

The Platonic Solids

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Chapter 1: Platonic Solid

Definition:

The most basic definition is to say that a Platonic Solid is an object where all faces are identical and the same number of faces meet at each vertex.

In solid geometry and some ancient physical theories, a Platonic solid is a convex polyhedron with all its faces being regular polygons of the same size and shape, and* the same number of faces meeting at each of its vertices.¹

Platonic solid - any one of five solids whose faces are congruent regular polygons and whose polyhedral angles are all congruent²

Chapter 2: Types of Solids

Note: for a great overview and lesson plan on how to teach the platonic solids to students visit [this site](#).

¹ Definition from Google web definition
² definition from www.thefreedictionary.com

There are only 5 platonic Solids in existence. For those who already know what Platonic Solids are, one might wonder as to whether there are any other 5 Platonic Solids than the ones which we already know?

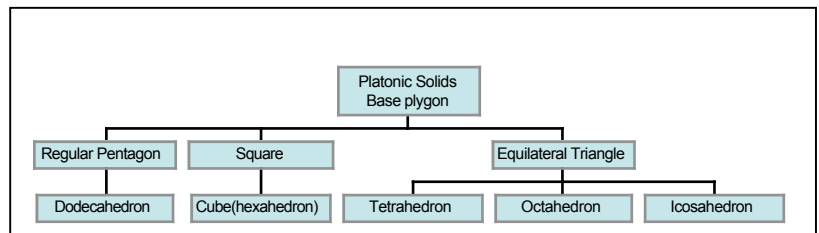
For those who do not know what Platonic Solids are, they are those solid objects whose sides are made of a unique regular polygon i.e. they are regular polyhedra.

The 5 platonic solids are:

Number	Platonic Solid	# of sides	Base polygon
1	Tetrahedron	4	Equilateral Triangle
2	Cube	6	Square
3	Octahedron	8	Equilateral Triangle
4	Dodecahedron	12	Regular Pentagon
5	Icosahedron	20	Equilateral triangle

Here's another way to look at it:

Chapter 3: Why only 5?



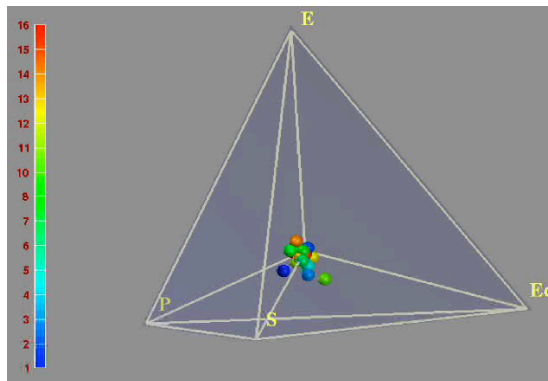
Now the question arises. Why do we have only 5 Platonic Solids? Are there any other regular polyhedra which haven't been discovered yet?

In mathematics we always make a logical analysis to find an equation to solve a particular type of problem with In the above case the puzzle is the existence of only 5 regular polyhedra. To solve this puzzle, first we shall look at the available data and build a formula to verify the existence of a given regular polyhedra with a given number of sides with a given regular polygon.

The data available is that for a given polyhedra there is only one regular polygon which makes up its sides. The other data which is required to form any solid object regular or irregular (i.e. with irregular and/or different polygons) is that the sum of the angles between the sides of the polygons at any given vertex of the polyhedra should always be less than 360 degrees else the polyhedra will flatten out at that particular vertex and the formation of a closed solid object would be impossible.³

So we have two sets of data:
Sum of the angles of all the sides at any given vertex of the regular polyhedra has to be less than 360°

available data or to find a solution to a puzzle.



regular polyhedra) and the angles between all sides and the total angle at any vertex is the same at all vertices (as all the sides are made up of regular polygons).

Thus if 'n' is the total number of the sides which meet at a vertex of the polyhedra and 'Ø' is the angle between any two sides of the regular polygon, then the sum of the angles between all the sides at any vertex of the polyhedra is given by

$$\text{Ø} + \text{Ø} + \text{Ø} + \text{Ø} + \text{Ø} + \text{Ø} \dots n \text{ times} = n\text{Ø}$$

We know that for the polyhedra to exist, nØ has to be less than 360

$$\text{i.e. } n\text{Ø} < 360 \quad - (1)$$

Now we shall see how many regular

Platonic Solids	Face Shapes	# of Faces	# of Vertices	# of Edges
Tetrahedron				
Hexahedron				
Octahedron				
Dodecahedron				
Icosahedron				

Please Complete the Table above through your own investigation.

The number of sides which meet at a vertex is same for all vertices (for a

polyhedra are possible which satisfy the above condition.

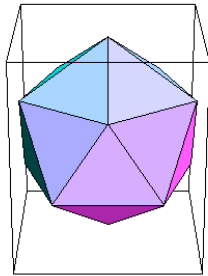
We know that a minimum of 3 sides are required to meet at a vertex to form a polyhedra. Thus n>2 is also an essential condition.

³ text copied from <http://www.hitxp.com/math/geo/euclid/210503.htm>

Therefore, let us start with $n=3$, i.e. let 3 regular polygons meet at each vertex. Let the first regular polygon we start with be the one with the smallest number of sides i.e. an equilateral triangle. So we have $\theta = 60$. Thus $n\theta = 3 \times 60 = 180 < 360$. Hence a regular polyhedra with the above combination is possible as this satisfies our condition (1) and we know that this a tetrahedron.

Now let $n=4$ and we continue with an equilateral triangle as the regular polygon. We have $n\theta = 4 \times 60 = 240 < 360$. Since our condition is satisfied, we have another regular polygon i.e. an Octahedron.

Let $n=5$ and we still continue with an equilateral triangle as our regular polygon and we have $n\theta = 5 \times 60 = 300$ which still satisfies our condition and here we have an Icosahedron.



Let $n=6$ and we continue with the equilateral triangle i.e. $\theta=60$. Thus $n\theta = 6 \times 60 = 360 = 360$. Here our condition (1) is not satisfied and hence regular polyhedra with this combination cannot exist in nature. With $\theta=60$ no other regular polyhedra is possible for $n=6$ or greater.

So now we switch to the next smallest regular polygon i.e. a square. Hence we have $\theta=90$. As usual, we shall start with $n=3$. We have $\theta=90$ for a square. Thus $n\theta = 3 \times 90 = 270 < 360$. Thus it is possible to have a regular polyhedra made up of squares with 3 squares meeting at each vertex. This is our most familiar Platonic Solid i.e. a cube.

Now let $n=4$ and $\theta=90$ i.e. let us continue with the square. Now $n\theta = 4 \times 90 = 360 = 360$. Our condition (1) is not satisfied. Hence a regular polyhedra with this combination is not possible nor is it possible for higher values of n with $\theta = 90$.

Let us now switch to the next polygon i.e. a pentagon which has 5 equal sides and 5 interior angles with each angle being 108 degrees i.e. $\theta=108$. Starting with $n=3$ we have $n\theta = 3 \times 108 = 324 < 360$. Hence such a regular polyhedra can exist and it is a Dodecahedron.

Now when $n=4$ and $\theta=108$ we have $n\theta = 4 \times 108 = 432 > 360$. Hence this regular polyhedra cannot exist as our condition (1) is not satisfied. Similarly it cannot exist for higher values of n with $\theta=108$.

Now we move on to hexagon as our regular polygon i.e. $\theta=120$. starting with $n=3$ we see that $n\theta = 3 \times 120 = 360 = 360$. Our condition (1) is not satisfied and this regular polyhedra cannot exist nor can it exist for any higher values of n or θ as all such combinations will be greater than 360.

Thus we have only 5 Platonic Solids whose existence we have proved in the above reasoning and we also have proved that no other regular polyhedra can exist in nature.

Chapter 4: Types of Platonic Solids

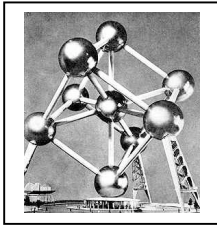
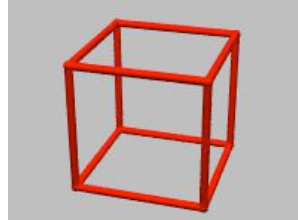
Tetrahedron



tetrahedron (plural: **tetrahedra**) is a **polyhedron** composed of four triangular faces, three of which meet at each **vertex**. A **regular tetrahedron** is one in which the four triangles are regular, or "equilateral," and is one of the **Platonic solids**.

Hexahedron (Cube)

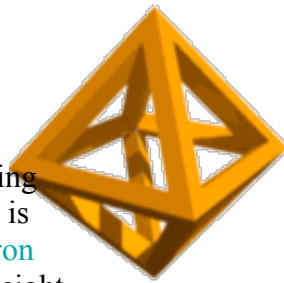
A hexahedron is a [polyhedron](#) with six faces. It is composed of six [square](#) faces that meet each other at [right angles](#) and has eight vertices and 12 edges.



Sodium chloride (NaCl; common table salt) naturally forms cubic crystals.

Octahedron

An octahedron is a [polyhedron](#) having eight faces. The regular octahedron is the [Platonic solid](#) with six [polyhedron vertices](#), 12 [polyhedron edges](#), and eight equivalent [equilateral triangular](#) faces,



Dodecahedron

A general dodecahedron is a [polyhedron](#) having 12 faces. The regular dodecahedron is the [Platonic solid](#) composed of 20 [polyhedron vertices](#), 30 [polyhedron edges](#), and 12 [pentagonal faces](#)



Icosahedron

An icosahedron is a 20-faced [polyhedron](#). The icosahedron is the [Platonic solid](#) having 12 [polyhedron vertices](#), 30 [polyhedron edges](#), and 20 equivalent [equilateral triangle](#) faces.



⁴ info about the platonic solids found at [Platonic solid](#)