A game theory for recidivism and rehabilitation of criminal offenders

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Let us consider a population of individuals that belong to a specific class $P, N_0, N_1, \cdots, N_R$. Here P represent ideal citizens that will not commit crimes, whereas N_k are individuals that can potentially commit crimes, and that have a record of past k crimes. In this context, the difference between P and N_0 is that an individual belonging to P will not commit crimes, while an individual belonging to N_0 has not committed any crimes – yet – but may, if the occasion presents itself. We wish to introduce a game theory for these individuals and follow their dynamics from the neutral state N_0 to the subsequent states N_k or P and by assuming their dynamics depends not only on the punishment given them, but also on possible forms of rehabilitation - in the sense of vocational or educational programs, counseling, anger management classes or a mentor.

We assume that N_R is the set of irreducible criminals that have committed a threshold level of crimes and that considered incorrigible. Once an individual has reached either P or N_R they do not partcipate in the game anymore, if not as bystanders and giving a certain positive (P) or negative (N_R) imprint to society.

The game is played so that individuals may shift between categories N_k depending on their histories and at any time, may decide to turn into law abiding citizens P, regardless of their past history if the proper incentives or punishments are set in place and as long as $k < R$. In some sense the possibility for a criminal with a history of k crimes to join the population P is to be intended as a form of permanent rehabilitation that can lead criminals to turn their lives around. However, once the state $k = R$ is reached, we don't allow for the transition to P to occur any longer. These are criminals that are considered incorrible. We also assume that $\sum_k N_k + P = N_{tot}$, that is, the number of players is constant. We will be concerned with studying the time evolution of P and N_R and, most importantly the ratio P/N_R at the end of the game, which represents the level of uprighteousness attained in society.

To study the problem of societal evolution given these categories and the inherent dilemma of whether, from a society standpoint, to punish or not, to give incentives or not we will now contruct our evolutionary game. At each round an individual i is selected from the pool of the N_k categories at random, with $0 \leq k < R$, and with unitary payoff. The player is confronted with a crime of entity $0 < \delta < 1$, where δ measures the severity of the crime.

We assume the individual in the group N_k has a history of punished k_p and unpunished k_u crimes, so that $k = k_p + k_u$ where each punished crime was associated with a punishment $\gamma > \delta$. As we shall see, it will be useful to consider the effective punishment $\theta = \gamma - \delta$, we therefore use θ and not γ, δ individually. Everytime the criminal was apprehended, he was given not only a punishment but also some educational and employment opportunities of entity h and with duration of τ units of time to rehabilitate the offender. We assign to each criminal a history record consisting of his or her crimes and the time at which the conviction occurred. We will use the notation that each convicted crime is denoted by 1 and each not convicted crime is denoted by 0. For example, if a criminal is in the pool N_3 this implies there have been 3 crimes, committed at times t_ℓ where $1 \leq \ell \leq 3$ of which, say, the first two have been left unpunished while for the last there has been a punishment and a possible rehabilitation, so that the history string associated with this criminal is $(t_1, 0, t_2, 0, t_3, 1)$. In this example $k_p = 1$ and $k_u = 2$.

The individual i with his or her own history is now faced with the choice of whether to commit a new crime or not. We assume this occurs with a probability that depends on various factors: resources given in the past after convictions by society to discourage criminal actions, measured by a fixed amplitude h and time duration τ so that the probability of offending is given by

$$p_{\rm crime} = \frac{p_i + a_i s_i}{2} = \frac{1}{2}\left[\frac{p_0 + k_u}{k_u + \theta k_p + p_0} + \frac{\sum_{k \neq 0} N_k}{N_{\rm tot}}(1 - h e^{-(t-t_k)/\tau})\right]. \tag{1}$$

We choose this form – given by the sum of two terms - to represent the idea that an individual may commit crimes depending on their own personal history and on the surrounding community, in equal manner and that these two tendencies are totally independent of each other. Also note that at $t = 0$ when $N_k = k_u, k_p = 0$, the overall probability to commit a crime is $1/2$, so that individuals are not inherently "good" or "bad". Let us now examine the terms in the above, each of which carries a certain societal meaning. The first term p_i is the probability of offending for criminal i that strictly depends on his or her past history via the term

$$p_i = \frac{p_0 + k_u}{k_u + \theta k_p + p_0} \tag{2}$$

so that previous unpunished crimes k_u embolden the criminal and where the probability of committing crimes for the first time, when $k_u = k_p = 0$ is one, and similarly if the criminal was never caught and

Figure 1: A schematic of the p_i curve as a function of k_p, where we have set $p_0 = 2$ and where $k_u = 4$. The above, darker curve is for low punishment $\theta = 0.1$, the lower, lighter curve is for high punishment $\theta = 0.8$. Note the difference between the two as the number of arrests increases.

only $k_p = 0$. As soon as θk_p is greater than one p_i is less than one. Note the term p_0 which represents the "stepness" of the p_i curve, and note the term θk_p so that each occurrence of past crime is weighted by how strong the effective punishment was. If $\theta = 0$, $p_i = 1$, which means that since there was no punishment, the criminal inherently will always want to offend.

In Fig. 1 we plot p_i as a function of k_p, where we have set $p_0 = 2$ and where $k_u = 4$. The above, darker curve is for low punishment $\theta = 0.1$, the lower, lighter curve is for high punishment $\theta = 0.8$. Note the difference between the two as the number of arrests increases.

The next term is a product of two terms, s_i, a_i representing external influences. Of these, the first is the societal pressure term, represented by

$$s_i = \frac{\sum_{k \neq 0} N_k}{N_{\text{tot}}} \tag{3}$$

so that if the community is made of upright P or "neutral" citizens P_0, there is no pressure and the probability of committing a crime is very small. In the limit of $P \to N_{tot}$ the probability goes to zero. We may include individual i in the enumeration of the N_k. The quantity s_i is meant as a "copycat" term: seeing or knowing about crimes will increase the likelihood that crimes are committed.

This tendency is attenuated by the factor a_i due to societal intervention evalutated at the last time the criminal committed a crime t_k and given by

$$a_i = (1 - he^{-(t-t_k)/\tau}). \tag{4}$$

This term is meant to represent interventions and help from the community such as having a job or an education opportunity, a counselor, a mentor, a family member to rely on. The duration of this intervention is τ and its amplitude is h. Note that if $\tau \gg t - t_k$ and the intervention did not last long enough the exponent tends to zero, the term is 1 and there is no attenuation. On the other hand, if $\tau \ll t - t_k$ the attenuation is the most effective at $1 - h$. Here we assume, of course, $0 \leq h \leq 1$, so that $h = 0$ represents no resources given to attenuate crime, while $h = 1$ represents the most resources possible. In principle we could have both h, τ be dependent on the crime number, but we choose to keep them constant.

Now, depending on p_i, a_i, s_i a crime is committed or not.

If the crime is not committed the game proceeds to the change strategy phase, if it is committed then an arrest phase playes out. Here, we simply assume that the crime goes unpunished with rate $1-r$ and that the arrest and punishment probability is r. Furthermore, if the crime is arrested and punished, we assume that be default, resoures h, τ will be given, regardless of the criminal's past.

The next step of the game is for the criminal to decide whether or not to change his or her own strategy and keep the criminal life or decide to reform after the punishment phase is over. There are several cases to be considered:

- No crime was committed: In this case we let the criminal turn into a citizen P with probability

$$p_{\text{reform}} = \frac{rP}{\sum_k N_k + P} \tag{5}$$

  where we assume that the criminal will recommit to turning his or life around after having been 'tempted' and not having caved in to crime. We assume this occurs also by observing the virtuous society around him or her, modulated by a factor r, the probability of arrest.

- A crime was committed but the criminal was not caught: In this case the criminal stays a criminal and individual i moves from pool N_k to pool N_{k+1} with probability 1. There is no chance to turn into a law abiding citizen.

- A crime was committed, the criminal was caught and resources was assigned: The criminal decides to turn into a law abiding citizen via the probability

$$p_{\text{reform}} = \frac{1}{2}\left[\frac{hrP}{\sum_k N_k + P} + \frac{\theta k_p}{\theta k_p + k_u + p_0}\right] \tag{6}$$

  Viceversa, he or she will turn into a criminal with probability $1 - p_{\text{reform}}$.

Finally, the game will end when all players are either in pool P or in pool N_R. These two types of individuals do not play actively in the game and are sinks of the population types. They represent a good or a bad society.

To summarize, the parameter space we have is made of the following h, τ, θ, p_0, r. However, r can be set at around 1/4, consistent with police estimates, so we effectively need to consider only the h, τ, θ, p_0 parameters.