GEOL 452 - Mathematical Tools in Geology
Lab Assignment # 7 March 11, 2010 (Due March 23, 2010) Name:

# Power Functions in Crystal Melts and Drainage Basins

## A. Crystallization in Melts

Magma which resides in a magma chamber may cool slowly over time and begin to crystallize. The Bowen's reaction series shows that minerals crystallize in a certain order which describes them as *compatible* or *incompatible* (with the crystallizing solid). For example, olivine will be the first to crystallize, then pyroxenes, followed by amphibole, biotite, feldspar, and quartz. The magma melt, likewise, will change in composition as these minerals crystallize to the solid phase. The composition of lavas which subsequently participate in a volcanic eruption are determined by this crystallzation process. Each element has a unique "distribution coefficient" that determines its degree of compatibility, which can be measured using this equation

$$D = \frac{M_i^C}{M_i^{Lq}}, \tag{1}$$

where $M_i^C$ and $M_i^{Lq}$ are the weight % of the element $i$ in the crystal and the liquid, respectively. The value $D$ generally depends on compositional factors, but can also be influenced by temperature, pressure, melt composition, and crystals present. Even for one element, it's distribution coefficient can change as the melt and crystallization evolves. In general the concentration of an element in the liquid melt ($C$) is given by

$$C = C_o F^{D-1} \tag{2}$$

where $C_o$ is the initial concentration of that element in the liquid and $F$ is the fraction of the liquid remaining.

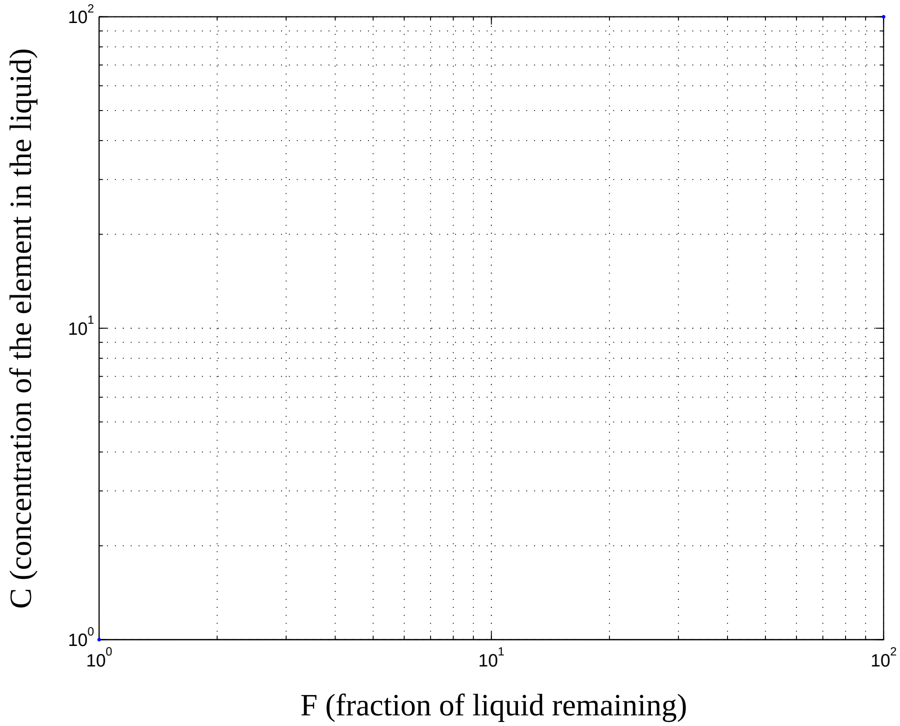
1. Let's consider the case of zircon ($Zr$) dissolved in a basaltic melt. Over most of the basalts crystallization history, $Zr$ behaves as a highly incompatible element where $D = 0.01$. Even after 90% crystallization, $Zr$ still remains dissolved in the liquid melt. Assuming the original concentration of $Zr$ in the melt was 400 ppm, determine the concentration of $Zr$ in the remaining melt after 10% crystallization.

2. From what we learned about power functions in class, can you say whether the *slope* of this function (when plotted on a normal linear graph) will be steadily increasing or decreasing as fractionation procedes ? Will this plot be concave up or concave down ? In order to answer this question, what will you have to do to equation (2) ? (Hint from the calculus lesson: What is a slope ? How can you study the *slopes* of an equation...)

3. Draw a very general normal graph (no number necessary) with $F$ labeled on the horizontal axis and $C$ labeled on the vertical axis. Draw the shape of the curve which you expect the data to produce (concave up or down).

4. If you were to plot values for your result in problem #1 on a log-log scale, what would be the shape of the curve ? What would be the value of the slope and y intercept ?

5. After crystallization proceeds in the melt to 98% crystallization, the $Zr$ begins to crystallize. At this point, its distribution coefficient changes to a value that is greater than 1. Let's assume it has reached a value where $D = 2$. Using the equation you determined from problem #1, what is the exponent in the power function ? If you were to plot the concentration versus melt fraction on a log log scale, what would be the slope of the curve ? On the log-log graph below, draw a line (of any length) which shows this slope (don't worry about the y intercept for now).




6. The magma has now reached 99% crystallization, more $Zr$ has crystallized out of the melt and the distribution coefficient for $Zr$ has increased to $D = 3$. Rewrite equation (2) with these new values. What is the exponent in the power function for this stage of melting and how will the slope of the data change if plotted on a log-log scale ? Draw this slope on the log-log graph above and label it's slope value (again don't worry about the length of this line or the y intercept).

7. Let's see what your data looks like. Make a table of data for concentration ($C$) and fractionation ($F$) of zircon assuming an initial concentration of 400 ppm and a constant distribution coefficient, $D = 2$. Consider the melt percent in 10% increments from 0 to 100%. Plot this on a normal linear graph using Excel or Octave.

8. Plot your data for $Zr$ on a log-log plot using Excel or Octave. Verify that the slope is what you predicted in problem #5. What is the value of the y intercept ?

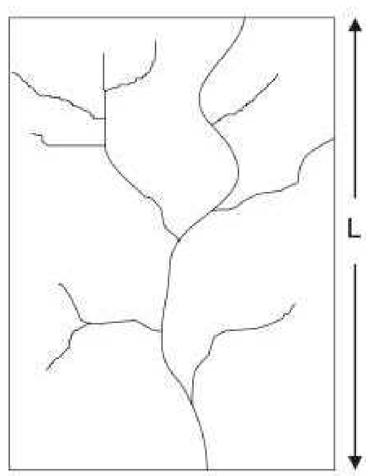
9. In your data table, add a third column for the concentration if the distribution coefficient increases to $D = 3$. Plot this data on the same graph (perhaps with a new symbol or color) so you can compare the change in slope. Varify that the slope is what you predicted in problem #6. Just so ya' know, near the end of the crystallization process, the distribution coefficient between zircon ($ZrSiO_4$) and melt can get quite large reaching values for $D$ of 100-1000! That's quite a slope!

**B. Drainage Basins**

For most drainage systems, the relationship between the length of the stream and the area of the drainage basin can be expressed as:

$$L = cA^k \tag{3}$$

where $L$ is the length of the stream, $A$ is the area of the drainage basin, $c$ is a constant that usually lies between 0.5-2.0 and $k$ is a constant that is generally below 0.6



10. For the example data given, draw a best-fit line (in the log-log graph below on the right) and determine the constant values $c$ and $k$.