Interpreting a Distance-Time Graph

The runner at the bottom of the Gizmo is about to run a 40-yard dash. The graph on the right plots the runner's distance from the starting line on the vertical axis, and time on the horizontal. You can drag the points on the graph and then watch how changes to the graph affect the runner.

1. In the Gizmo, make sure Number of Points is set to 2, and Show graph and Show animation for Runner 1 are on. Drag the points on the graph to (0 seconds, 0 yards) and to (4 seconds, 40 yards). On the stopwatch, click the green start button to see Runner 1 run the 40-yard dash. (If you'd like to see a replay of the race, click the red reset button, and then click start.)
   1. After you watch the runner run, consider individual points on the graph. What does the point (4, 40) on the graph tell you about the runner? Be specific, and write your answer in a complete sentence.
   2. What does the point (0, 0) tell you about the runner?
   3. Where on the track is the runner after 2 seconds of running? Explain how you can tell from the graph.
   4. When is the runner on the 30-yard line? Explain how the graph tells you.
   5. Drag the vertical green probe across the graph to check your answers. As you do this, you can see a "snapshot" of the runner at a particular moment. (The time of the snapshot is shown on the stopwatch. This time matches where you placed the green probe on the graph.)

2. Click reset (red button) and then lower the point at (4 seconds, 40 yards) to (4 seconds, 20 yards).
   1. How does the steepness of this new graph compare to the previous one?
   2. Press start (green button) and watch the runner run. How far does Runner 1 now run in 4 seconds?
   3. Is his speed faster or slower than in the previous example?
   4. What do you think the steepness of the graph tells you about the runner?

3. Click reset. Next, drag the point currently located at (0, 0) on the graph and slowly drag it vertically. Watch Runner 1 as you do this.
   1. What does Runner 1 do as you drag the y-intercept of the graph upward?
   2. In general, what does the y-intercept tell you about the runner?

4. Click reset. Under Runner 2, turn on Show graph and Show animation. Make sure Runner 1's graph contains the points (0 seconds, 0 yards) and (4 seconds, 20 yards). Drag the points on Runner 2's graph to (0 seconds, 0 yards) and (4 seconds, 40 yards). Then run the animation.
   1. Which runner runs the furthest in 4 seconds?
   2. Which runner runs the fastest?
   3. Does each runner run at a constant speed? Explain your reasoning.
   4. How does the gap between the runners' graphs change over the 4 seconds? Why does the gap change when both runners are running at constant speeds?
   5. On paper, write down where the runners are located on the track at \( t = 0, t = 1, t = 2, t = 3, \) and \( t = 4 \) seconds.
   6. Then, use your answers to the previous question to write a function rule for each runner. Use time \( (t) \) as the input variable and distance from starting line \( (d) \) as the output variable.

5. Turn off Show graph and Show animation for Runner 2. For each of the following problems, adjust the points until the animation of Runner 1 matches the given description.
   1. The runner starts at the 10-yard line and runs forward at a constant speed, reaching the 40-yard line in 4 seconds.
2. The runner starts at the 40-yard line and runs backwards at a constant speed, reaching the 0-yard line in 4 seconds.

3. The runner starts at the 20-yard line and stands still for 4 seconds.

6. With Show graph and Show animation on for Runner 1, turn on only Show animation for Runner 2. (It is important that Show graph for Runner 2 is off.) Click New to generate a new random graph for Runner 2, which will remain hidden until you turn on Show graph for this runner.

1. Click the play button and watch the animation for Runner 2. Then, adjust the points in the graph of Runner 1 so that both runners run exactly the same race, side-by-side on the track. (Feel free to re-play the race as often as you like.) When you think you have it, turn on Runner 2’s graph using Show graph to check your answer.

2. To increase the challenge, increase the value of the Number of Points slider. Notice that the graph is now divided into segments. Now runners can slow down, speed up, or even change direction.