = 86$ and the standard deviation is $s = 3$.

15. a. The definitional formula is easy to use when the mean is a whole number and there are relatively few scores.
    b. The computational formula is preferred when the mean is not a whole number.

16. For sample A the mean is $M = 4.25$. No, the definitional formula would not be easy. For sample B the mean is $M = 4$. Yes, the definitional formula would be easy.

17. a. Mean = 21/7 = 3.0

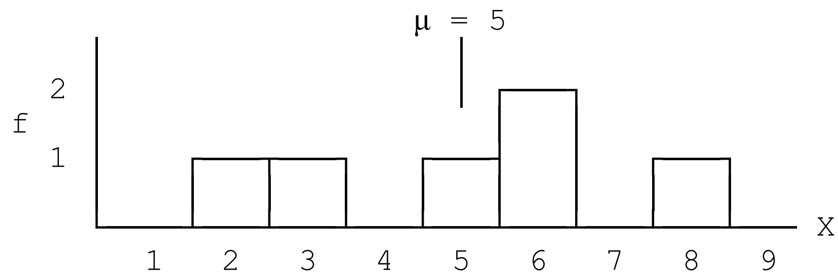
b. and c.

X	Deviation	Squared Deviation
3	0	0
2	−1	1
5	+2	4
0	−3	9
1	−2	4
2	−1	1
8	+5	25
	0	44 = SS

18. a. $\Sigma X = 10$ and $\Sigma X^2 = 36$.

b. $SS = 11$

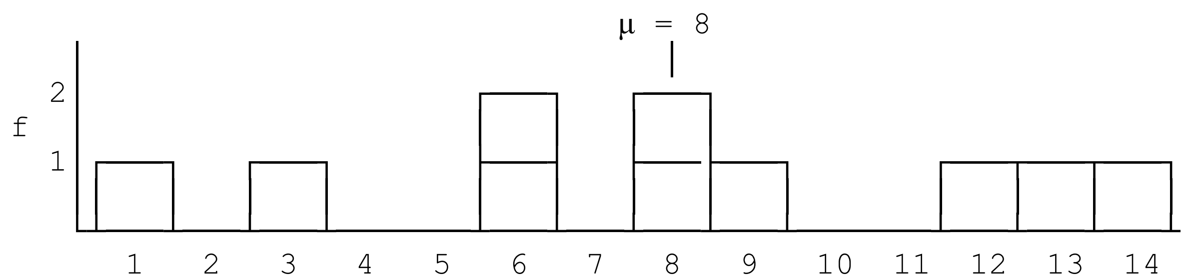
19. a.



b. The mean is 30/6 = 5. The score $X = 5$ is exactly equal to the mean. The scores $X = 2$ and $X = 8$ are farthest from the mean (3 points). The standard deviation should be between 0 and 3 points.

c. SS = 24, $\sigma^2 = 4$, $\sigma = 2$

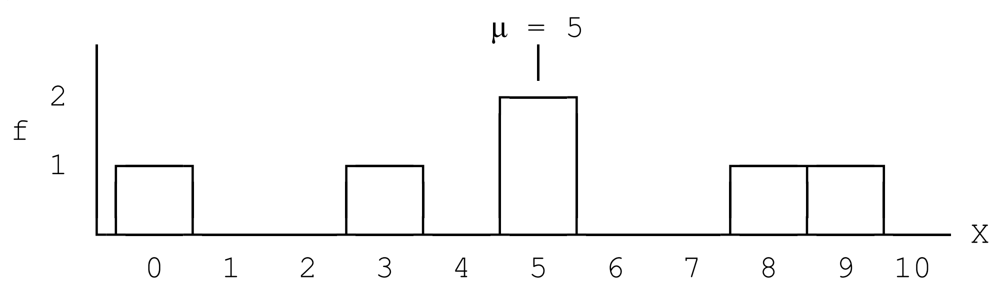
20. a.



b. The mean is 80/10 = 8. The two scores of $X = 8$ are exactly equal to the mean. The score $X = 1$ is farthest from the mean (7 points). The standard deviation should be between 0 and 7 points.

c. $SS = 160$, $\sigma^2 = 16$, and $\sigma = 4$.

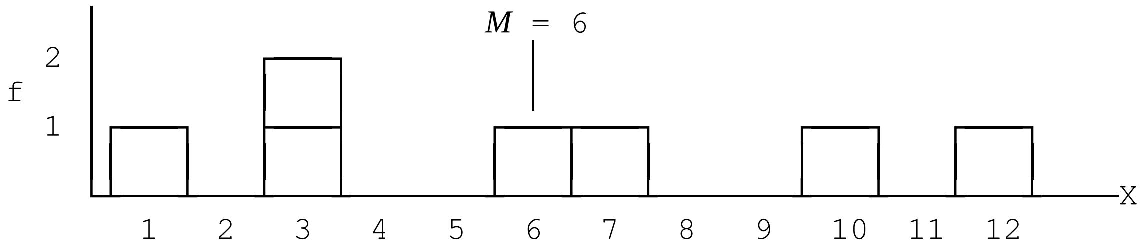
21. a.



b. The mean is $\mu = 30/6 = 5$ and the standard deviation appears to be about 3 points.

c. $SS = 54$, $\sigma^2 = 9$, $\sigma = 3$

22. a.



b. The mean is 42/7 = 6 and the standard deviation appears to be about 4 or 5.
c. SS = 96, $s^2 = 16$, $s = 4$.

23. *SS* = 80, the sample variance is 16 and the standard deviation is 4.

24. *SS* = 64, the sample variance is 16 and the standard deviation is 4.

25. *SS* = 9, the sample variance is 3 and the standard deviation is 1.73.

26. *SS* = 3, the sample variance is 1 and the standard deviation is = 1.

27. *SS* = 32, the population variance is 4 and the standard deviation is 2.

28. a. For the females, M = 10 and s = 1.60. For the males, M = 8 and s = 2.45.
b. The males and females have the same mean IQ score but the male's scores are more variable.

29. a. For the younger woman, the variance is $s^2 = 0.786$. For the older woman, the variance is $s^2 = 1.696$.
b. The variance for the younger woman is only half as large as for the older woman. The younger woman's scores are much more consistent.