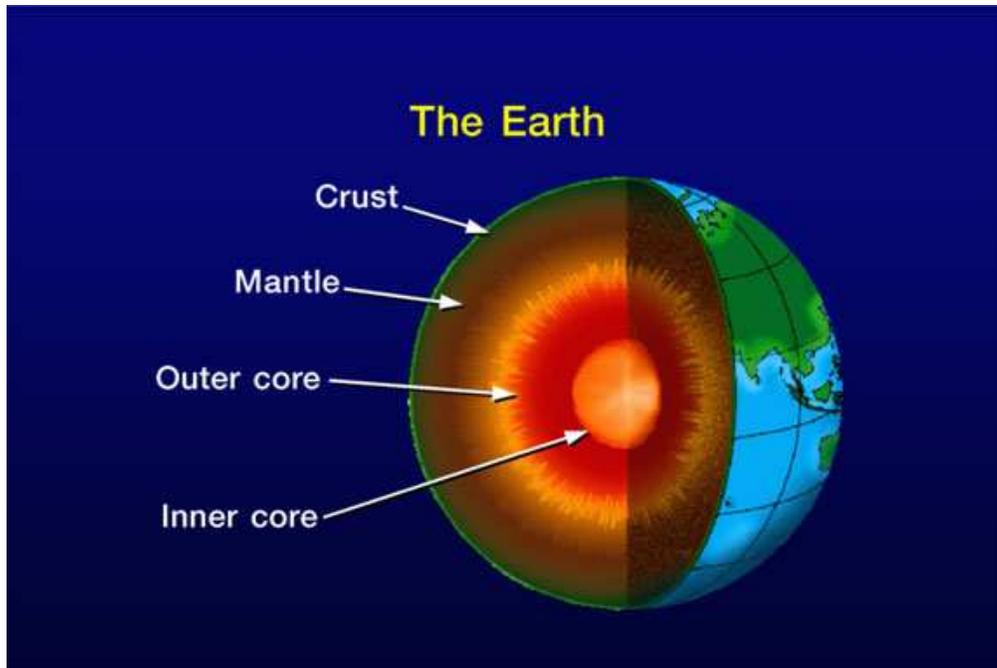
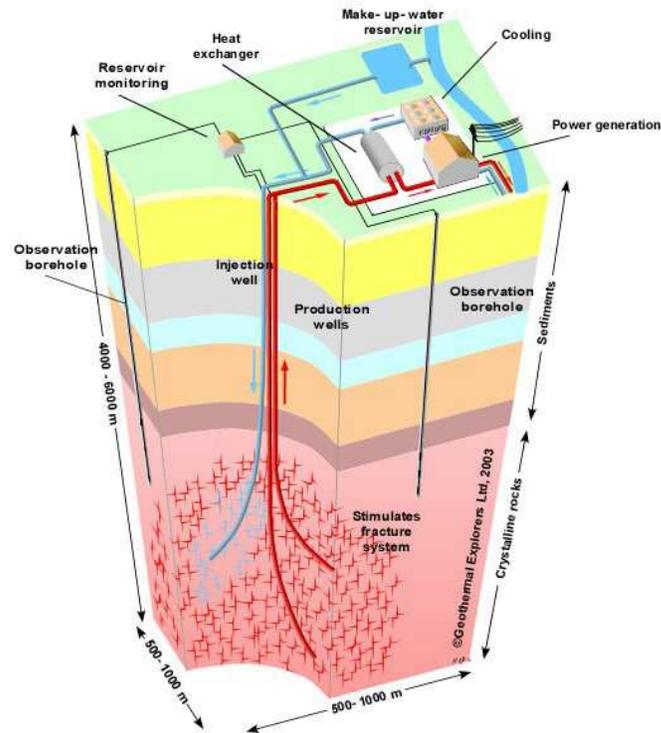

Polynomials, Taylor Series, and the Temperature of the Earth
How Hot is it ?



A. Linear Temperature Gradients

Near the surface of the Earth, borehole drilling is used to measure the temperature as a function of depth in the Earth's crust, much like the 3D picture you see below. These types of measurements tell us that there is a constant gradient in temperature with depth.

1. Write the form of a linear equation for temperature (T) as a function of depth (z) that depends on the temperature gradient (G). Assume the temperature at the surface is known (T_o). Also draw a picture of how temperature might change with depth in a borehole, and label these variables.



2. Measurements in boreholes tell us that the average temperature gradient increases by 20°C for every kilometer in depth (written: $20^{\circ}\text{C km}^{-1}$). This holds pretty well for the upper 100 km on continental crust. Determine the temperature below the surface at 45 km depth if the surface temperature, (T_o) is 10°C .

3. Extrapolate this observation of a linear temperature gradient to determine the temperature at the base of the mantle (core-mantle boundary) at 2700 km depth and also at the center of the Earth at 6378 km depth. Draw a graph (schematically - by hand) of T versus z and label each of these layers in the Earth's interior.

4. The estimated temperature at the Earth's core is 4300°C . How does this compare to your estimate? If it is different speculate as to the cause of the discrepancy. Could it be a problem with propagation of errors?

B. Temperature Gradients using Polynomials

The problem as you may have guessed, is that below 100 km, the temperature gradient is not exactly linear. It increases much slower at deeper depths in the Earth, e.g. temperature changes by 2000°C in the first 100 km, but only 350°C in the following 300 km. Through the inner core, temperature is virtually constant.

5. Using a polynomial can allow for changes in gradient (or slope) along the depth profile of the Earth's interior. Here write a general equation for temperature (T) represented by a second order polynomial function which has a variable, z , for depth and coefficients c_0 , c_1 , and c_2 .

6. In order to get closer to the expected value of temperature at the Earth's center, review your class notes about concave functions and determine whether the coefficient for c_2 should be positive or negative and whether it should be a large number (greater or less than 1.0)?

7. Make a table of values for temperature in the Earth at depths increasing every 500 km using a second order polynomial function that has $c_0 = 1110$, $c_1 = 1.05$, and $c_2 = -8.255 \times 10^{-5}$. Substitute these values into your equation. Plot these values on a graph of depth versus temperature.

8. Using this equation, what is the temperature at the inner-outer core boundary at 5100 km depth ? What temperature does this give for the surface of the Earth ? Are both of these values reasonable ?

9. While a second order polynomial helps characterize the changing gradients in the Earth's deep interior, it does not accurately represent the change to a constant slope at the surface. Thus, we can use the linear equation for the upper 100 km of the Earth, and a polynomial for temperatures at deeper depths. Plot your values for temperature versus depth using your linear equation from problem #2 - for the upper 100 km at 10 km increments. Plot these values on top of the polynomial plot you have for problem #7 (perhaps use different colors or symbols to show the 2 different data sets).

C. Find the Depth of a Particular Temperature

10. Find the depth where the Earth's interior temperature reaches 1500°C . To do this, rewrite your equation from problem #7 and substitute this value in the appropriate place.

11. To accomplish your task, you will have to find the *roots* of this polynomial equation. To do this rearrange your terms so that everything is on one side, and a "zero" is on the other.

You can use general rules for finding the roots of a quadratic equation. For a quick review, rename your coefficients c_0 , c_1 , and c_2 as a, b , and c , which may look more familiar to you. These roots can be found using the general equation,

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (1)$$

Using this equation, find the value of z where the temperature is 1500°C . You may have 2 results using the \pm sign - decide which makes more sense in the Earth. Where will these distance values be measured from ?