Stress, Strain, and Viscosity

Solids and Liquids

- **Solid Behavior:**
  - elastic
  - rebound
  - retain original shape
  - small deformations are temporary
  - (e.g. Steel, concrete, wood, rock, lithosphere)

- **Liquid Behavior:**
  - fluid
  - no rebound
  - shape changes
  - permanent deformation
  - (e.g. Water, oil, melted chocolate, lava)

Solids and Liquids

Linear Viscous Fluid:

- Rate of deformation is proportional to the applied stress
- A "**linear**" viscous fluid (*w.r.t. stress and strain rate*)
- Also known as a "Newtonian Fluid"

Solids > ? > Liquids

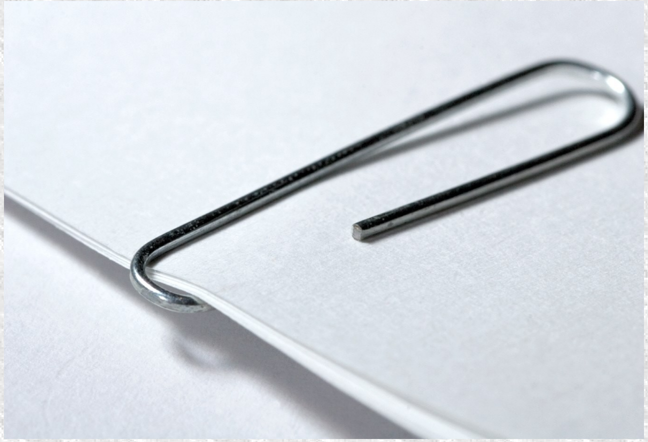
- Is there anything that behaves in a way ***between*** solid and liquid ?

- **Plastic Material**
  - solid
  - but deforms permanently
  - malleable
  - ductile

- Ductile or malleable materials are "non-linear"

Solids and Liquids

- Are all materials either a ***solid*** or a ***fluid, all the time*** ?
- Applied heat can cause solid materials to behave like a fluid
- Some material may be ***elastic*** when small forces are applied but ***deform permanently*** with larger applied forces



- **Elastic:** Deformed material returns to original shape

- **Ductile:** Stress exceeds the ***elastic limit*** and deforms material permanently

This rock responded to stress by folding and flowing by *ductile* deformation.

Occurs under ***high heat*** and ***high pressure***

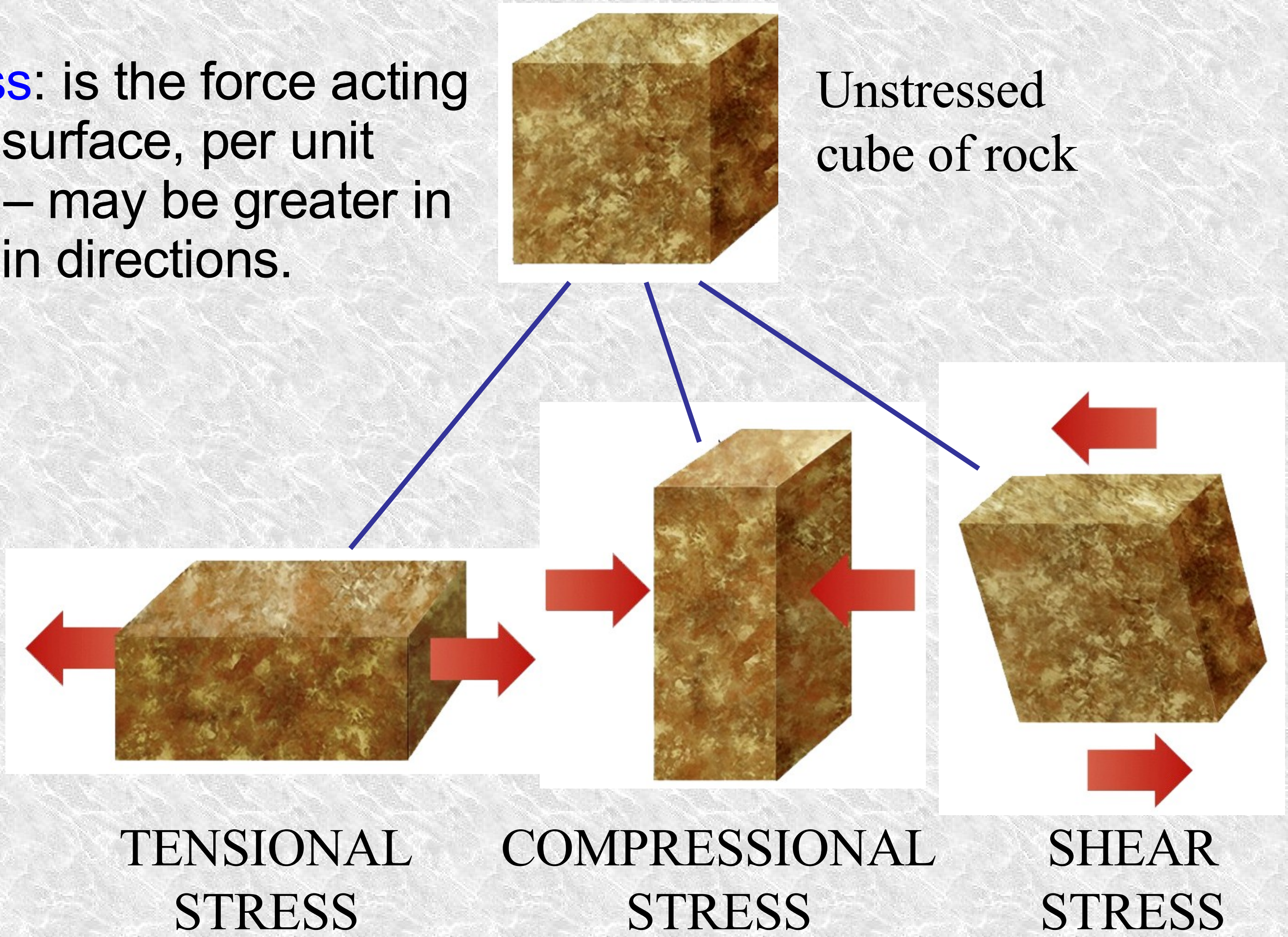
This rock fractured under stress by *brittle* deformation.

Occurs under low heat and at shallow depths in the Earth's crust.

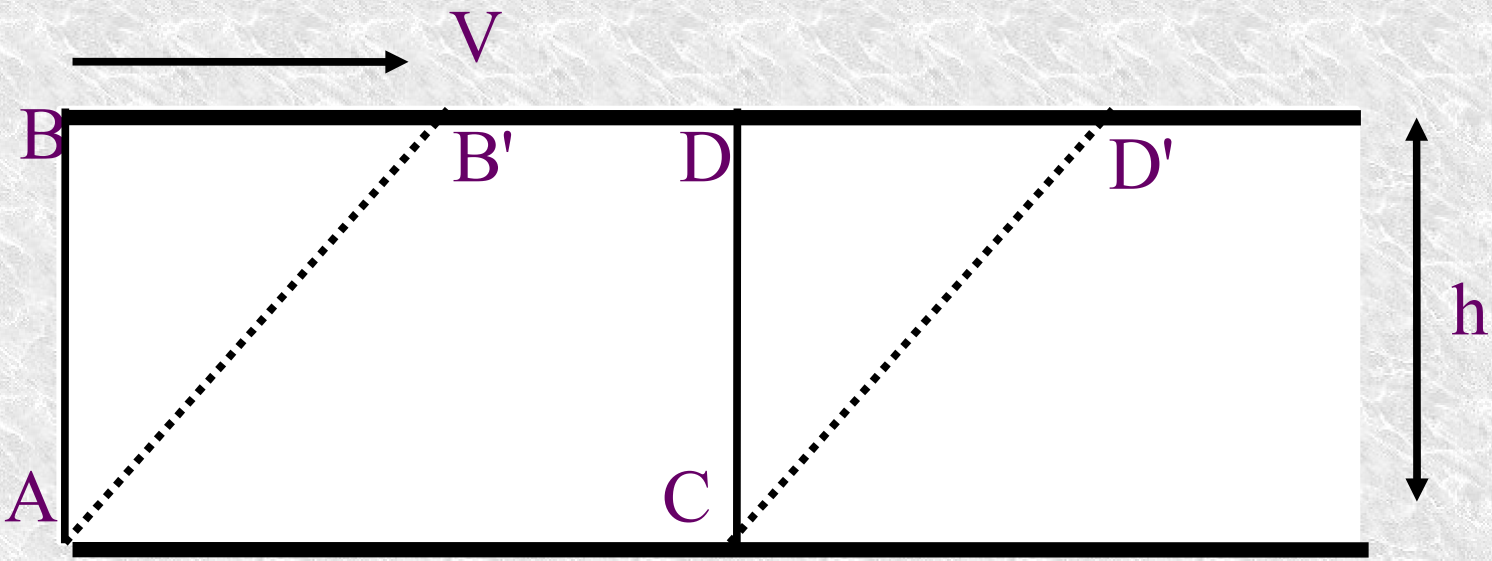
Three types of stress

Figure 9.2

Stress: is the force acting on a surface, per unit area – may be greater in certain directions.



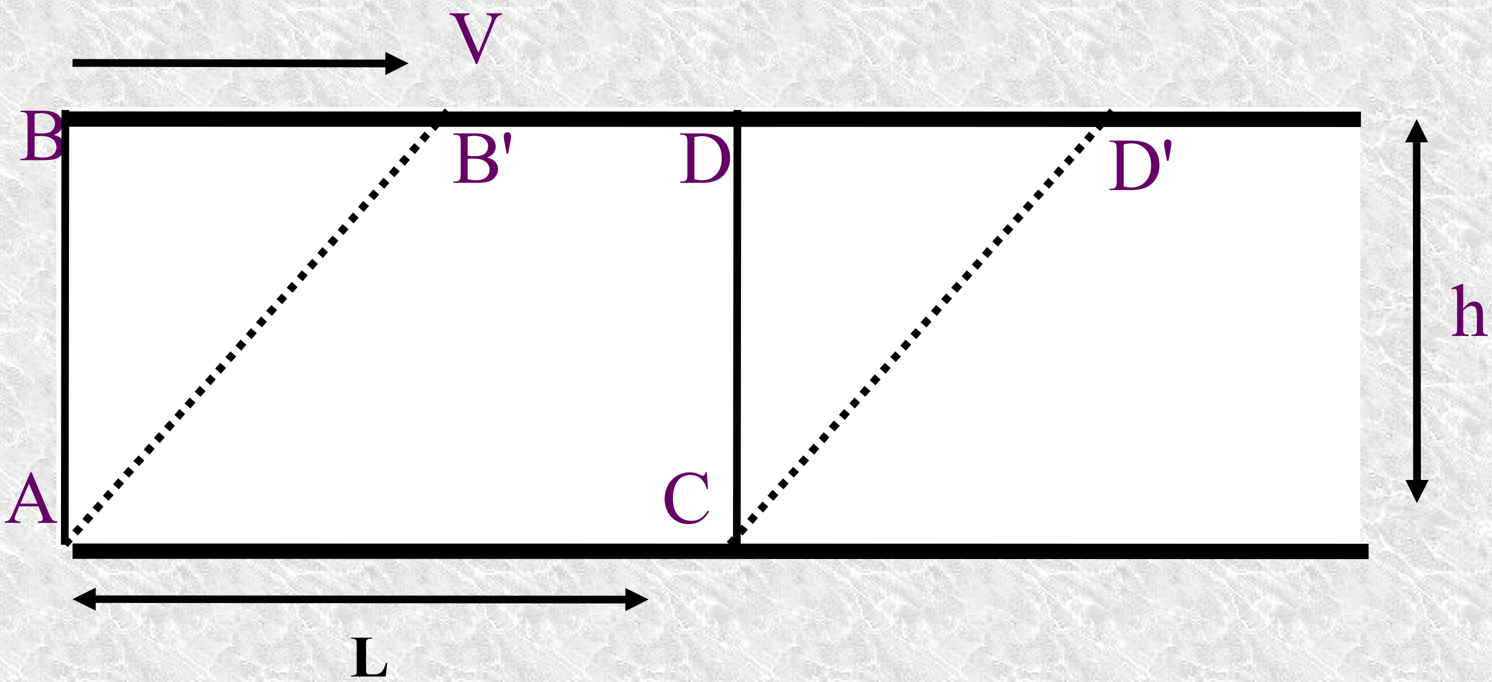
Strain



- Consider a layer of fluid between 2 plates
- The top plate moves with velocity, V
- The ***shape change*** can be written as $V \Delta t / H$
- The rate of shape change is d/dt(shape change)

rate of deformation = $\dot{\varepsilon}$ = V/H *(units of 1/time)*

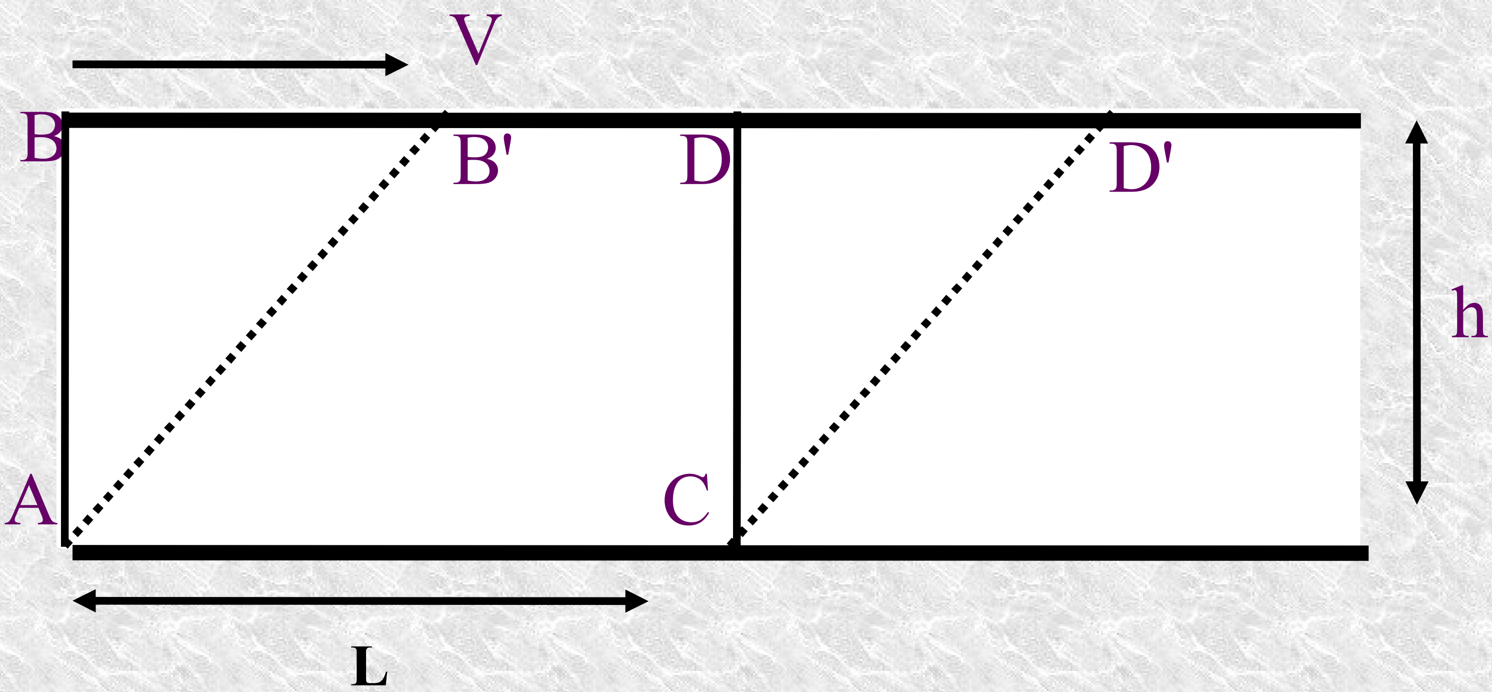
Stress



- Pressure is applied to move the fluid
- Stress is described as ***force per unit area*** *(units of pressure, Pa)*

$$\boldsymbol{\sigma = \textit{force/area} = F/LW}$$

Viscous Fluid



- A viscous fluid is defined by the relationship of stress to strain rate

$$\boxed{\sigma = 2\mu\dot{\varepsilon}}$$

- Viscosity ($\boldsymbol{\mu}$) is the constant of proportionality *(units of Pas)*
- This "constitutive" equation describes the mechanical properties of A material

Viscous Fluid

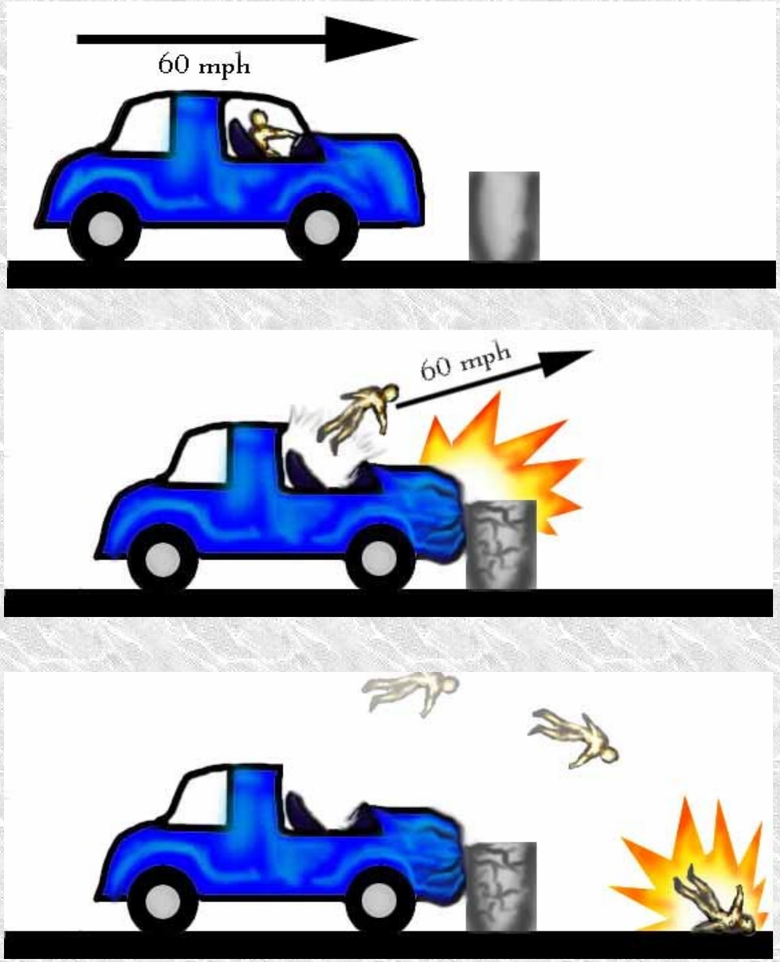
$$\boxed{\sigma = 2\mu\dot{\varepsilon}}$$

- If material has a high viscosity (μ),
  it will strain less for a given applied stress (σ)

$$\mu = \sigma / 2\varepsilon$$

Physical Laws of Motion

- **Newton's 1st Law:** Object in motion stays in motion; Action and reaction, Velocity motion imparted by the top plate induces a reaction of the fluid below



Physical Laws of Motion

- **Newton's 2nd Law:**
  Acceleration of fluid is proportional to the net force

What does this mean?

- If there is no acceleration, then forces balance, that is things move but don't accelerate. In this case, forces balance and there is "**no net force**"

- For all viscous fluids, the ***net force* = 0**

- Velocities in the Earth's mantle are small and ***accelerations*** are negligible

- ***Momentum*** is also negligible in slow viscous fluids

Conservation of Mass

- Mass is conserved in fluid flow
  (density changes are negligible)

- Fluid is "***incompressible***".

- Rate of fluid flow into box = flow out of box

- No net accumulation of material

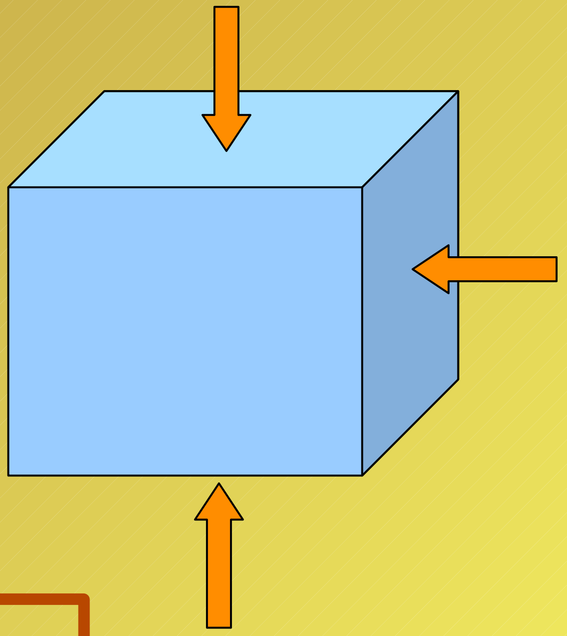
- See **Class notes** – Formal treatment of stress and strain

  - Stress >
  - Strain >
  - Viscosity >

Hydrostatic Stress

- ***Hydrostatic stress*** is defined as confining pressure
- Normal stresses acting on a particle are equal on all sides
- Known as "isotropic"

$$\boxed{\sigma_{11} = \sigma_{22} = \sigma_{33} = -P}$$



- With no tangential components

$$\boxed{\sigma_{12} = \sigma_{13} = \sigma_{23} = 0}$$

Deviatoric Stress

- Mantle flow is **not** driven by hydrostatic pressure
- But is driven by deviations from it, known as ***deviatoric stress***
- Let's consider an *average* normal stress

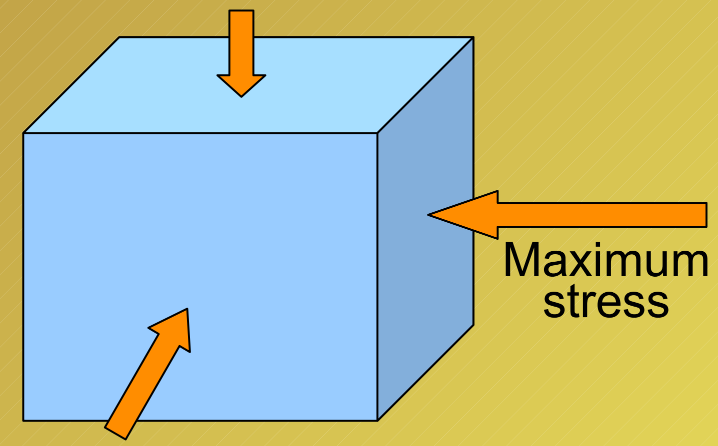
$$\sigma_{ave} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = \sigma_{ii} / 3$$

- Then ***deviatoric stress*** is given by

$$\sigma_{dev} = \sigma_{ij} - \sigma_{ave}$$

the applied stress - average normal stress = difference (deviatoric stress)

Deviatoric Stress



$$\sigma_{dev} = \sigma_{ij} - \sigma_{ave}$$

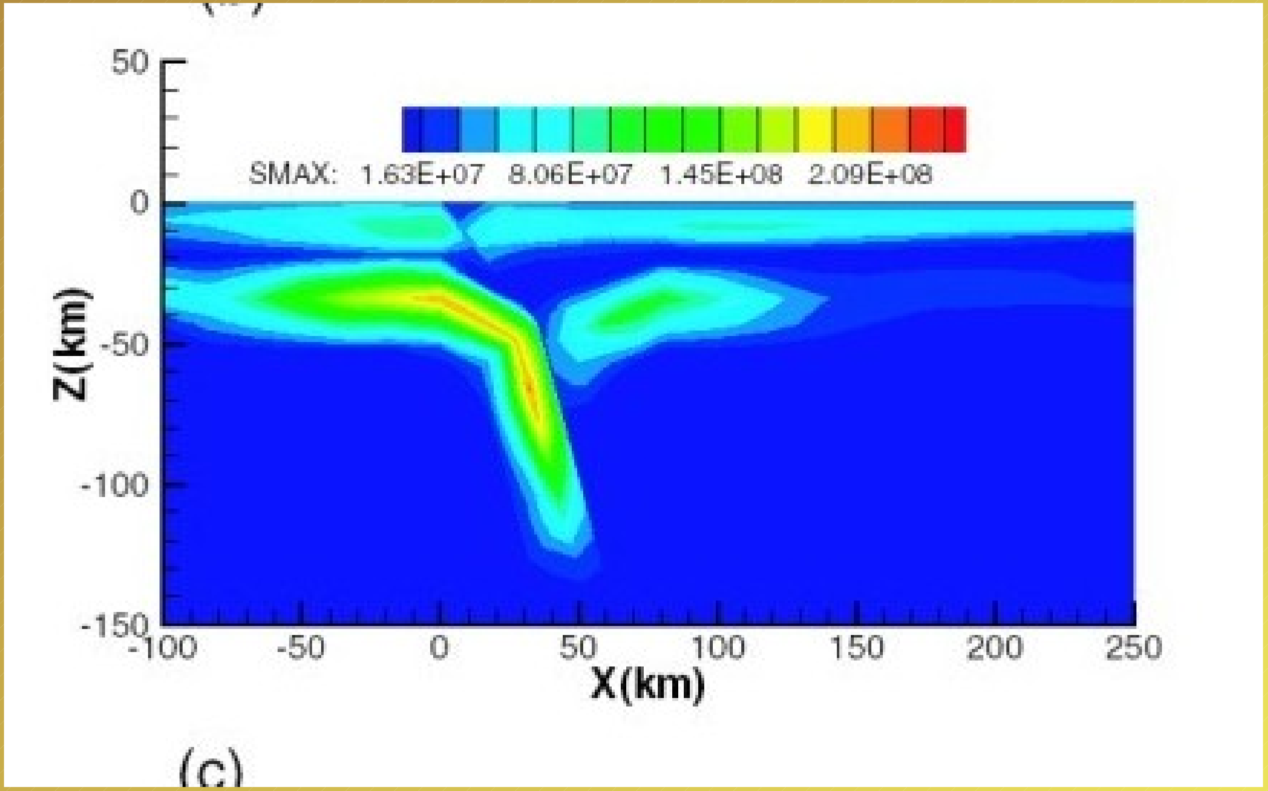
- If ***deviatoric stress*** is non-zero, than fluid flow proceeds

$$\begin{pmatrix} \sigma_{11} - \sigma_{ave} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma_{ave} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_{ave} \end{pmatrix}$$

- If the diagonals are all equal, then there is *no deviatoric stress*
- And there is no fluid flow

Lithostatic Stress

- A special case of hydrostatic stress
- Hydrostatic stress increases with depth in the Earth
- But at each depth, normal stresses are *balanced*



- As a slab sinks into the Earth's interior, it experiences progressively increasing hydrostatic stress with depth known as ***lithostatic stress***.

Measuring Lithostatic Stress

- Near the surface, stress measurements are strongly effected by faults, joints, etc.
- At deeper depths, pressure closes faults and fractures and stresses are transmitted across faults
- Activity: calculate deviatoric stress in a continental block

Normal faults in El Salvador

Jointing in a rock

Measuring Lithostatic Stress

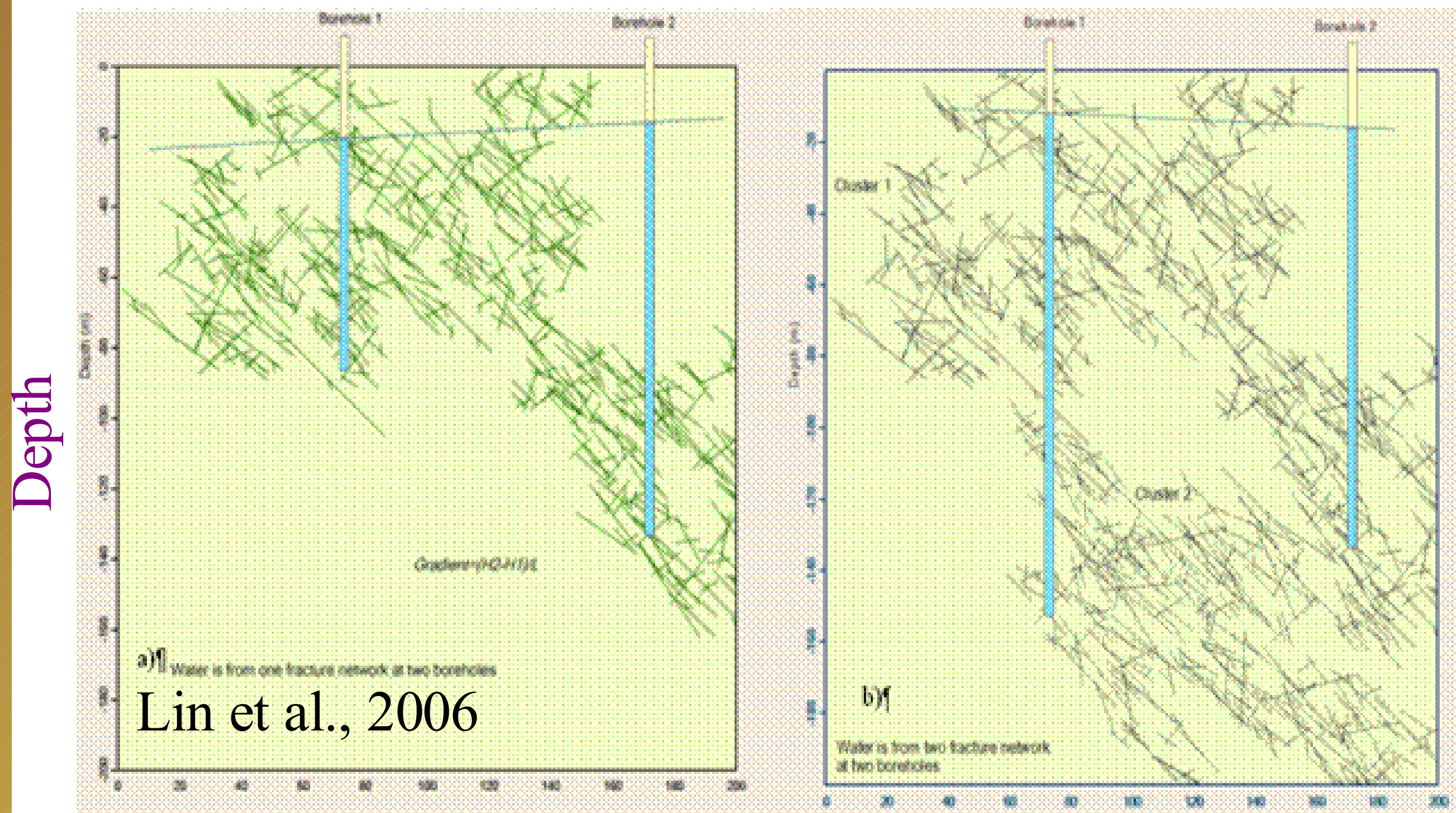
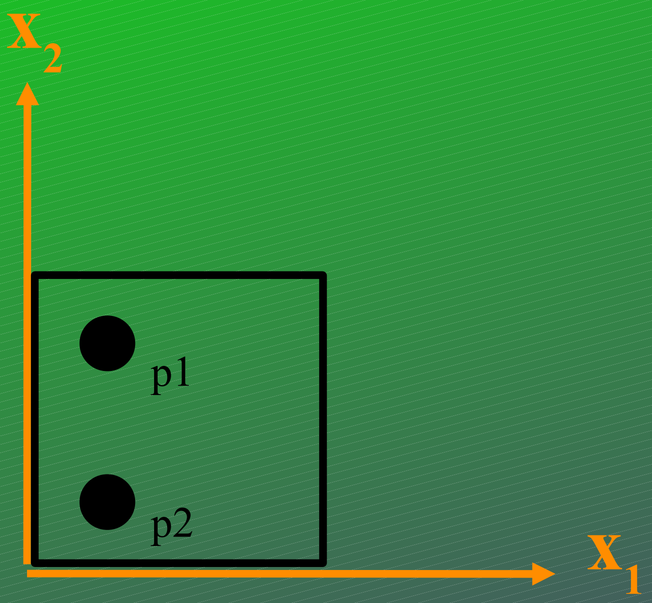


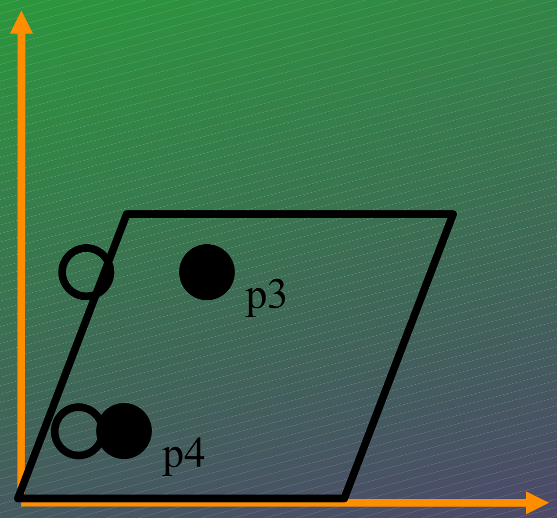
Fig.9 Interconnected fractures generated based on surface measurement of TMG sandstones at Eseljagberg, Villesdorp, Western Cape, at a size of 200×200 m, with the involvement of two boreholes, indicating the change of groundwater situation before and after the extension of borehole 1.

- Fracture density measured in the field in south Africa
- Boreholes show reduced hydraulic conductivity at deeper depths
- Indicating that fractures close with increasing ***lithostatic stress***

Strain

- ***Strain*** is a measure of deformation
- When deformation occurs, different parts of a body are displaced by different amounts

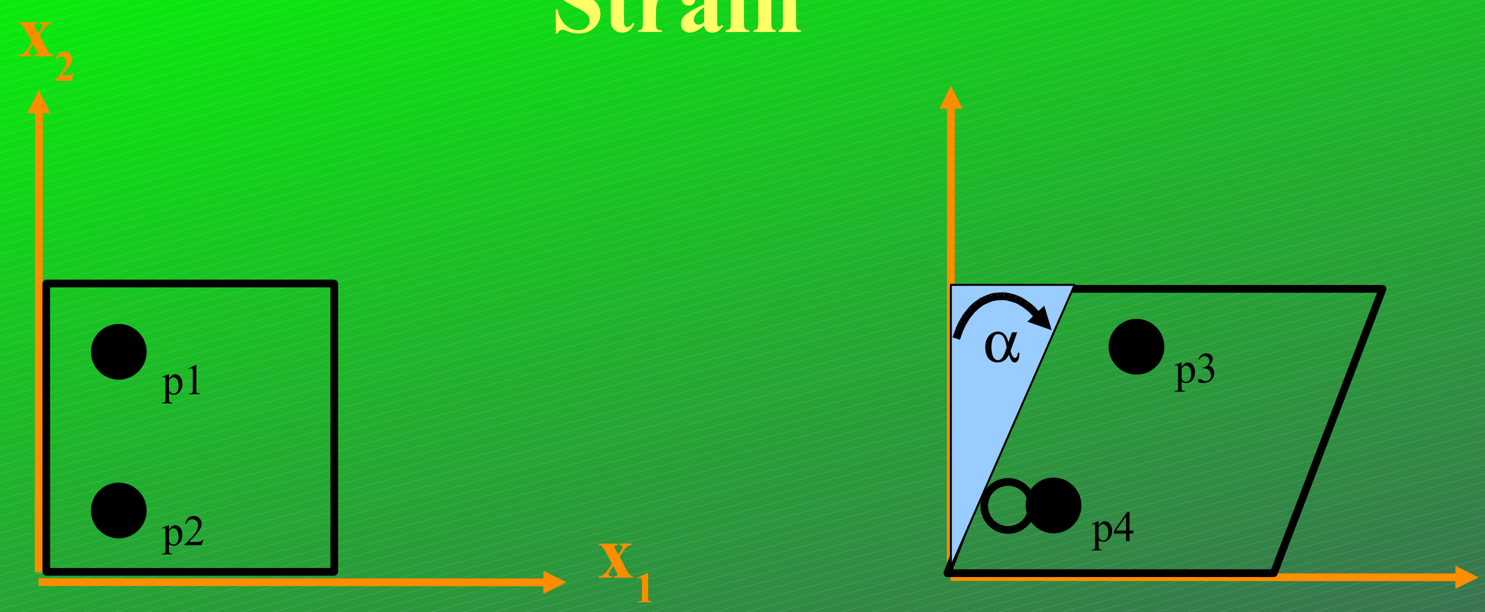




$$\mathbf{u}_1 = b\,\mathbf{x}_2$$

- Displacements ($\mathbf{u}$) in $\mathbf{x}_1$ direction increase with increasing $\mathbf{x}_2$.
- No displacement in $\mathbf{x}_2$ direction: $\mathbf{u}_2 = 0$

Strain

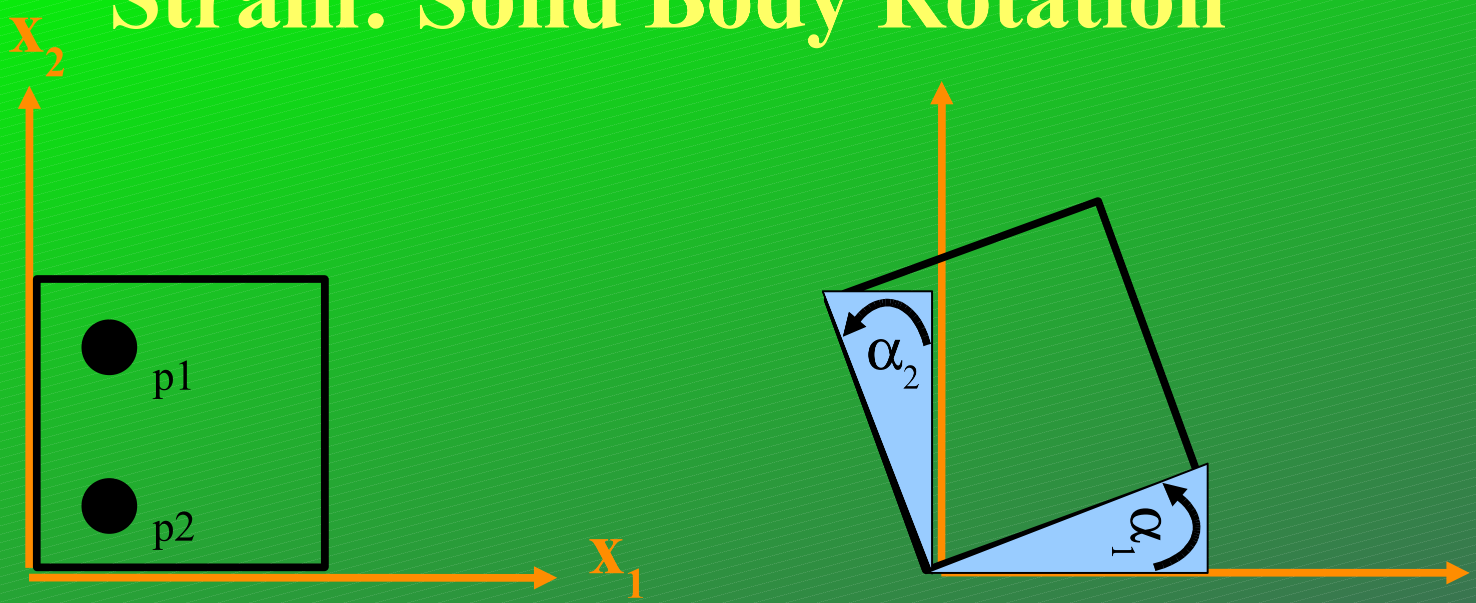


- Displacement of p1 and p2 are not equal
- This implies a ***gradient*** of displacement

$$\frac{d\,\mathbf{u}_1}{d\,\mathbf{x}_2} = \boldsymbol{b} = -\,\boldsymbol{tan}\ \boldsymbol{\alpha}$$

- Where α is the angle through which the body deforms
- Shearing deformation changes along the $\mathbf{x}_2$ direction.

Strain: Solid Body Rotation



- If $\alpha_1 = \alpha_2$, then we have rotation only and no deformation
- In this case, the ***gradient*** in deformation can be written as:

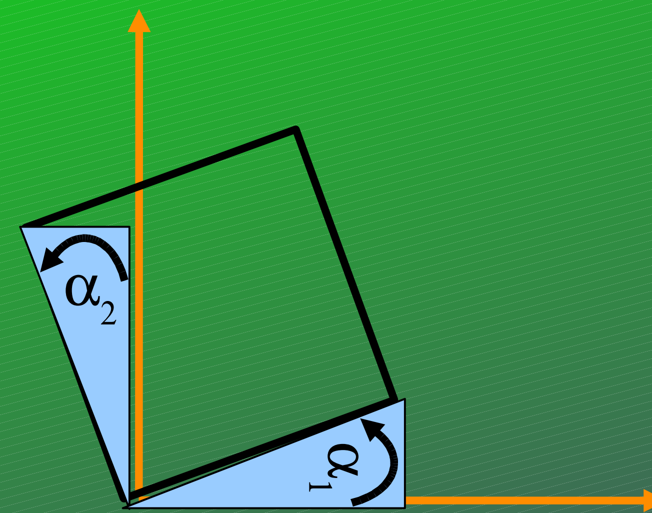
$$\tan \alpha_1 = \frac{d\,\mathbf{u}_2}{d\,\mathbf{x}_1}$$

$$\tan \alpha_2 = \frac{-\,d\,\mathbf{u}_1}{d\,\mathbf{x}_2}$$

- ***Solid body*** rotation suggests the gradient of deformations are equal

Strain:
How do we distinguish *deformation* and *rotation* ?

- Think about how α_1 and α_2 relate to each other



$$(\alpha_1 - \alpha_2) = 0$$

Rotation only

$$(\alpha_1 - \alpha_2) = nonzero$$

Deformation

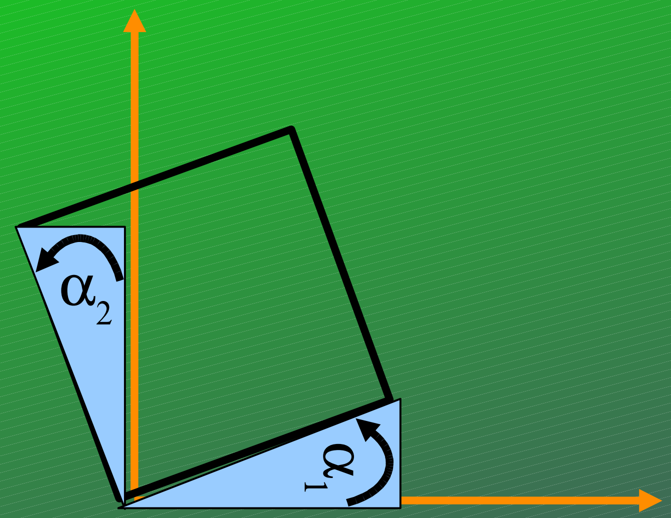
$$(\alpha_1 + \alpha_2) = 0$$

Pure shear

$$(\alpha_1 + \cancelto{0}{\alpha_2})$$

Simple shear

Strain:
How do we distinguish *deformation* and *rotation* ?



- We generally write this in terms of the tangent of the rotation angle

Rotation:

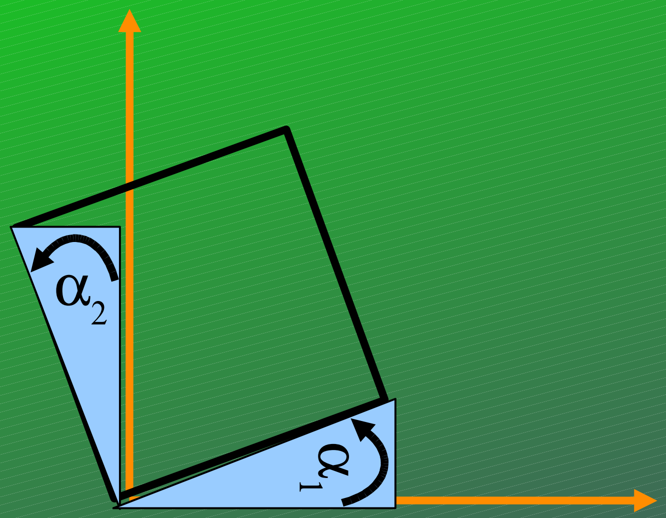
$$\omega_{12} = ½\,(\tan\alpha_1 + \tan\alpha_2) = ½\left(\frac{d\,\mathbf{u}_2}{d\,\mathbf{x}_1} - \frac{d\,\mathbf{u}_1}{d\,\mathbf{x}_2}\right)$$

Deformation:

$$\varepsilon_{12} = ½\,(\tan\alpha_1 - \tan\alpha_2) = ½\left(\frac{d\,\mathbf{u}_1}{d\,\mathbf{x}_2} + \frac{d\,\mathbf{u}_2}{d\,\mathbf{x}_1}\right)$$

The Strain Tensor

- The ***strain tensor*** can them be written



$$\varepsilon_{ij} = \tfrac{1}{2} \left(\frac{d\,\mathbf{u}_i}{d\,\mathbf{x}_j} + \frac{d\,\mathbf{u}_j}{d\,\mathbf{x}_i} \right)$$

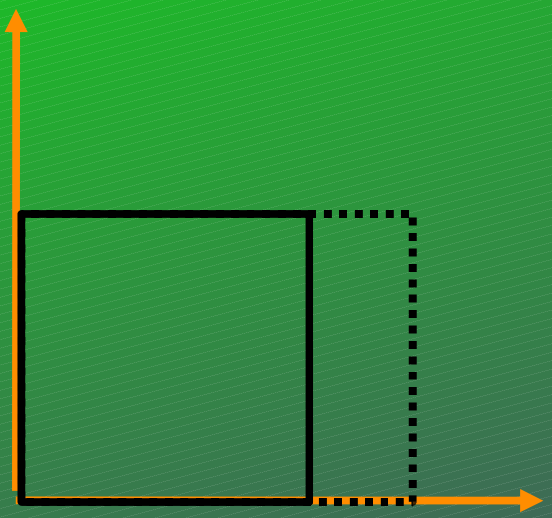
- And the rotation ***tensor*** can them be written

$$\omega_{ij} = \tfrac{1}{2} \left(\frac{d\,\mathbf{u}_j}{d\,\mathbf{x}_i} - \frac{d\,\mathbf{u}_i}{d\,\mathbf{x}_j} \right)$$

The Strain Tensor : Test

- The ***strain tensor*** can them be written

$$\boldsymbol{\varepsilon}_{ij} = \tfrac{1}{2}\left(\frac{d\,\mathbf{u}_i}{d\,\mathbf{x}_j} + \frac{d\,\mathbf{u}_j}{d\,\mathbf{x}_i} \right)$$

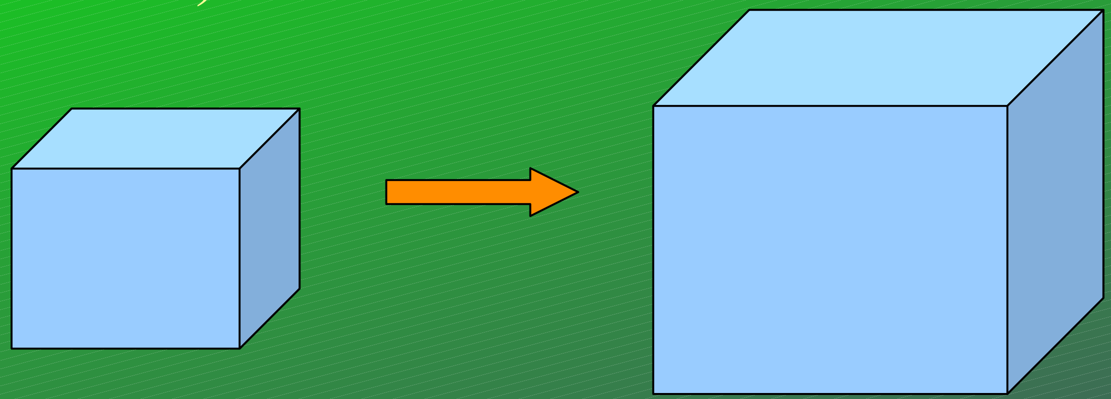


Let's try a test: In the case of a box that is "stretched"

- What is $\boldsymbol{i}$ and $\boldsymbol{j}$ in ε_{ij} ?
- Then find the solution

The Strain Tensor : Case of Volume Change

- If a cube expands (or shrinks)

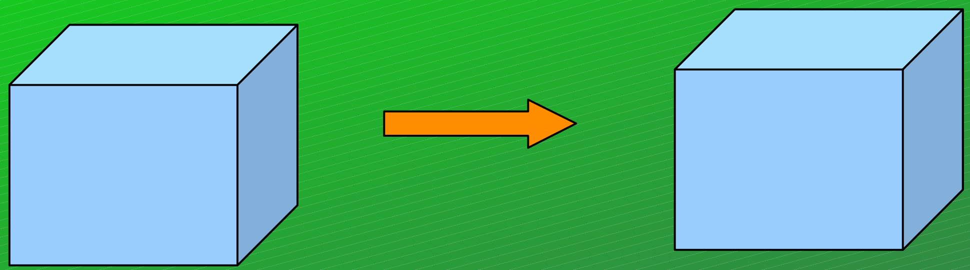


$$\frac{V - Vo}{Vo} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$E_{ij} = \frac{d\,\mathbf{u}_i}{d\,\mathbf{x}_i} = \nabla \cdot u$$

$\nabla \cdot u$ Indicates volume change, dilitation, divergence

The Strain Tensor : Case of Volume Change



$\nabla \cdot u = 0$

Means there is no volume change and fluid is ***incompressible***

Strain Rate

- Strain rate describes deformation ***change over time***
- How fast can a material deform ?
- How fast can fluid flow ?

Strain Rate

- Strain rate describes deformation ***change over time***
- How fast can a material deform ?
- How fast can fluid flow ?
- Rates are concerned with ***velocity***

$$\dot{\varepsilon}_{ij} = \tfrac{1}{2} \left(\frac{d V_i}{d \mathbf{x}_j} + \frac{d V_j}{d \mathbf{x}_i} \right)$$

- We can use velocity gradient to measure the rate of fluid flow or shear

Viscous Fluids

- ***Viscous fluids*** resist shearing deformation
- Have a linear relationship between stress and strain rate (known as Newtonian fluids)
- Fluid examples:
  - air, water viscosities are low
  - honey, thick oil viscosities are high

Viscous Fluids

- ***Viscous fluids*** resist shearing deformation
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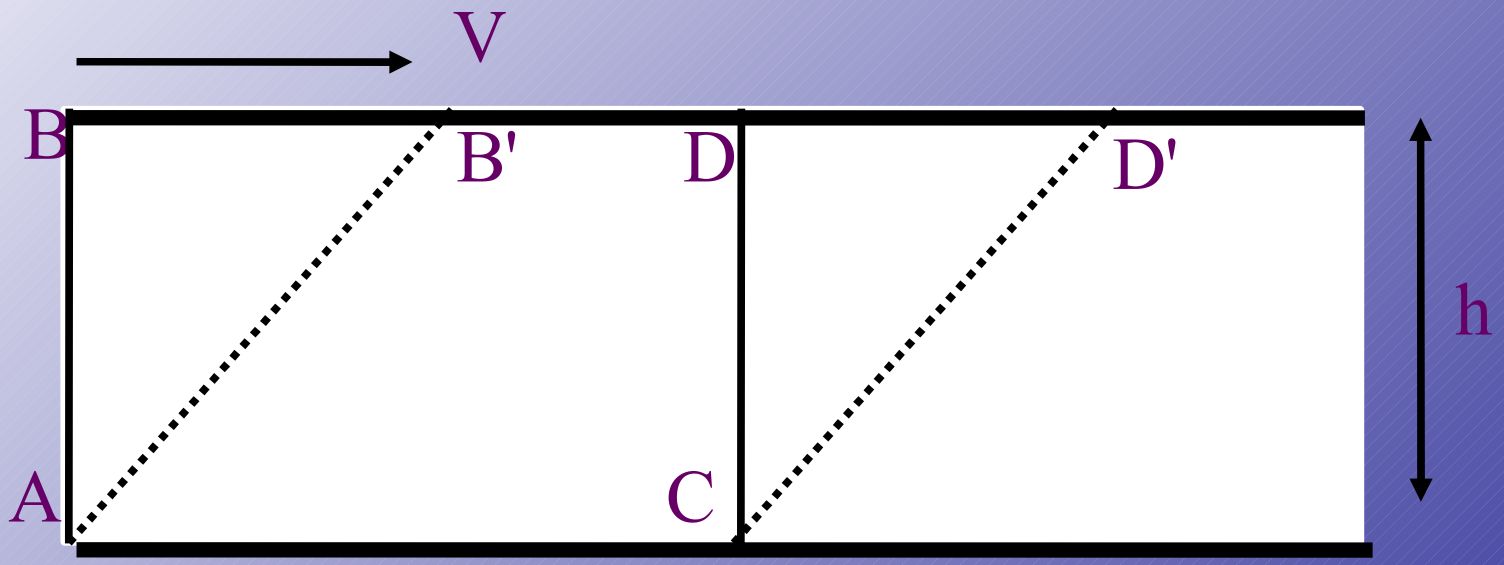
- See Class Notes:

Viscous Fluids



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