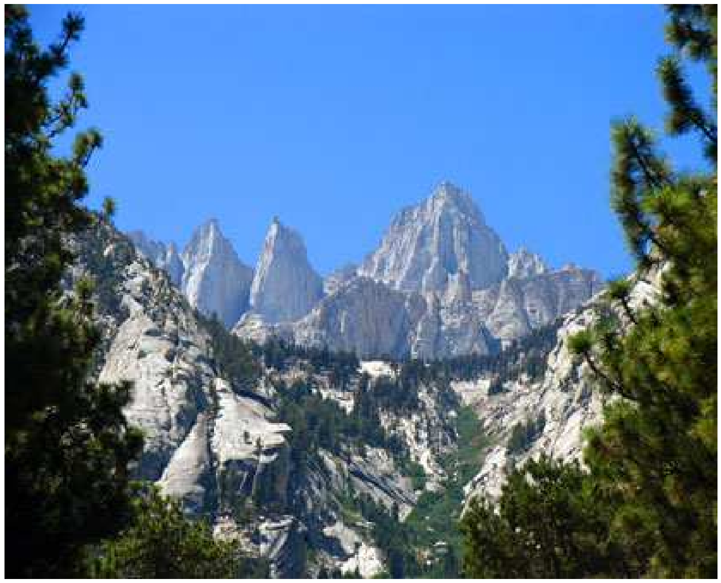
Name:

Significant Figures & Logarithms

A. Practice with Significant Figures



Mt. Whitney - the highest mountain in the contermenous U.S.

1. Use the topo map of the Mt. Whitney quadrangle to determine the surface gradient. Use the city of Lone Pine and the top of Mt. Whitney for your end points and the mapped contour intervals for elevation. What two measurements do you need to determine a gradient (show the equation) ? Estimate your errors for each measurement and give your final answer with the correct number of significant figures. Indicate error estimates for the final result by using "propagation of error" techniques if necessary. (Give your answer for gradient in units of ft/ft)

2. Convert 33.0 meters to millimeters with the correct accuracy.

3. Show how many miles are equal to 102.6 kilometers with the correct significant figures.

4. You have discovered a natural aquifer near Independence Peak. To measure it's flow rate, you install 2 wells: One at Independence Peak which shows a potentiometric surface 100.0 ft below the peak elevation, and the Second at Slim Lake which shows a pot. surface 100.0 ft below the lake elevation. Determine the hydraulic conductivity, K, if the average porosity, n, is assumed to be 0.17 and you measure a tracer velocity, v, of 0.10 ft/day. You can use the Darcy flow equation below where Δh is the difference in elevation of the potentiometric surfaces in the two wells and e Δs is the distance between them. Please give your final answer with the correct significant figures and indicate how well you know this result (Give your final answer for K in units of ft/day).

$$v = -\frac{K\Delta h}{n\Delta s}. \tag{1}$$

B. Logarithms

5a. Given the following logarithmic expression, rewrite these 3 variables in its exponential form (relating the base (10) to N).

$$log_{10}N = x \tag{2}$$

5b. Do the same for this logarithmic expression.

$$log_b N = x \tag{3}$$

6. Use your Noggin! WITHOUT using your calculator, try to find the answer to the following four problems.

$$log_{10} 100 = \tag{4}$$

$$log_{10} 1000 = \tag{5}$$

$$log_3 27 = \tag{6}$$

$$log_2 32 = \tag{7}$$

7. If you are given the exponential function below, rewrite this in terms of a logarithm which solves for x.

$$Y = a^x \tag{8}$$

8. Porosity of Buried Sediments
Water contained in sediments is usually squeezed out as the sediments are buried. We might assume that a certain proportion of water is released for each kilometer of burial. In general, it is observed that for each additional kilometer of burial, half of the remaining water is released. In this scenario, porosity always decreases with increasing burial but never actually reaches zero. What kind of mathematical function does this type of behavior represent ?

Try to write a mathematical equation for porosity according to this function using ϕ for porosity, ϕ_o for the starting porosity at the surface and z for depth. Think about your signs and about how porosity changes as depth increases.

9. Using your equation, make a table of values for porosity as a function of depth assuming surface porosity, ϕ_o, is 0.7 and depth changes from 0 to 10 km (in 1 km increments). Plot these on a graph and describe the shape of the curve and how porosity changes with depth.

10. Radioactive decay of elements occurs by a similar process. The term *radiation* refers to the emission of particles such as electrons, neutrons, or alpha particles. In 1900, Rutherford discovered that the rate of emission of radioactive particles is not constant over time but decreases exponentially. This suggests a statistical process. Because the nucleus is shielded from others by the atomic electrons, pressure and temperature changes have little or no effect on the rate of radioactive decay.
Write an equation that describes the decay of a parent isotope (N_p) over time considering the original concentration (N_o) and a decay constant, λ.

11. Draw a sketch of what this graph would like labeling time on the x axis and parent isotope concentration on the y axis (you don't need to plot points, just a sketch which shows a the shape of the curve going in the right direction). Describe the decay of N_p over time in words.

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12. Let's look at a specific example for Potassium, K, which has three natural isotopes: ^{39}K, ^{40}K, and ^{41}K. Only one of these, ^{40}K, is radioactive and has a half-life of 1.3 billion years. ^{40}K has two different decay schemes. 12 percent of ^{40}K decays to ^{40}Ar by electron capture and 88 percent decays to ^{40}Ca by beta decay. This decay ratio is constant for each newly formed isotope and is independent of pressure and temperature. This is can be described below:

$$^{40}K + electron \rightarrow ^{40}Ar \tag{9}$$

and

$$^{40}K \rightarrow ^{40}Ca + \beta^-. \tag{10}$$

We can use these isotopes to estimate the age of eruption of an igneous rock. Choosing a potassium bearing mineraal we only have to measure ^{40}K and the concentration of one of the daughter atoms (^{40}Ar or ^{40}Ca,) because we know their proportions are constant. Let's assume that you've done the chemical analysis on a sample of potassium feldspar and determined that ^{40}K is made up of 20,000 parent atoms. How many daughter atoms, N_d, should be found for ^{40}Ar and ^{40}Ca ? (Hint: Don't forget that only half of the parent isotope will decay.)

13. For the equation given below for decay of ^{40}K, determine the concentration of the original parent isotope ,N_o, when the mineral formed if $N_o = N_p + N_d$, where N_d is the total daughter atoms.

$$\frac{N_p}{N_o} = (1 - \lambda)^{\gamma} \tag{11}$$

14. Simplify the equation above by using 0.5 (half-life) for the decay constant, λ. Can you guess what γ stands for ?

15. Simplify your equation further by taking the logarithm of both sides. Remember your logging rules.

16. Given the information you have thus far, determine the time of eruption of this igneous rock. Solve for gamma (the number of time units or number of half-lives). How many years ago was this ?

17. Aqueous geochemists use the concept of *Law of Mass Action* (LMA) to determine equilibrium constants. Logarithms make this calculation infinitely simpler as it involves multiplication and division of variables which vary over many orders of magnitude. If you can express these numbers logarithmically, the law of logarithms (which follow from exponentiation) greatly simplify the calculations.

As an example let's consider the reaction for the dissolution and precipitation of calcite:

$$CaCO_3 \rightleftharpoons Ca^{2+} + CO_3^{2-}. \tag{12}$$

The solubility product determined for this reaction is $K_{calcite} = 10^{-8.48}$. In natural waters, this reaction becomes a bit more complicated.

$$CaCO_3 + CO_2 + H_2O \rightleftharpoons Ca^{2+} + 2HCO_3^-. \tag{13}$$

Several solubility products describe this complex reaction as listed below.

$$CaCO_3 \rightleftharpoons Ca^{2+} + CO_3^{2-}, \quad K_{calcite} = 10^{-8.48} \tag{14}$$

$$CO_2 + H_2O \rightleftharpoons H_2CO_3, \quad K_{CO_2} = 10^{-1.47} \tag{15}$$

$$H_2CO_3 \rightleftharpoons H^+HCO_3^-, \quad K_{H_2CO_3} = 10^{-6.35} \tag{16}$$

$$CO_3^{2-} + H^+ \rightleftharpoons HCO_3^-, \quad 1/K_{HCO_3} = 10 + 10.33 \tag{17}$$

Determine the equilibrium constant of the sum of these reactions. This is obtained by multiplication of the equilibrium constant of each of these constituent reactions.

The point of this exercise is to realize that although the reactions can be quite complicated, the advent of logarithmic notation makes this calculation incredibly simple, don't you think ?