MECHANICS LABORATORY
AM 317

EXPERIMENT 5
COLUMN BEHAVIOR
BUCKLING

I. OBJECTIVES

1.1 To determine the effect the slenderness ratio has on the load carrying capacity of pin ended columns.

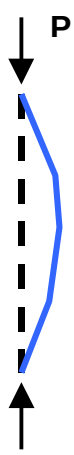
1.2 To observe short, intermediate and long column behavior under the application of a compressive load.

1.3 To compare experimentally observed values of critical stress with the theoretical values.

II. BACKGROUND

A straight slender member subjected to an axial compressive load is called a column. If such a member is relatively short, it will remain straight when loaded, and failure will occur by yielding of the material (in the case of wood crushing of the fibers will occur). However, if the member is relatively long a different type of behavior will be observed. When the compressive load reaches a so called "critical load" a long column will undergo a bending action in which the lateral deflection will become very large with little increase in load. This behavior is called "buckling" and can occur even though the maximum stress in the column is less than the yield stress of the material. The load at which a column will buckle is affected by material properties, column length and cross section, and end conditions.

Lateral deflection of column produced by buckling



If a long slender bar of constant cross section is pinned at each end, the applied compressive load, P_{cr} that will cause buckling is:

$$Pcr = \frac{\pi^2 EI}{(KL)^2} \qquad 5.1$$

Where:

E = Modulus of elasticity.
I = Minimum moment of inertia of cross-sectional area about an axis through the centroid.
L = Length of the bar.
K = Effective length constant (KL = effective length)

This formula was first obtained by the Swiss mathematician, Leonard Euler (1707-1783) and the load P_{cr} is called the Euler buckling load (see Appendix A for the derivation).

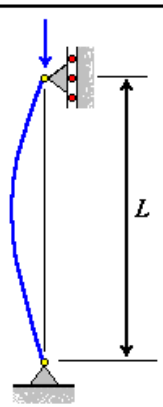
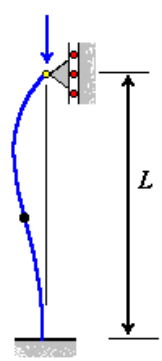
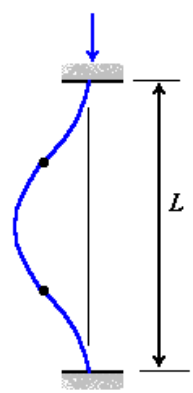
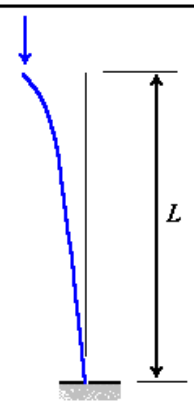
Euler's formula defines a boundary above which elastic instability occurs in a compression member. To make it independent of the size of the member it is frequently written in terms of stress rather than load.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{(KL)^2 A} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad 5.2$$

where L/r is called the slenderness ratio and r is the radius of gyration. The radius of gyration can be computed from the equation:

$$r = \sqrt{\frac{I}{A}} \qquad 5.3$$

The effective length L_e = KL depends on the support boundary conditions which are summarized in the table below:

(a) Pinned - pinned column	(b) Fixed - pinned column	(c) Fixed - fixed column	(d) Fixed - free column
			
$L_e = L$	$L_e = 0.699L$	$L_e = 0.5L$	$L_e = 2L$
K = 1	K = 0.699	K = 0.5	K = 2

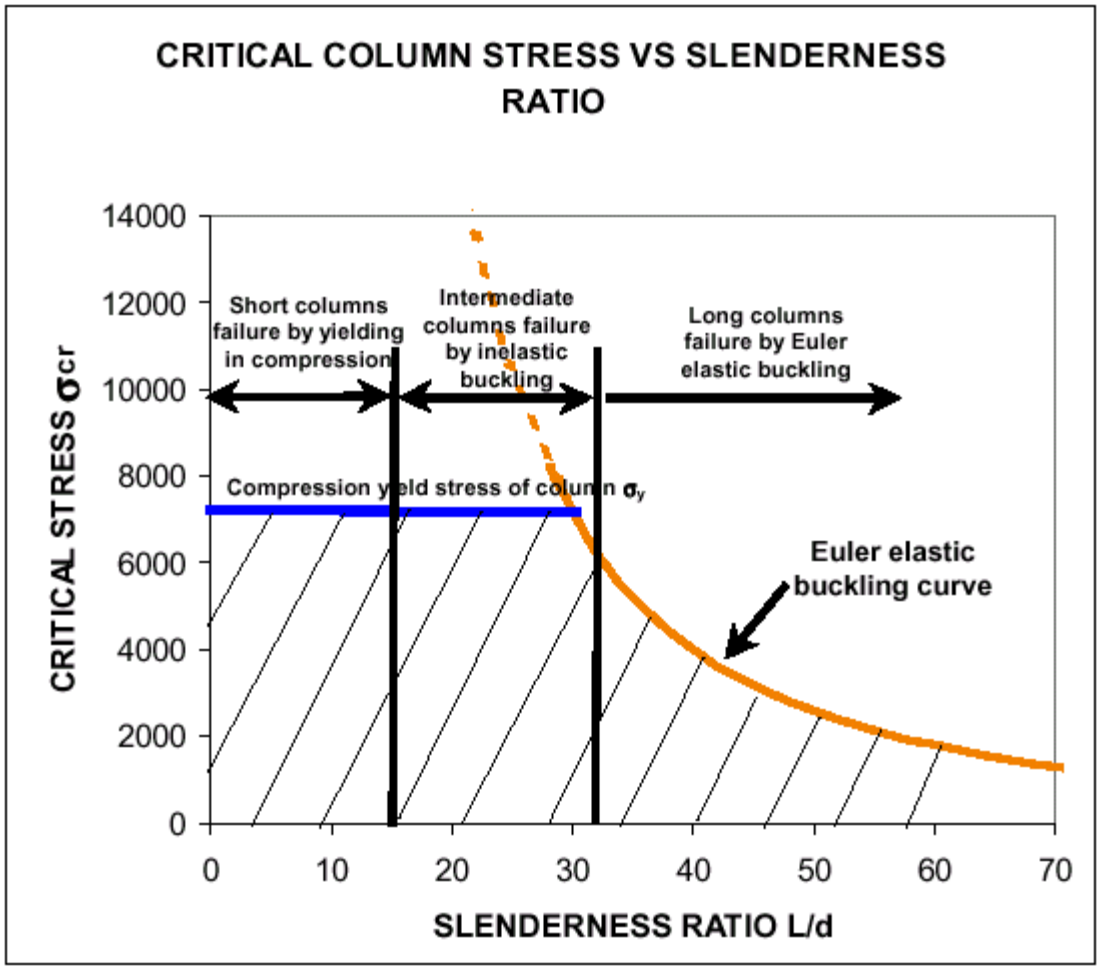
In this experiment, the members tested are made of Douglas Fir and have a cross section of approximately 0.7 x 0.7 inches. This radius of gyration can then be written in terms of the minimum thickness (d_{min}) of the member. It is the minimum cross section dimension (dmin) that usually defines the axis about which buckling takes place experimentally.

$$\sigma = \frac{\pi^2 E}{12\left(\frac{KL}{dmin}\right)^2} \qquad 5.4$$

where L/d is an alternate definition of slenderness ratio for members with a rectangular cross section.

If the cross section is square, the moment of inertia and the radius of gyration are the same about any axis through the centroid and buckling is likely to occur about any axis. During the testing of each specimen try to observe the axis about which bending is taking place. If the member bends about a diagonal then the moment required to reach the proportional limit is a minimum because the distance to the extreme fiber is one half the diagonal rather than one half the thickness.

A second boundary to the safe stress range for a compression member is the yield stress or crushing strength. The experimental data points may therefore be expected to fall below both Euler curve and the yield stress line as shown in the following figure.



The two material properties of interest in this experiment are the crushing strength and modulus of elasticity in bending for Douglas Fir. Mark's handbook implies, based on data from the U.S. Forest Products Laboratory, that both of these material properties are directly related to the specific gravity of the wood. Average values at 12% moisture content, for Douglas Fir are:

specific gravity (SG)	**=**	**0.51**
modulus of elasticity in bending (E)	**=**	**1,950,000 psi**
maximum crushing strength (σ_{ys})	**=**	**7430 psi**

Since the modulus and the crushing strength vary directly with the specific gravity, the critical stress values obtained from the experiment may be adjusted using the values you determine for the specific gravity:

$$\sigma = \left(\frac{0.51}{SG_{exp}}\right)\sigma_{exp} \qquad 5.5$$

III. EQUIPMENT

3.1 Instron test machine.

3.2 Assorted small tools.

MATERIALS

Six specimens of nominal 0.7 in x 0.7 in. Douglas Fir of varying lengths.

These specimens used in this experiment are selected at random from the wood shop. No attempt has been made to cut each length member from the same piece of wood. Thus the results may be expected to vary considerably; however, by adjusting the data for specific gravity an attempt will be made to reduce the scatter and explain the results.

The weight of each member, in ounces, will be needed to calculate the specific gravity of each piece. First, the weight per cubic inch of each member is obtained by dividing its weight by its actual volume in cubic inches (cu. in.) then, dividing by standard weight of water, 0.5778 oz/in^3, the specific gravity of each member is obtained.

IV. PROCEDURE

4.0 Measure and weigh the wood samples.

4.1 Calculate the theoretical Euler buckling load and stress. Calculate the theoretical maximum crushing load (the maximum crushing stress is given).

4.2 Turn the computer on.

4.3 Pull out the emergency stop button (red button) on the right hand side of the Instron machine. Two green arrows will light.

4.4 Check that the upper and lower bearing plates are aligned vertically then place the longest column in the bearing plates.

4.5 Place the wire cage around the machine.

4.7 Computer Procedure

A. login to the QTEST program

B. **From the main menu select Method (M) use the arrow keys to select the test titled Compression Roberts. <enter>**

C. Go to "Test-menu"

1. **Select Specimen (S) and input Sample ID#: wood <enter>**

2. **Input the cross section dimensions (Th and BREADTH) <enter>**
3. **Select Run (R), Load range X1 (no input required)**

4. **Hit <enter> again (no input required)**

5. **"Warning load cell has not been calibrated" should appear <enter>**

6. **Input the pretest specimen dimensions (Th and BREADTH, two significant figures) <enter>**

7. **** Warning ** Test is about to begin. <enter>**

   If for any reason you want to stop the test now in progress PRESS <SPACE> BAR TO ABORT THE EXPERIMENT.

8. **A load deflection plot will be created until the column fails. Observe the manner in which each specimen failed; for buckling, the specimen bows out before fracturing, for axial compression yielding, the specimen fractures before bowing out, or by a combination of both (bowing and fracturing occur simultaneously). Sketch the sample at fracture.**

9. **Upon failure the program returns to Specimen Results Menu. Record the peak load and peak stress measured.**

10. **Select Graph (G) to see the plot. Use the mouse cursor to click on Print to obtain a printout of the plot. If you click on Plot, the program will abort!**

11. **Esc brings you back to the Specimen Results Menu. Select Next (N) <u>prior</u> to inserting a new column between the supports.**

12. Select run to test the new column and repeat the above steps until all the columns are tested.

D. To exit, use the ESC key to get to the main menu then Exit (X). Turn off the computer. Push in the red knob to turn off the instron machine.

V. REPORT

5.1 Plot the following curves on the same graph using $\sigma = P_{cr}/A$ as ordinate and L/d as abscissa:

a. Two experimental curves, using P_{cr}/A and L/d as obtained and measured in the laboratory. Plot the "raw" data and the adjusted data.

b. The theoretical curve plotted from the Euler column formula (Equation 5.4). If using Excel, double click on the y axis and set the maximum scale value to 14,000 (psi). This is required because the Euler buckling stress will approach infinity as the length approaches zero.

c. The horizontal line indicating the maximum crushing stress.

5.2 Indicate directly on the figure (as in the figure in the text) the range of values of L/d corresponding to short columns, intermediate columns, and long columns.

5.3 Discuss the results and draw appropriate conclusions.

SELECTED REFERENCES

6.1 Statics and Strength of Materials. Stevens, Ch. 10.

6.2 Mark's Standard Handbook for Mechanical Engineers. 8th Edition. McGraw Hill, Chapter 6, pp. 123, 125.

6.3 An Introduction to the Mechanics of Solids. Crandel and Dahl.

DATA SHEET EXPERIMENT # 5
BUCKLING

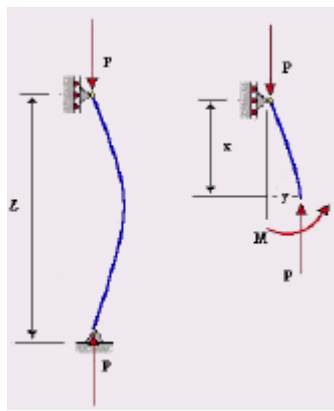
Student Name _____ Group #_____

Date Exp. Performed _____Instructor's Name_____

Specimen Number	1	2	3	4	5	6
Specimen Length (Inches)						
Effective Length KL (Inches)						
Least Cross Section Dimension dmin (Inches)						
Greatest Cross Section Dimension dmax (Inches)						
Cross. Section Area (sq. in)						
Slenderness Ratio (L/dmin)						
Weight (Oz.)						
Specific Gravity = Density/0.5778						
Max. Crushing Strength σ_{ys} (psi)						
Max. Compressive Expected Load (lb)						
Calculated Euler Pcr (lb)						
Pcr/A Euler Calculated (psi)						
Failure Load (Pcr) Experimental (lb)						
Experimental Failure Stress Pcr/A (psi)						
Adjusted Experimental Stress Pcr/A (psi)						

Appendix A
Derivation of Euler's Equation

Euler's Equation: In 1757, Leonard Euler (pronounced Oiler) developed a relationship for the critical column load which would produce buckling. A very brief derivation of Euler's equation goes as follows:



A loaded pinned-pinned column is shown in the diagram. A top section of the diagram is shown with the bending moment indicated. In terms of the load P, and the lateral deflection y, we can write an expression for the bending moment M:

$$M = - P\, y(x) \qquad 1$$

We can also state that for beams and columns, the bending moment is proportional to the curvature of the beam, which, for small deflection can be expressed as:

$$(M / EI) = (d^2y / dx^2) \qquad 2$$

Where E = Young's modulus and I = moment of Inertia. Substituting Equation 1 into Equation 2, we obtain the following differential equation:

$$(d^2y / dx^2) = -(P/EI)y$$

or

$$(d^2y / dx^2) + (P/EI)y = 0 \qquad 3$$

This is a second order differential equation which has the general solution of:

$$y = A \sin\sqrt{\frac{P}{EI}}\; x + B \cos\sqrt{\frac{P}{EI}}\; x \qquad 4$$

We next apply boundary conditions: at x = 0, y = 0 and at x = L, y = 0. That is, the deflection of the column must be zero at each end since it is pinned. Applying the first boundary condition, it is noted that B must be zero since cos(0) = 1. The second boundary condition implies that either A must be zero (which leaves us with no equation at all) or that:

$$\sin\sqrt{\frac{P}{EI}}\,L = 0 \qquad 5$$

Noting that sin(π) = 0, we can solve for P:

$$\sqrt{\frac{P}{EI}}\,L = \pi$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad 6$$

where P_{cr} stands for the critical load at which the column is predicted to buckle.

By replacing L with the effective length, KL, we can generalize the formula to predict the critical load for Fixed-Pinned, Fixed-Fixed, and Fixed-Free columns.

It should be noted that buckling is a complicated phenomena, and the buckling in any individual column may be influenced by misalignment in loading, variations in straightness of the member, presence of initial unknown stresses in the column, and defects in the material.